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Recent Developments in Axiomatic Studies of Marxian Exploitation Theory

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Recent Developments in Axiomatic Studies of Marxian Exploitation Theory*

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Abstract

This paper reviews and extends recent axiomatic work on Marxian exploitation theory. In the context of production economies with general production technology and heterogeneous agents, a property dubbed **Labour Exploitation** is derived which precisely identifies the domain of definitions of exploitation as the unequal exchange of labour. Then, within this rather large domain, the set of definitions that preserve two fundamental tenets of Marxian theory – the **Profit-Exploitation Correspondence Principle** and the **Class-Exploitation Correspondence Principle** – is characterised. It is shown that, among the main definitions in the literature, only the New Interpretation proposed by Duménil [5] and Foley [8] satisfies both axioms. Finally, it is argued that this result may be due to the fact that, unlike other approaches, the New Interpretation does not crucially depend on the existence of well-defined labour values of individual commodities.

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Keywords: Axiomatic analysis; UE Exploitation; Profit-Exploitation Correspondence Principle; Class-Exploitation Correspondence Principle; Labour Values.

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1 Introduction

The concept of exploitation is central in the Marxist analysis of capitalism. According to Karl Marx [14, 15], although the relation between capitalists and workers is mediated by formally free contracts in labour markets that workers can enter and exit, it is nevertheless marred by injustice and actual unfreedom. Workers are 'free' in the sense of not owning the means necessary to their subsistence, and have no access to the means of production. Hence, they cannot but enter the labour contract and work for capitalists, accepting the prevailing wage which rewards only part of the labour performed. The remaining part is, in Marx's view, uncompensated, and thus expropriated by capitalists whose labour contribution to production is negligible.

This *unequal exchange of labour* (UE) is a general, persistent, defining feature of capitalist economies. It plays a key role in Marxist analyses of the injustice of capitalism which resides in the systematic differences between the amount of labour that individuals contribute to the economy in some relevant sense and the amount of labour they receive in some relevant sense.[1] But the capitalist appropriation of the surplus labour produced by workers lies at the heart of the Marxist explanation of the origin of profits and of the class structure of capitalism. Thus, the concept of exploitation plays a key role in Marxist analyses of structural change, political economy, macro-economic dynamics, and international relations.

The application of the notion of UE exploitation to capitalist economies, however, involves a fundamental difficulty. In feudal societies, exploitative relations are out in the open: serfs spend part of their week working for themselves while the rest is uncompensated work performed for the feudal lord. In capitalism, the division of a worker's labour into the compensated and uncompensated portions is not immediately observable. Theoretical analysis and a proper measure of the amounts labour expended and, especially, received by agents are required in order to uncover the existence of UE exploitation.

In Marx [14, 15], an operational measure of UE was proposed by means of the notion of *labour values*. While the amount of labour workers contribute to the economy is simply their working time, the labour workers receive is defined as the labour embodied in the consumption bundle that they (can) purchase via their earnings. This classic definition was operationalised thanks to Okishio's [23] formal definition of labour values as the solution to the system of equations in a simple model of economies with a single Leontief production technique, and neither joint production nor fixed capital.

If these strong simplifying assumptions are dropped, however, the definition of labour values is unclear, and the standard approach focusing on Leontief multipliers leads to paradoxical results (such as negative labour values). In the

[1] We focus on the so-called "contribution view" of UE exploitation which identifies the distributive injustice of capitalism in a disproportionality between contribution and reward. This notion is related to the *proportional solution* (Roemer and Silvestre [30]); its philosophical foundations can be traced back to Aristotle; and it can be justified in terms of the Kantian categorical imperative (Roemer [29]).

received view, both in the mainstream but also in most heterodox quarters, this represents a fatal flaw of the labour theory of value and, by implication, of Marxian exploitation theory which is widely considered to be a purely metaphysical construct resting on logically flawed foundations.

In response to this view, and in order to provide a general theory of UE exploitation a number of approaches have been proposed either as extensions of Okishio [23] to production economies with choice of techniques and joint production (see, for example, Morishima [21], Roemer [28], Flaschel [6, 7]); or as an alternative to Okishio [23] (see, for example, the so-called 'New Interpretation' proposed by Duménil [5] and Foley [8]).

The wealth of definitions proposed in the literature, and the debate around them, immediately raise two issues. Substantively, different definitions have rather different properties, with very different implications for both theoretical and empirical analysis of actual capitalist economies. For example, while the New Interpretation can be directly taken to the data based on observable magnitudes (Mohun [16]), the approaches proposed by Morishima [21] and Roemer [28] require knowledge of the whole production set, including all of the (counterfactual) techniques that might have been adopted by capitalists. Further, while the definitions proposed by Okishio [23] and Morishima [21] allow one to identify exploitation independent of price information, most of the alternative definitions require prior knowledge of monetary magnitudes. And so on. These differences are so marked that it is unclear whether these definitions can be considered as part of the same broad conceptual family. Or indeed whether it is even possible to talk about a UE approach to exploitation theory.

Methodologically, the haphazard fashion in which the debate has proceeded has not helped. Since the problems of the classic approach based on the labour theory of value came to light, the alternative definitions proposed have been assessed based on their ability to capture at least *some*, core features of Marxian exploitation theory. However, shortly after a definition was proposed, a counterexample was typically found highlighting the limitations of the approach. This led to the proposal of a new definition, until another counterexample was identified. And so on.

For example, Morishima [21] famously proposed an approach that did not require individual labour values to define exploitation but preserved what he considered to be the most important insight of Marxian exploitation theory, namely the relation between exploitation and profits as formalised in the *Fundamental Marxian Theorem* (FMT), which had originally been proved by Okishio [23]. Yet, Petri [24] and Roemer [27] showed that in more general economies with a more general equilibrium concept, the FMT did not hold under Morishima's [21] definition. One gets the impression that the debate on the FMT could go on forever, in an infinite cycle of definition-counterexample-new definition.

Only recently has a new, axiomatic approach to Marxian exploitation theory been proposed which tackles both issues. Substantively, the axiomatic literature precisely identifies the domain of admissible UE definitions and characterises the subsets of definitions that preserve some core insights of Marxian exploitation theory. Methodologically, the axiomatic approach allows one to clarify the the-

oretical foundations of exploitation theory by starting from first principles.

This paper provides a comprehensive, critical survey of this literature which points to some important avenues for further research, and presents some new results, especially concerning the relation between exploitation and the labour theory of value.

The survey focuses on production economies with rather general production technology and heterogeneous agents (section 2). In this context, section 3 reviews axiomatic work that has derived a property – called *Labour Exploitation* – that precisely identifies the contours of the Marxian theory of exploitation – formally, the *domain* of admissible UE definitions.

This domain is rather large indeed – it includes *all* of the main approaches in the literature – and the issue arises as to how to discriminate among alternative definitions. What features should the appropriate theory of exploitation possess? Two key tenets of Marxist theory have been emphasised in the literature. As discussed above, one is the FMT. The other concerns the relation between the class and exploitation structure of capitalist economies: in Marxian theory, exploitative relations do not follow some random pattern, as members of the capitalist class are exploiters while members of the working classes are exploited. This is such an important insight that it has been formalised as the *Class-Exploitation Correspondence Principle* (CECP) by Roemer [28], who proved that it holds in simple Leontief economies.

While FMT and CECP are formally derived in the literature as results that hold under certain definitions within certain economies, their epistemological status is as postulates: a UE definition of exploitation should be such that both insights hold. Sections 4 and 5 survey recent contributions that have formalised both properties as axioms: FMT has been formulated as a new axiom called *Profit-Exploitation Correspondence Principle* (PECP). This axiom requires that for any capitalist economy and any market equilibrium, total profits are positive if and only if any propertyless employee is exploited. CECP has also been formulated as an axiom which requires that for any capitalist economy and any market equilibrium associated with a positive maximal profit rate, every member of the capitalist class is an exploiter while every member of the working class is exploited.

The contributions reviewed in sections 4 and 5 characterise the set of definitions that satisfy PECP and CECP and, perhaps surprisingly, show that the *only* definition in the literature that satisfies both properties is the 'New Interpretation' proposed by Duménil [5] and Foley [8].

Section 6 reviews (and extends) recent axiomatic work on the relation between exploitation and the existence of well-defined labour values. Theorem 1 proves that in the set of economies in which individual labour values of commodities are well defined, all of the main definitions of exploitation satisfy PECP. This seems to suggest that the reason why most of the main definitions fail to preserve PECP in general is that the properties of these definitions crucially hinge on the existence of well-defined labour values.

Section 7 concludes. All formal proofs are in the Appendix.

2 The Framework

The economy comprises n different commodities that can be produced and consumed. Let $\mathbb{R}$ (resp., $\mathbb{R}_+, \mathbb{R}_-$) be the set of (resp., non-negative, non-positive) real numbers. Technology is described by a production possibility set P, which contains activities $\alpha \equiv (-\alpha_l, -\underline{\alpha}, \overline{\alpha}) \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+$, where α_l and $\underline{\alpha}$ describe the amount of *effective* labour and the vector of produced goods used as inputs in order to produce a vector of outputs $\overline{\alpha}$.[2] Denote the vector of net outputs (outputs minus inputs) arising from α as $\widehat{\alpha} \equiv \overline{\alpha} - \underline{\alpha}$. Assume that technology displays constant returns to scale.[3]

A *von Neumann* technology is a production set, $P_{(B,A,L)}$, such that there exists a profile (A, B, L) such that:

$$P_{(A,B,L)} \equiv \left\{\alpha \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+ \mid \exists y \in \mathbb{R}^m_+ : \alpha \leqq (-Ly, -Ay, By)\right\}.$$

where A is an $n \times m$ non-negative matrix whose generic component, $a_{ij} \geqq 0$, is the amount of good i used as input to operate one unit of the j-th production process; B is an $n \times m$ matrix, whose generic component, $b_{ij} \geqq 0$, is the amount of output of good i produced by operating one unit of the j-th production process; and $L > \mathbf{0}$ is a $1 \times m$ row vector of labour input coefficients. The j-th column of A (resp. B) is denoted as A_j (resp. B_j).

A *Leontief* technology, $P_{(A,L)}$, is special case of a von Neumann technology, $P_{(B,A,L)}$, such that $m = n$ and $B = I$, where I is the $n \times n$ identity matrix:

$$P_{(A,L)} \equiv \left\{\alpha \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+ \mid \exists y \in \mathbb{R}^n_+ \ : \alpha \leqq (-Ly, -Ay, y)\right\},$$

and A is assumed to be productive and indecomposable.[4]

An economy comprises a set of N agents, $\mathcal{N} = \{1, .., N\}$, with generic element $\nu \in \mathcal{N}$. For each $\nu \in \mathcal{N}$, let $s^\nu > 0$ be ν's skill level: if ν spends an amount of *time*, λ^ν, in production, then $\Lambda^\nu = s^\nu \lambda^\nu$ is the *effective* labour performed by ν.[5] Let $\overline{l} > 0$ be the total amount of *time* each agent can spend in productive activities. Then, each ν's *effective* labour endowment is $l^\nu = s^\nu \overline{l}$. Let $\omega^\nu \in \mathbb{R}^n_+$ denote ν's endowment of productive assets (capital goods), and let $\omega = \sum_{\nu \in \mathcal{N}} \omega^\nu$ be the vector of social endowments.

An economy is a list $E = \langle \mathcal{N}, P, \boldsymbol{\omega}, \boldsymbol{l} \rangle$, where $\boldsymbol{\omega} \equiv (\omega^\nu)_{\nu \in \mathcal{N}}$ and $\boldsymbol{l} \equiv (l^\nu)_{\nu \in \mathcal{N}}$. Denote the class of all such economies by $\mathcal{E}$.

[2] Vector inequalities: for all $x, y \in \mathbb{R}^n$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, n)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; and $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, n)$.

[3] Formally, P is a closed, convex cone such that (i) $\mathbf{0} = (0, ..., 0) \in P$; (ii) for any $\alpha \in P$ with $\overline{\alpha} \geq \mathbf{0}$, $\alpha_l > 0$ holds; and (iii) for any $c \in \mathbb{R}^n_+$, there exists $\alpha \in P$ such that $\widehat{\alpha} \geqq c$. Property (ii) states that labour is indispensable to produce some output. Property (iii) says that any non-negative vector of goods can be produced as a net output.

[4] Similar properties can be defined in the more general case of joint production. For a discussion, see section 6 below.

[5] This is equivalent to Marx's (and Ricardo's) reduction of complex to simple labour in an economy with heterogeneous *skills*. For a comprehensive axiomatic analysis of the reduction of complex to simple labour in economies with heterogeneous *labour*, see Yoshihara and Veneziani [50]. For a discussion of how to measure UE exploitation in economies with heterogenous labour, focusing in particular on the relation between exploitation and profit, see Veneziani and Yoshihara [37].

An allocation of productive factors and produced goods in the economy is specified by a vector $\boldsymbol{x} \equiv (\boldsymbol{c}, \boldsymbol{\Lambda}) \in \mathbb{R}_+^{nN+N}$, where $\boldsymbol{c} = (c^\nu)_{\nu \in \mathcal{N}}$ represents an assignment of the n commodities to the N agents and $\boldsymbol{\Lambda} = (\Lambda^\nu)_{\nu \in \mathcal{N}}$ is a profile of (effective) labour supplied by each agent. The allocation $\boldsymbol{x}$ is called *feasible* for the economy E if and only if there exists an activity $\alpha \in P$ satisfying $\underline{\alpha} \leqq \omega$, $\alpha_l = \sum_{\nu \in \mathcal{N}} \Lambda^\nu$, $\widehat{\alpha} = \sum_{\nu \in \mathcal{N}} c^\nu$, and $0 \leqq \Lambda^\nu \leqq l^\nu$ for each agent $\nu \in \mathcal{N}$. In other words, an allocation is feasible if there exists an aggregate activity that can be activated given the aggregate endowments of capital and labour, and whose net output is equal to aggregate consumption for each good.

A feasible allocation $\boldsymbol{x}$ for E is called *balanced* if and only if there exist a vector (p, w, r) of commodity prices $p \in \mathbb{R}_+^n \setminus \{\mathbf{0}\}$, a wage rate $w \in \mathbb{R}_+$, and a profit rate $r \in \mathbb{R}_+$ such that $p\overline{\alpha} = (1+r)\, p\underline{\alpha} + w\alpha_l$ and $p\overline{\alpha}' \leqq (1+r)\, p\underline{\alpha}' + w\alpha_l'$ for any $\alpha' \in P$; and $pc^\nu = w\Lambda^\nu + rp\omega^\nu$ holds for each agent $\nu \in \mathcal{N}$. A balanced allocation can be decentralised via markets in that the aggregate activity $\alpha \in P$ maximises profits and every agent's consumption bundle c^ν is affordable.

A balanced allocation $\boldsymbol{x}$ and its associated price vector (p, w, r) are an *equilibrium* if and only if no agent would (strictly) prefer another balanced allocation at (p, w, r) – that is, no agent would like to deviate from the allocation at the given prices. For each economy $E \in \mathcal{E}$, let $\mathcal{S}_E$ be the set of equilibria in E.

3 UE exploitation: the appropriate domain

A preliminary, foundational issue concerns the very object of analysis. For the concept of exploitation is used in many different contexts in order to denote very different phenomena – ranging from issues of surrogate motherhood to medical testing in developing countries to international relations. Furthermore, there are a plethora of different theoretical approaches which incorporate very different normative intuitions – ranging from neoclassical and libertarian approaches to liberal egaliatarian and Marxist ones.

The starting point of any investigation in exploitation theory should therefore be not the dearth but the exceptional abundance of conceivable approaches. The first question to answer, then, is: what principles characterise the set of definitions that aim to capture *exploitation as the unequal exchange of labour* (rather than, say, the deviation of market prices from the competitive benchmark)? This question is tackled by Yoshihara and Veneziani [49] using the axiomatic method.

A *definition of exploitation* is a rule that specifies in each economy $E \in \mathcal{E}$ and at each equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, a partition of $\mathcal{N}$ into the set of *exploited agents* $\mathcal{N}^{ted}$, the set of *exploiters* $\mathcal{N}^{ter}$, and the set of agents who are *neither exploited nor exploiters* $\mathcal{N}^{ne}$. Different theories of exploitation correspond to different ways of partitioning agents, and there are in principle infinitely many rules to partition $\mathcal{N}$ into the three subsets at each equilibrium and each economy. In addition to the main definitions in the literature on Marxian economics (e.g., Morishima [21], Roemer [28], Duménil [5], Foley [8]), one may formalise, for example, a libertarian approach whereby $\mathcal{N} = \mathcal{N}^{ne}$ at all equilibria of each

economy: according to libertarians, there can be no exploitation at the competitive equilibrium emerging from voluntary transactions.

Which properties characterise the theory of exploitation as the unequal exchange of labour? Axiom 1 captures the most basic essence of UE theory:

Axiom 1 (UE): For any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each agent $\nu \in \mathcal{N}$, there exist two numbers $L^\nu_{\min}, L^\nu_{\max} \in \mathbb{R}$ with $L^\nu_{\min} \leqq L^\nu_{\max}$, such that agent ν is an exploiter if and only if $\Lambda^\nu < L^\nu_{\min}$; and is exploited if and only if $\Lambda^\nu > L^\nu_{\max}$.

By Axiom 1, UE exploitation arises from a discrepancy between the amount of labour contributed and the amount of labour 'received' by agents. In all UE approaches, the former magnitude corresponds to the amount of (effective) labour spent in production. Theoretical differences revolve around the definition of the latter magnitude – which may be in principle an interval, identified by the upper and lower bounds, $L^\nu_{\max}, L^\nu_{\min}$. The following axioms embody some general principles – shared by all of the main approaches – to identify the two bounds of the labour received, or *reference labour amounts*.

How do agents 'receive' labour? In a feudal society, the exploitative relation is immediately visible and agents 'receive' the part of their own labour that they manage to retain. In capitalist economies, exploitation is veiled by the exchange of commodities. Axiom 2 captures this idea: it stipulates that the amount of labour received by agents, $(L^\nu_{\min}, L^\nu_{\max})_{\nu \in \mathcal{N}}$, is determined by some reference commodity bundles, called *exploitation reference bundles* (ERBs).

Axiom 2 (Exploitation Reference Bundles, ERBs): For any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each agent $\nu \in \mathcal{N}$, there exist $c^\nu_{\min}, c^\nu_{\max} \in \mathbb{R}^n_+$ and a function f^ν such that $L^\nu_{\min} = f^\nu(c^\nu_{\min})$ and $L^\nu_{\max} = f^\nu(c^\nu_{\max})$.

If the relation of exploitation occurs under the veil of the exchange of commodities, as Axiom 2 states, then a theory of exploitation should contribute to unveil the underlying injustice as well as the social relations that allow such injustice to occur. This has to do with the determination of the ERBs: in UE theory, the injustice of exploitative relations is captured by ERBs being linked with the agents' incomes, while the social relations conducive to exploitation are captured by a focus on productive conditions.

Formally, for all $\nu \in \mathcal{N}$ and all $\Lambda^\nu \leqq l^\nu$, let $B(\omega^\nu, \Lambda^\nu; p, w, r)$ be the set of consumption bundles that agent ν can purchase at prices (p, w, r), if she supplies Λ^ν units of labour, given her endowment ω^ν.[6] Axiom 3 stipulates that the ERBs be at least affordable via market exchanges at the given equilibrium prices.

Axiom 3 (Economic Feasibility of ERBs): For any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each agent $\nu \in \mathcal{N}$, $c^\nu_{\min}, c^\nu_{\max} \in B(\omega^\nu, \Lambda^\nu; p, w, r)$.

[6] Formally: $B(\omega^\nu, \Lambda^\nu; p, w, r) \equiv \left\{c^\nu \in \mathbb{R}^n_+ \mid pc^\nu = w\Lambda^\nu + rp\omega^\nu\right\}$.

Axiom 4 requires the ERBs to be producible as net outputs given the existing technology.

Axiom 4 (Technological Feasibility of ERBs): For any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each agent $\nu \in \mathcal{N}$, there exist α'^{ν}, $\alpha''^{\nu} \in P$ such that $\widehat{\alpha}'^{\nu} \geqq c^{\nu}_{\min}$ and $\widehat{\alpha}''^{\nu} \geqq c^{\nu}_{\max}$.

While Axioms 3-4 require that the ERBs be linked to agents' incomes and to productive conditions, they do not specify the relationship between the reference labour amounts and the ERBs. Axiom 5 bridges that gap by further emphasising the relevance of social relations embodied in productive conditions. To introduce it, let $\partial P \equiv \{\alpha \in P \mid \nexists \alpha' \in P : \alpha' > \alpha\}$ be the frontier of the production possibility set P. Then:

Axiom 5 (Reference Labour Amounts of ERBs): For any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each agent $\nu \in \mathcal{N}$, $L^{\nu}_{\min} = f^{\nu}(c^{\nu}_{\min})$ and $L^{\nu}_{\max} = f^{\nu}(c^{\nu}_{\max})$ hold if and only if there exist $\alpha^{\nu}_{\min}$, $\alpha^{\nu}_{\max} \in \partial P$ such that $\widehat{\alpha}^{\nu}_{\min} \geqq c^{\nu}_{\min}$ with $\widehat{\alpha}^{\nu}_{\min} \ngtr c^{\nu}_{\min}$, $\widehat{\alpha}^{\nu}_{\max} \geqq c^{\nu}_{\max}$ with $\widehat{\alpha}^{\nu}_{\max} \ngtr c^{\nu}_{\max}$, $L^{\nu}_{\min} = \alpha^{\nu}_{l\,\min}$, and $L^{\nu}_{\max} = \alpha^{\nu}_{l\,\max}$.

In other words, the upper and lower bounds of the labour received by each agent should be the amounts of labour necessary for the production of the ERBs given the current production technology.

Yoshihara and Veneziani [49] show that Axioms 1-5 are equivalent to the following property which therefore defines the domain of UE exploitation theory:[7]

Labour Exploitation (LE): For any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, for each agent $\nu \in \mathcal{N}$, there exist $c^{\nu}_{\min}, c^{\nu}_{\max} \in B(\omega^{\nu}, \Lambda^{\nu}; p, w, r)$ and $\alpha^{\nu}_{\min}$, $\alpha^{\nu}_{\max} \in \partial P$ satisfying $\widehat{\alpha}^{\nu}_{\min} \geqq c^{\nu}_{\min}$ with $\widehat{\alpha}^{\nu}_{\min} \ngtr c^{\nu}_{\min}$, $\widehat{\alpha}^{\nu}_{\max} \geqq c^{\nu}_{\max}$ with $\widehat{\alpha}^{\nu}_{\max} \ngtr c^{\nu}_{\max}$, and $\alpha^{\nu}_{l\,\min} \leqq \alpha^{\nu}_{l\,\max}$ such that:

$$\begin{aligned} \nu &\in \mathcal{N}^{ted} \text{ if and only if } \alpha^{\nu}_{l\,\max} < \Lambda^{\nu}, \\ \nu &\in \mathcal{N}^{ter} \text{ if and only if } \alpha^{\nu}_{l\,\min} > \Lambda^{\nu}. \end{aligned}$$

Axioms 1-5 capture the core tenets of UE theory and so **LE** should be interpreted as a *domain condition*: it identifies the space of admissible UE approaches, that is, the foundational properties that any definition of exploitation should satisfy in order to capture the notion of exploitation as the unequal exchange of labour.

LE requires that for each economy and each equilibrium, the exploitation status of every agent ν be determined by the difference between the labour

[7] The equivalence between **LE** and Axioms 1-5 is stated formally as a Theorem 1 in Yoshihara and Veneziani [49]. However, the proof of this characterisation is straightforward because **LE** simply summarises Axioms 1-5.

that ν 'contributes' to the economy, and the labour she 'receives'.[8] Whereas the former quantity is given by the amount of (effective) labour Λ^ν the agent supplies in equilibrium, the labour received by ν is determined by identifying at most two affordable and technically feasible bundles, $c^\nu_{\min}$, $c^\nu_{\max}$, *and* their labour content which is equal to the labour necessary to produce them as net output, $\alpha^\nu_{l\,\min}$, $\alpha^\nu_{l\,\max}$. The amount of labour that ν receives is the (possibly degenerate) interval $[\alpha^\nu_{l\,\min}, \alpha^\nu_{l\,\max}]$, and so, for any $\nu \in \mathcal{N}$, if Λ^ν is greater (resp., lower) than $\alpha^\nu_{l\,\max}$ (resp., $\alpha^\nu_{l\,\min}$) then ν is regarded as 'giving' more (resp., less) labour than ν 'receives' and therefore a member of $\mathcal{N}^{ted}$ (resp., $\mathcal{N}^{ter}$).

Armed with **LE**, we can analyse whether the main definitions in the literature do capture the foundational properties of UE theory.

First, consider Morishima's [21] celebrated definition. For any commodity bundle $c \in \mathbb{R}^n_+$, let $l.v.\,(c)$ denote the *minimum* amount of (effective) labour necessary to produce c as net output.[9] Morishima [21] focuses on the bundle actually purchased by an agent, c^ν, and defines its labour content as $l.v.\,(c^\nu)$:

Definition 1. [Morishima [21]]: For any economy $E \in \mathcal{E}$ and any balanced allocation $\boldsymbol{x}$ for E, an agent $\nu \in \mathcal{N}$, who supplies Λ^ν and consumes c^ν, is *exploited* if and only if $\Lambda^\nu > l.v.\,(c^\nu)$ and *an exploiter* if and only if $\Lambda^\nu < l.v.\,(c^\nu)$.

Definition 1 identifies the exploitation status of each agent prior to and independent of price information, as in the standard Marxian approach, focusing only on production data. However, as Roemer [28] noted, it identifies exploitation status based on production techniques that may never be used by profit-maximising capitalists, and is therefore independent of the social relations of production.

In order to remedy these shortcomings, Roemer [28] proposed an alternative definition. Given an equilibrium $(p, w, r; \boldsymbol{x})$ in an economy E, for any commodity bundle $c \in \mathbb{R}^n_+$, let $l.v.\,(c; p, w, r)$ be the *minimum* amount of (effective) labour necessary to produce c as net output with a profit-rate-maximising activity at given equilibrium prices.[10] Then:

Definition 2. [Roemer [28]]: For any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, an agent $\nu \in \mathcal{N}$, who supplies Λ^ν and consumes c^ν, is *exploited* if and only if $\Lambda^\nu > l.v.\,(c^\nu; p, w, r)$ and *an exploiter* if and only if $\Lambda^\nu < l.v.\,(c^\nu; p, w, r)$.

[8] To be precise, exploitation involves the unequal exchange of *abstract, homogeneous and socialised quantities of labour*. By construction, and given our assumptions on agents and technology, as well as the definition of equilibrium, the amounts of labour in Axioms 1-5 and **LE** are indeed abstract, homogeneous, and social (in that they emerge from the working of the capitalist economy). Therefore, in what follows we shall simply refer to 'labour' for brevity. We are grateful to an anonymous referee for raising this issue.

[9] Formally, $l.v.\,(c) \equiv \min\{\alpha_l \mid \alpha \in P, \widehat{\alpha} \geqq c\}$. Under mild assumptions on technology, $l.v.\,(c)$ is unique, well-defined and positive whenever $c \neq \mathbf{0}$.

[10] Formally, let $P(p, w, r) \equiv \left\{\alpha \in P \mid \frac{p\widehat{\alpha} - rp\underline{\alpha} - w\alpha_l}{p\underline{\alpha}} = \max_{\alpha' \in P} \frac{p\widehat{\alpha}' - rp\underline{\alpha}' - w\alpha'_l}{p\underline{\alpha}'}\right\}$ be the set of activities that maximise the rate of return on capital at (p, w, r). Then, $l.v.\,(c; p, w, r) \equiv \min\{\alpha_l \mid \alpha \in P(p, w, r), \widehat{\alpha} \geqq c\}$.

In Definition 2, exploitation status cannot be determined prior to and independent of price information as in classic Marxian exploitation theory. However, by requiring that the labour content of the bundle purchased by each agent be determined focusing on profit-rate-maximising activities, Definition 2 identifies exploitation status based on production techniques that can actually be used under capitalistic relations of production.

Finally, consider a definition of exploitation based on the "New Interpretation" proposed by Duménil [5] and Foley [8]. Given an equilibrium $(p, w, r; \boldsymbol{x})$ in an economy E, let $\alpha^{p,w,r} \in P$ be the *aggregate (profit-rate-maximising) production activity* at that equilibrium. The vector of net outputs corresponding to $\alpha^{p,w,r}$ is denoted by $\widehat{\alpha}^{p,w,r}$. Then, for any $c \in \mathbb{R}^n_+$ with $pc \leqq p\widehat{\alpha}^{p,w,r}$, let $\tau^c \in [0,1]$ be defined by $\tau^c \equiv \frac{pc}{p\widehat{\alpha}^{p,w,r}}$. Then:

Definition 3. [Duménil [5]; Foley [8]]: For any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, an agent $\nu \in \mathcal{N}$, who supplies Λ^ν and consumes c^ν, is *exploited* if and only if $\Lambda^\nu > \tau^{c^\nu} \alpha_l^{p,w,r}$ and *an exploiter* if and only if $\Lambda^\nu < \tau^{c^\nu} \alpha_l^{p,w,r}$.

Definition 3 incorporates the intuition that the notion of exploitation should be related to the production and distribution of national income and the distribution of social labour expended in actual production activities. For in Definition 3, the labour content of aggregate net output, $\widehat{\alpha}^{p,w,r}$, is equal to total social labour, $\alpha_l^{p,w,r}$, and for any bundle c whose monetary value does not exceed national income $p\widehat{\alpha}^{p,w,r}$, the labour contained in c is equal to the fraction τ^c of social labour necessary to produce a fraction of aggregate net output, $\tau^c \widehat{\alpha}^{p,w,r}$. Here, social relations of production play a more central role than in Definition 2, because the definition of labour content requires a prior knowledge of social production $\alpha^{p,w,r}$ and labour content is explicitly linked to the redistribution of total social labour.

The next result shows that all three of the definitions presented satisfy the domain condition (for a proof see Appendix A):

Proposition 1 [Yoshihara and Veneziani [49]]. *Definitions 1-3 satisfy **LE**.*

It is natural that all three definitions satisfy **LE** because **LE** is a domain axiom that is not meant to single out a unique approach. Rather, it identifies the boundaries of an entire field and the fact that all of the main UE approaches in the literature satisfy **LE** is indirect proof that it is indeed an appropriate domain condition.[11] In what follows, if a definition of exploitation satisfies **LE**, we shall call it a *UE definition*.

[11] For a proof that the alternative definition proposed by Roemer [28] in which agents' exploitation status is independent of their consumption choices satisfies **LE**, see Appendix B. Yoshihara and Veneziani [49] argue that **LE** is also satisfied by: the input-output approach proposed by Flaschel [6, 7]; and the variant of Definition 2 that focuses on any activities in $P(p, w, r)$ such that $\widehat{\alpha} \geqq c$, and not only labour minimizing ones (Roemer [28], pp.164-168).

Given that there exist many UE definitions, including Definitions 1-3, the natural question is how to select among them. Which UE definition, for example, captures the core insights of Marx's theory? This question is tackled, axiomatically, in the next sections.

4 Exploitation and profits: the PECP

A core tenet of Marxian exploitation theory concerns the central relevance of the appropriation of (aggregate) surplus value created in the capitalist production process in the generation of (aggregate) positive profits at a competitive equilibrium. Alternative UE definitions have indeed been evaluated based on whether they capture this insight, as formalised in the *Fundamental Marxian Theorem* (FMT) originally proved by Okishio [23]. FMT shows that the capitalist economy is profitable if and only if the (average) rate of exploitation of the working class by the capitalist class is positive.

While FMT is proved as a *result*, its epistemological status is as an axiom. Veneziani and Yoshihara [36, 37] make this explicit and formalise the intuition of FMT in the **Profit-Exploitation Correspondence Principle (PECP)**.

Given an economy $E = \langle \mathcal{N}, P, \boldsymbol{\omega}, \boldsymbol{l} \rangle \in \mathcal{E}$, an agent $\nu \in \mathcal{N}$ is *propertyless* if and only if $\omega^\nu = \mathbf{0}$. Moreover, given an equilibrium $(p, w, r; \boldsymbol{x})$ in the economy E, a propertyless agent $\nu \in \mathcal{N}$ is *employed* if and only if $\Lambda^\nu > 0$. Then, following Veneziani and Yoshihara [36, 37], **PECP** is given as follows:

Profit-Exploitation Correspondence Principle (PECP): For any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, it follows that:

$$r > 0 \Leftrightarrow \text{every propertyless employed agent is exploited,}$$

in terms of a given definition of exploitation.

The intuition behind **PECP** is the same as that of FMT. If a definition of UE exploitation is appropriate, it should identify the existence of a mechanism which mediates the unequal exchange of labour, namely the transfer of (a part of) the fruits of the productive efforts of the exploited to the exploiter.

Two features of **PECP** are worth stressing. First, the correspondence between profits and exploitation is required for a large class of economies which potentially include fixed capital goods, joint production, technical change, and so on.

Second, however, **PECP** *per se* is not a very demanding property. For one, it does not rule out the possibility that some propertyless employees are exploited in an equilibrium with zero profit.[12] This implies that *in the context of our model*, in which the exploitation of each agent is defined, **PECP** is logically weaker than the FMT, as the latter implies that no propertyless employee is exploited in any equilibrium with zero profit. In contrast, while the FMT implies that the rate of exploitation for the whole working class is positive in any

[12] This is no longer true if the definition of exploitation also satisfies **LE**.

equilibrium with positive total profits, **PECP** requests that every propertyless worker is exploited, which is a stronger claim than that of the FMT.

Veneziani and Yoshihara [36, 37] study the necessary and sufficient condition for **PECP**, as stated in the following proposition:

Proposition 2 [Veneziani and Yoshihara [36, 37]]: For any UE definition, the following two statements are equivalent for any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$:
(1) the **PECP** holds;
(2) if $r > 0$, then for any propertyless employed agent $\nu \in \mathcal{N}$, there exists a production activity $\alpha^{\nu}_{\pi} \in P$ with $\alpha^{\nu}_{\pi l} = \Lambda^{\nu}$ such that $\widehat{\alpha}^{\nu}_{\pi} \in \mathbb{R}^{n}_{+}$, $p\widehat{\alpha}^{\nu}_{\pi} > w\Lambda^{\nu}$, and $(\alpha^{\nu}_{l\pi}, \underline{\alpha}^{\nu}_{\pi}, \overline{\alpha}^{\nu}_{\pi}) \geqq \eta^{\nu} (\alpha^{\nu}_{l\,\max}, \underline{\alpha}^{\nu}_{\max}, \overline{\alpha}^{\nu}_{\max})$ hold for some $\eta^{\nu} > 1$.

Proposition 2 identifies a necessary and sufficient condition for a UE definition to preserve **PECP**. Condition (2) states that if the equilibrium profit rate is positive, then for each propertyless employee $\nu \in \mathcal{N}$, there exists a suitable production point, $\alpha^{\nu}_{\pi} \in P$, which can be used to check whether agent ν is exploited by comparing it with activity $\alpha^{\nu}_{\max}$. (Recall that by **LE**, the labour input $\alpha^{\nu}_{l\,\max}$ associated with $\alpha^{\nu}_{\max}$ represents the upper bound of the labour "received" by agent ν in a given UE definition.) How is $\alpha^{\nu}_{\pi} \in P$ determined? It is defined as the proportional expansion of $\alpha^{\nu}_{\max}$ up to the point where $\alpha^{\nu}_{\pi l} = \Lambda^{\nu}$, which produces a non-negative net output ($\widehat{\alpha}^{\nu}_{\pi} \geqq \mathbf{0}$) that agent ν cannot afford at equilibrium prices ($p\widehat{\alpha}^{\nu}_{\pi} > w\Lambda^{\nu}$). If $\Lambda^{\nu} = \alpha^{\nu}_{\pi l} > \alpha^{\nu}_{l\,\max}$, then agent ν is exploited.

Proposition 2 provides a demarcation line by which one can test which of infinitely many potential definitions preserves the essential relation of exploitation and profits in capitalist economies: if a UE definition does not meet condition (2), then it will not satisfy **PECP**, raising some doubts of its adequacy.

Armed with Proposition 2, it is possible to show that, among Definitions 1-3, only Definition 3 satisfies **PECP** (for a proof see Appendix C):

Proposition 3 [Yoshihara and Veneziani [49]]. *Among Definitions 1-3, only Definition 3 satisfies **PECP**.*

5 The Class-Exploitation Correspondence Principle

A second, central claim of Marxian exploitation theory concerns the relation between an agent's position in the class hierarchy and exploitation status. Given capitalistic social relations, members of the capitalist class extract surplus from the productive contributions of members of the proletariat.

This intuition has been originally formalised and proved as a result – dubbed the **Class-Exploitation Correspondence Principle (CECP)** – by Roemer [28]. In the context of simple Leontief economies, he proved that in equilibrium,

class membership and exploitation status emerge endogenously: the wealthy can rationally choose to belong to the capitalist class and become exploiters, while the poor have no other option than being in the working class and are exploited.

We acknowledge the central epistemological status of the CECP in exploitation theory and state it formally as an axiom, according to which in any capitalist economy and at any equilibrium with a positive profit rate, every member of the capitalist class is an exploiter while every member of the working class is exploited.

Let us introduce some additional information about the actions of agents at an equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$. Let $(\alpha^\nu, \beta^\nu, \gamma^\nu) \in P \times P \times [0, l^\nu]$ denote a profile of actions taken by agent ν in equilibrium, where $\alpha^\nu \in P$ is an activity operated by ν using their own assets $p\omega^\nu$ and employing their own labour; $\beta^\nu \in P$ is an activity operated by agent ν using their assets $p\omega^\nu$ to hire others; and $\gamma^\nu \in [0, l^\nu]$ is the amount of effective labour that ν sells on the market. By the definition of equilibrium, $\sum_{\nu\in\mathcal{N}} \alpha^\nu + \sum_{\nu\in\mathcal{N}} \beta^\nu = \alpha^{p,w,r}$ and since $\alpha_l^\nu + \gamma^\nu = \Lambda^\nu$ for all $\nu \in \mathcal{N}$, it follows that $\sum_{\nu\in\mathcal{N}} \beta_l^\nu = \sum_{\nu\in\mathcal{N}} \gamma^\nu$ in equilibrium.

In this section, for simplicity and without loss of generality, we focus on economies $E = \langle \mathcal{N}, P, \boldsymbol{\omega}, \boldsymbol{l} \rangle \in \mathcal{E}$ such that all agents have the same level of skill, so that $s^\nu = 1$ for each $\nu \in \mathcal{N}$. Therefore, $\Lambda^\nu = \lambda^\nu$ holds for each $\nu \in \mathcal{N}$. Moreover, all agents are assumed to be indifferent to the increase in leisure. In this case, for any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, $\Lambda^\nu = \bar{l}$ holds for any $\nu \in \mathcal{N}$. Without loss of generality, let $\bar{l} = 1$. Then, $pc^\nu = rp\omega^\nu + w$ holds for any $\nu \in \mathcal{N}$ in equilibrium.

Following Roemer [28], in order to describe the class structure of a capitalist economy, let (a_1, a_2, a_3) be such that $a_i \in \{+, \mathbf{0}\}$, $i = 1, 2$, and $a_3 \in \{+, 0\}$, where "+" represents a non-negative and non-zero vector or scalar. Then:

Definition 4 [Roemer ([28]; Chapter 5)]: For any economy $E \in \mathcal{E}$ and any equilibrium $\left(p, w, r; \boldsymbol{x}; (\alpha^\nu, \beta^\nu, \gamma^\nu)_{\nu\in\mathcal{N}}\right) \in \mathcal{S}_E$, agent ν belongs to: (i) $C^H \subseteq \mathcal{N}$ if and only if $(\alpha^\nu, \beta^\nu, \gamma^\nu)$ has the form $(+, +, 0)$; (ii) $C^{PB} \subseteq \mathcal{N}$ if and only if $(\alpha^\nu, \beta^\nu, \gamma^\nu)$ has the form $(+, \mathbf{0}, 0)$; (iii) $C^S \subseteq \mathcal{N}$ if and only if $(\alpha^\nu, \beta^\nu, \gamma^\nu)$ has the form $(+, \mathbf{0}, +)$; (iv) $C^P \subseteq \mathcal{N}$ if and only if $(\alpha^\nu, \beta^\nu, \gamma^\nu)$ has the form $(\mathbf{0}, \mathbf{0}, +)$.

C^H is the capitalist class: in equilibrium, every member of C^H optimally chooses to invest their own capital and hire workers; C^{PB} is the middle class, whose members are self-employed producers who use their own capital; C^S is the class of part-time workers: they sell their labour power but also act as self-employed producers using their own capital; and C^P is the working class, the set of agents who earn their living working for others.

Roemer [28] characterises the class structure of the capitalist society based on the unequal ownership structure of capital assets. Consider an equilibrium $\left(p, w, r; \boldsymbol{x}; (\alpha^\nu, \beta^\nu, \gamma^\nu)_{\nu\in\mathcal{N}}\right) \in \mathcal{S}_E$. Recall that $P(p, w, r)$ is the set of production activities which achieve the maximal profit rate r at the equilibrium prices.

Let $\alpha^{\max} = (-\alpha_l^{\max}, -\underline{\alpha}^{\max}, \overline{\alpha}^{\max})$ and $\alpha^{\min} = \left(-\alpha_l^{\min}, -\underline{\alpha}^{\min}, \overline{\alpha}^{\min}\right)$ be the solutions, respectively, of $\max_{\alpha \in P(p,w,r); \alpha_l = 1} p\underline{\alpha}$ and $\min_{\alpha \in P(p,w,r);\ \alpha_l = 1} p\underline{\alpha}$: $\alpha^{\max}$ (resp., $\alpha^{\min}$) can be interpreted as the *most capital-intensive* (resp., *most labour intensive*) activity among those that maximise the profit rate and use exactly one unit of labour.

Lemma 1 establishes a correspondence between the class structure and the unequal distribution of privately owned capital assets:

Lemma 1 (Wealth-Class Correspondence) [Roemer ([28]; Chapter 5)]: For any economy $E \in \mathcal{E}$ and any equilibrium $\left(p, w, r; \boldsymbol{x}; (\alpha^\nu, \beta^\nu, \gamma^\nu)_{\nu \in \mathcal{N}}\right) \in \mathcal{S}_E$, for any $\nu \in \mathcal{N}$:
(i) $\nu \in C^H \Leftrightarrow p\underline{\alpha}^{\max} < p\omega^\nu$;
(ii) $\nu \in C^{PB} \Leftrightarrow p\underline{\alpha}^{\min} \leqq p\omega^\nu \leqq p\underline{\alpha}^{\max}$;
(iii) $\nu \in C^S \Leftrightarrow 0 < p\omega^\nu < p\underline{\alpha}^{\min}$;
(iv) $\nu \in C^P \Leftrightarrow p\omega^\nu = 0$.

In other words, members of the capitalist class C^H are the wealthiest. By condition (i), their capital holdings are so large that more than one unit of labour is necessary to activate their assets in production and so they must hire others in order to optimise. A similar argument holds for the agents in the other classes lower down in the wealth hierarchy.

Following Yoshihara [42] and Veneziani and Yoshihara [38], the **Class-Exploitation Correspondence Principle** is given as follows:

Class-Exploitation Correspondence Principle (CECP): For any economy $E \in \mathcal{E}$ and any equilibrium $\left(p, w, r; \boldsymbol{x}; (\alpha^\nu, \beta^\nu, \gamma^\nu)_{\nu \in \mathcal{N}}\right) \in \mathcal{S}_E$ with $r > 0$, for any $\nu \in \mathcal{N}$:

$$\nu \in C^H \Rightarrow \nu \in \mathcal{N}^{ter};$$
$$\nu \in C^S \cup C^P \Rightarrow \nu \in \mathcal{N}^{ted}.$$

in terms of a given definition of exploitation.

In other words, for any economy and any equilibrium with a positive profit rate, every capitalist is an exploiter, while every agent in the working classes is exploited.

Yoshihara [42] and Veneziani and Yoshihara [38] derive a necessary and sufficient condition for **CECP** to hold. (For a formal proof of Proposition 4, see Appendix D.)

Proposition 4 [Yoshihara [42]; Veneziani and Yoshihara [38]]: For any UE definition, the following two statements are equivalent:
(1) **CECP** holds under this definition of exploitation;
(2) For any economy $E = \langle \mathcal{N}, P, \boldsymbol{\omega}, \mathbf{1} \rangle \in \mathcal{E}$ and any equilibrium $\left(p, w, r; \boldsymbol{x}; (\alpha^\nu, \beta^\nu, \gamma^\nu)_{\nu \in \mathcal{N}}\right) \in \mathcal{S}_E$, there exist at most two bundles $\overline{c}_\pi, \underline{c}_\pi \in \mathbb{R}^n_+$ and their corresponding activity vectors $\alpha^{\overline{c}_\pi}, \alpha^{\underline{c}_\pi} \in \partial P$ such that $\widehat{\alpha}^{\overline{c}_\pi} \geqq \overline{c}_\pi$, $\widehat{\alpha}^{\underline{c}_\pi} \geqq \underline{c}_\pi$, $\alpha_l^{\overline{c}_\pi} = 1 = \alpha_l^{\underline{c}_\pi}$, and

satisfying: $rp\underline{\alpha}^{\max} + w \geqq p\overline{c}_\pi \geqq p\underline{c}_\pi \geqq rp\underline{\alpha}^{\min} + w$. Moreover, for any $\nu \in \mathcal{N}$:

$$\begin{aligned} \nu &\in \mathcal{N}^{ter} \Leftarrow pc^\nu_{\min} > p\overline{c}_\pi; \\ \nu &\in \mathcal{N}^{ted} \Leftarrow pc^\nu_{\max} < p\underline{c}_\pi, \end{aligned}$$

where $c^\nu_{\min}$ and $c^\nu_{\max}$ are ERBs specified by this definition of exploitation.

Proposition 4 identifies a necessary and sufficient condition for a UE definition to satisy **CECP**. Given an equilibrium in which all agents supply one unit of labor, condition (2) requires that the maximum and minimum reference income levels of 'exploitation-neutral' agents (that is, agents who are *neither exploited nor exploiters*) can be identified. According to a given UE definition of exploitation, any agent whose income earned at the equilibrium is higher (*resp.* lower) than the maximum (*resp.* minimum) reference level is deemed an exploiter (*resp.* exploited). Condition (2) then specifies how to determine these reference income levels. First, at most two commodity bundles $\overline{c}_\pi, \underline{c}_\pi \in \mathbb{R}^n_+$ must be selected such that the expenditures, $p\overline{c}_\pi$, $p\underline{c}_\pi$, of these bundles at the equilibrium represent the maximum and minimum reference income levels of exploitation-neutral agents. Second, these bundles must be produced, respectively, as net outputs via the present production technology using one unit of labour (that is, $\alpha^{\overline{c}_\pi}, \alpha^{\underline{c}_\pi} \in \partial P$ such that $\widehat{\alpha}^{\overline{c}_\pi} \geqq \overline{c}_\pi$, $\widehat{\alpha}^{\underline{c}_\pi} \geqq \underline{c}_\pi$, and $\alpha_l^{\overline{c}_\pi} = 1 = \alpha_l^{\underline{c}_\pi}$). Furthermore, to satisfy **CECP**, condition (2) requires that these reference income levels, $p\overline{c}_\pi$, $p\underline{c}_\pi$, be bounded by the maximum and minimum income of middle-class agents.

Note that $rp\underline{\alpha}^{\max} + w$ (resp., $rp\underline{\alpha}^{\min} + w$) represents the maximal (resp., minimal) revenue available for the middle class at the equilibrium prices (p, w, r). By Lemma 1, every agent in C^H earns more than $rp\underline{\alpha}^{\max} + w$ and every agent in $C^S \cup C^P$ earns less than $rp\underline{\alpha}^{\min} + w$. Given this, for each agent ν in C^H, $pc^\nu_{\min} = pc^\nu_{\max} = rp\omega^\nu + w > rp\underline{\alpha}^{\max} + w \geqq p\overline{c}_\pi$ holds. Then, as condition (2) implies that an agent having $pc^\nu_{\min} > p\overline{c}_\pi$ is an exploiter, it follows that the agent ν in C^H is an exploiter. A similar argument holds for agents in $C^S \cup C^P$. Thus, condition (2) ensures that **CECP** holds for the given UE definition.

Based on this characterisation of **CECP**, Yoshihara [42] and Veneziani and Yoshihara [38] identify which of the main definitions proposed in the literature satisfy condition (2) and therefore **CECP** (see Appendix E for a proof).

Proposition 5 [Yoshihara [42] and Veneziani and Yoshihara [38]]: Among Definitions 1-3, only Definition 3 satisfies **CECP**.

Proposition 5 confirms the main conclusions of section 4: in general economies with a convex production set, none of the main definitions in the literature satisfy either **PECP** or **CECP**, except for Definition 3, which extends the New Interpretation (Duménil [5]; Foley [8]).

Why do Definitions 1-2, and most of the main approaches in the literature, fail to capture the key tenets of Marxian exploitation theory? In order to answer this question, one key point to note is that Propositions 3 and 5 no longer hold

if we restrict our attention to the class of Leontief economies. In this class, both **PECP** and **CECP** hold *under any UE definition*, as shown by Veneziani and Yoshihara [36] and Yoshihara [42], respectively.

This would seem to suggest that the problems of Definitions 1-2 (and most other approaches) arise from their sensitivity to the complexity of the technology, whose theoretical and normative relevance in Marxian exploitation theory is dubious at best. For it is unclear why the validity of **PECP** and **CECP** should hang on such morally arbitrary factors as, say, the existence of fixed capital or the possibility of joint production.

However, another interpretation may be advanced, according to which the problems of Definitions 1-2 do not arise from the complexity of the production technology, but from the difficulty of providing a coherent definition of individual labour values. This is the topic of the next section.

6 Exploitation and Labour Values

As noted in the Introduction, in the classical Marxian approach, individual *labour values* played a key role in the formal definition of exploitation. In the more recent literature, including Definitions 1-3, labour values played a much less central role, if at all. Ever since Steedman [32] showed that negative labour values and negative surplus value could emerge in economies with joint production, and Morishima [21] defined exploitation without relying on the labour values of individual goods, all of the main UE approaches have been independent of commodities' labour values.

However, recent work by Yoshihara [44, 45] suggests that the notion of individual labour values may have some relevance to the choice of the appropriate UE definition. For even when a more complex production technology is considered, a UE definition satisfies the **PECP** whenever the individual labour values of all goods are well-defined and the amount of labour received is defined by means of individual labour values. In this section, we will review these findings.

For the sake of simplicity, and without loss of generality, assume $n = 2$. Consider a von Neumann production set $P_{(A,B,L)}$: $\left(A^{(ij)}, B^{(ij)}, L^{(ij)}\right) \equiv \left(\left(A_i, A_j\right), \left(B_i, B_j\right), \left(L_i, L_j\right)\right)$ is called (i,j)-*von Neumann technique* if and only if it is a combination of the techniques of the i-th and j-th processes, (A_i, B_i, L_i), (A_j, B_j, L_j), where $i, j = 1, \ldots, m$ with $i \neq j$, such that there exists a bundle $c \in \mathbb{R}^2$ satisfying $\left(B^{(ij)} - A^{(ij)}\right) y \geqq c > \mathbf{0}$ for some activity vector $y \in \mathbb{R}^2_+$. An (i,j)-von Neumann technique is *all-productive* if and only if for any bundle $c \in \mathbb{R}^2_+$, there exists an activity vector $y \in \mathbb{R}^2_+$ such that $\left(B^{(ij)} - A^{(ij)}\right) y = c$ (Kurz and Salvadori [11], p.239). Let $\mathcal{T}_{(A,B,L)} \equiv \left\{ \left(A^{(ij)}, B^{(ij)}, L^{(ij)}\right) \mid i, j = 1, \ldots, m \text{ with } i \neq j \right\}$ be the set of all von Neumann techniques derived from $P_{(A,B,L)}$. The set $\mathcal{T}_{(A,B,L)}$ is all-productive if and only if every technique in the set is all-productive.[13]

[13] Mathematically, all-productiveness is rather stringent. From an economic viewpoint, however, it is quite reasonable. For example, if all of the 'jointly produced outputs' in every von Neumann technique come from the presence of depreciated fixed capital goods at the end of each production period, then such techniques can be all-productive, as argued by Yoshihara

Assuming that $\mathcal{T}_{(A,B,L)}$ is all-productive, Yoshihara [44] defines labour values of individual commodities for a technique $\left(A^{(ij)}, B^{(ij)}, L^{(ij)}\right) \in \mathcal{T}_{(A,B,L)}$ as the solution to the following system of equations:

$$vB^{(ij)} = vA^{(ij)} + L^{(ij)}. \tag{1}$$

The solution to equation (1) is $v^{(ij)} = L^{(ij)}(B^{(ij)} - A^{(ij)})^{-1} > \mathbf{0}$ and it exists for any von Neumann technique in $\mathcal{T}_{(A,B,L)}$ because the latter is all-productive. Therefore, let $\left(A^{(p,w,r)}, B^{(p,w,r)}, L^{(p,w,r)}\right) \in \mathcal{T}_{(A,B,L)}$ be the technique that maximises the profit-rate and is chosen at the equilibrium $(p, w, r; \boldsymbol{x})$: the labour values associated with this technique are given by $v^{(p,w,r)} = L^{(p,w,r)}(B^{(p,w,r)} - A^{(p,w,r)})^{-1} > \mathbf{0}$ and a UE definition can be proposed based on $v^{(p,w,r)}$.

We characterise the set of UE definitions which satisfy **PECP** in the set of economies $E = \langle \mathcal{N}, P_{(A,B,L)}, \boldsymbol{\omega}, \boldsymbol{l} \rangle \in \mathcal{E}$, where the corresponding set of techniques $\mathcal{T}_{(A,B,L)}$ is all-productive. To this aim, we generalise a property originally introduced by Yoshihara [45] in economies with multiple Leontief techniques.[14]

Labour Value Theory of Exploitation (LVE): Consider a UE definition. For any economy $E = \langle \mathcal{N}, P_{(A,B,L)}, \boldsymbol{\omega}, \boldsymbol{l} \rangle \in \mathcal{E}$, where the corresponding $\mathcal{T}_{(A,B,L)}$ is all-productive, and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, $\alpha^{\nu}_{l\max} \leqq v^{(p,w,r)} c^{\nu}_{\max}$ holds for every propertyless employed agent $\nu \in \mathcal{N}$.

LVE requires that the amount of labour received by propertyless agents – the socially necessary labour time for the reproduction of labour power – be measured according to the labour theory of value.

In an economy with a single Leontief technology, $v^{(p,w,r)}$ simply reduces to the standard labour value vector.[15] In economies with multiple Leontief techniques, or various all-productive von Neumann techniques, there is no consensus as to the appropriate definition of labour values (for a thorough discussion, see for example Mohun and Veneziani [17]). In this context, **LVE** identifies an upper bound to the admissible vectors of labour values by means of $v^{(p,w,r)}$. This upper bound is weak enough to allow for a rather broad class of admissible approaches, in that there are infinitely many definitions of labour values satisfying this constraint. Indeed, all of Definitions 1-3 satisfy **LVE**.

Let $\mathcal{E}_{vN} \equiv \left\{ \langle \mathcal{N}, P_{(A,B,L)}, \boldsymbol{\omega}, \boldsymbol{l} \rangle \in \mathcal{E} \mid \mathcal{T}_{(A,B,L)} \text{ is all-productive} \right\}$ be the set of economies with all-productive von Neumann techniques. We can show that **PECP** holds in $\mathcal{E}_{vN}$, for any UE definition which satisfies **LVE** (for a proof, see Appendix F):

[44].

[14] Observe that an all-productive von Neumann technique reduces to a productive Leontief technique if each of its processes produces only one output.

[15] Recent controversies in modern Marxian theory have given rise to alternative proposals regarding the formulation of labour values. These proposals criticise the standard value equation approach to labour values, as seen in the work of Wolf *et al.* [40] and Kliman and McGlone [10]. While a proper treatment of this topic goes well beyond the scope of this paper, we believe that these alternative proposals fail to capture some fundamental tenets of the labour theory of value, as argued by Veneziani [33] and Yoshihara [44]. See also Petri [25] for a comprehensive critique of these alternative approaches.

Theorem 1: Consider a UE definition. Then, **PECP** holds over $\mathcal{E}_{vN}$, whenever this definition satisfies **LVE**.

As Definitions 1-2 satisfy **LVE** on $\mathcal{E}_{vN}$, Theorem 1 implies that the differences between Definition 3 and the others in terms of **PECP** disappears within $\mathcal{E}_{vN}$, even when technology is complex. This may suggest that the real reason why Definitions 1-2 fail to preserve **PECP** is that the properties of these definitions crucially hinge on the existence of well-defined labour values. In contrast, Definition 3 robustly preserves the core tenets of Marxian exploitation theory, independently of whether individual labour values are well-defined or not – a property it inherits from the New Interpretation of Duménil [5] and Foley [8] whose key insights it captures.

Finally, Theorem 1 highlights some important links between the labour values and the measurement of exploitation, and so it would perhaps be interesting to explore the implications of the results presented here for debates on the recent alternative solutions to the so-called "transformation problem" – see, for example, Loranger [12], Montes-Rojas [18], and Montes-Rojas [19]. Though we will not delve deep into this topic as it goes beyond the scope of this paper, two important points should be noted.

First, according to Theorem 1, any definition of exploitation adopted in the literature on the transformation problem can pass the **PECP** test for a valid definition of exploitation. This is because in this literature the standard value equation approach is adopted to formulate labour values when focusing on economies with a single Leontief technology.

Second, from this paper's perspective, the controversy surrounding solutions to the transformation problem has arguably limited relevance since none of the solutions proposed in this literature can be straightforwardly applied to economies with more general technologies and it is unclear how they can be extended. The axiomatic analyses of **PECP** reviewed in this paper may provide a foundation for such solutions and point to possible generalisations that preserve the relationship between profit and exploitation across a broader range of economies.

7 Conclusions

This paper provides a comprehensive survey of the recent developments in the Marxian theory of exploitation, focusing in particular on axiomatic studies. In the context of production economies with general production technology and heterogeneous agents, a property dubbed **Labour Exploitation** is derived which precisely identifies the domain of definitions of exploitation as the unequal exchange of labour. Then, within this rather large domain, the set of definitions that preserve two fundamental tenets of Marxian theory – the **Profit-Exploitation Correspondence Principle** and the **Class-Exploitation Correspondence Principle** – is characterised. It is shown that, among the main definitions in the literature, only the New Interpretation

proposed by Duménil [5] and Foley [8] satisfies both axioms. Finally, it is argued that this result may be due to the fact that, unlike other approaches, the New Interpretation does not crucially depend on the existence of well-defined labour values of individual commodities.

Of course, the axiomatic results we have presented are not the last word on exploitation theory, and we do not mean to suggest that they conclusively support the New Interpretation. First, the literature reviewed in this paper focuses exclusively on static models, and does not consider the dynamics of exploitation. Some preliminary results derived by Veneziani and Yoshihara [38] suggest that the key insights can be extended to the intertemporal context. Nonetheless, more work is necessary in order to explore the dynamics of exploitative relations, and the appropriate definition when the agents' life span is properly considered. This is crucial if one wants to understand the nexus between exploitation, intersectoral competition, structural change, and technological innovations.

Second, the results presented in the paper have relevant implications for the empirical analysis of capitalist economies. For they lend support to the "New Interpretation", which is based on empirically observable magnitudes and provides a natural framework for the empirical analysis of exploitation in actual capitalist economies, including in the global context. While some recent contributions have indeed taken Definition 2 to the data providing an analysis of the distribution of exploitationin the US (Girardi et al [9]) and in international relations (Cogliano et al [3]), much remains to be done.[16]

Finally, the literature reviewed here analyses axiomatically the distributive dimension of exploitative relations. However, exploitation is characterised not only by the unequal exchange of labour, but also by the *asymmetric power structure in the capitalist-worker relationship*, as Wright [41] and Vrousalis [39] have forcefully argued. Power asymmetries are central both normatively and causally in the analysis of the mechanisms that ensure the persistent generation of exploitation in capitalist economies. Such mechanisms should play a central role in determining the distribution between equilibrium profits and equilibrium wage rates in competitive markets — an issue on which the literature reviewed in this paper does not shed much light.

On this issue, it should be noted that competitive mechanisms *alone* cannot ensure the persistence of exploitative relations in capitalist economies. To begin with, consider an intertemporal economy with (almost) infinitely-lived representative agents. In this context, intertemporal competitive equilibria are *generically determinate* and along an equilibrium trajectory, absent time preference, the persistence of exploitation is not guaranteed in the limit, as Veneziani [34, 35] showed.

Consider next an intertemporal economy with finitely-lived multiple agents. In this case, both in a simple Sraffian framework (Sraffa [31]) and in a stan-

[16] It is worth stressing that the robustness and consistency of the New Interpretation as a theoretical framework for empirical analysis of actual capitalist economies are not uncontested. Rieu [26] re-examined the quantitative issues raised by the New Interpretation, discussing, among other things, the problem of double counting and the relationship between the value of labour power and the bundle of wage goods.

dard Walrasian framework (Yoshihara and Kwak [47, 48] and Yoshihara [46]), the steady-state competitive equilibrium can be shown to be *generically indeterminate*. That is, a one-dimensional continuum of steady-state competitive equilibria emerges as the result of perfect competition in the capitalist economy, including a steady-state equilibrium with a zero profit rate. This implies that, without a proper specification of an *out-of-market* mechanism to determine the distribution between wages and profits, market competition *alone* may reach the zero-profit steady-state equilibrium, in which no exploitative relation emerges.

As is well known, classical Marxist literature suggests that the persistence of exploitation can be guaranteed by *capital-using and labour-saving* technical progress which can ensure capital scarcity via the reproduction of an industry reserve army which puts downward pressure on equilibrium wage rates. Such a structural change mechanism works because capitalists have effective decision-making power over their firms' production and investments, including whether to introduce such an innovation, while workers are essentially powerless in this respect. This classical exposition, and the underlying mechanism of structural change, have been recently analysed both computationally (see, e.g., Cogliano et al. [1, 2, 3]) and, mathematically, in a general equilibrium context (see Yoshihara and Veneziani [51]).

However, since the real history of the capitalist economy exhibits a consistent trend of positive profits even under nearly full employment, as in the golden age of capitalism, there must be an out-of-market mechanism guaranteeing the persistence of positive profits (and thus exploitation), which is associated with asymmetric power relations between capitalists and workers. Examining such a mechanism would enable the development of a general theory to predict the most likely distribution point over the wage-profit frontier and provide insight into the political economy of surplus labour extraction in capitalist economies. This is a promising avenue for further research into the power dimension of UE exploitation.

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A Proof of Proposition 1

To see that Definition 1 satisfies **LE** set $c^{\nu}_{\min} = c^{\nu}_{\max} = c^{\nu}$ and

$$\alpha^{\nu}_{\min} = \alpha^{\nu}_{\max} = \arg\min \{\alpha_l \mid \alpha \in P : \widehat{\alpha} \geqq c^{\nu}\}.$$

To see that Definition 2 satisfies **LE**, set $c^{\nu}_{\min} = c^{\nu}_{\max} = c^{\nu}$ and

$$\alpha^{\nu}_{\min} = \alpha^{\nu}_{\max} = \arg\min \{\alpha_l \mid \alpha \in P(p,w,r) : \widehat{\alpha} \geqq c^{\nu}\}.$$

To see that Definition 3 satisfies **LE**, set $c^{\nu}_{\min} = c^{\nu}_{\max} = \tau^{c^{\nu}} \widehat{\alpha}^{p,w,r}$ and

$$\alpha^{\nu}_{\min} = \alpha^{\nu}_{\max} = \tau^{c^{\nu}} \alpha^{p,w,r}.$$

B An alternative definition of exploitation

Roemer [28] proposed an alternative approach in which agents' exploitation status is independent of their preferences over bundles of produced goods, focusing on the *maximum* and the *minimum* amounts of labour embodied in the bundles that they can purchase.

Definition A.1. [Roemer [28]]: For any economy $E \in \mathcal{E}$ and any equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, an agent $\nu \in \mathcal{N}$, who supplies Λ^{ν} and possesses ω^{ν} at that equilibrium, is *exploited* if and only if $\Lambda^{\nu} > \max_{c \in B(\omega^{\nu}, \Lambda^{\nu}; p,w,r)} l.v.\,(c; p, w, r)$ and *an exploiter* if and only if $\Lambda^{\nu} < \min_{c \in B(\omega^{\nu}, \Lambda^{\nu}; p,w,r)} l.v.\,(c; p, w, r)$.

It can be shown that Definition A.1 satisfies **LE** by setting:

$$c^{\nu}_{\min} = \arg \min_{c \in B(\omega^{\nu}, \Lambda^{\nu}; p,w,r)} \min \{\alpha_l \mid \alpha \in P(p,w,r), \text{ with } \widehat{\alpha} \geqq c\},$$

$$c^{\nu}_{\max} = \arg \max_{c \in B(\omega^{\nu}, \Lambda^{\nu}; p,w,r)} \min \{\alpha_l \mid \alpha \in P(p,w,r), \text{ with } \widehat{\alpha} \geqq c\},$$

$$\alpha^{\nu}_{\min} = \arg\min\left\{\alpha_l \mid \alpha \in P(p,w,r),\ \text{with } \widehat{\alpha} \geqq c^{\nu}_{\min}\right\},$$

and

$$\alpha^{\nu}_{\max} = \arg\min\left\{\alpha_l \mid \alpha \in P(p,w,r),\ \text{with } \widehat{\alpha} \geqq c^{\nu}_{\max}\right\}.$$

The arguments in sections 4-6 show that the **CECP** does not hold if Definition A.1 is adopted. Further, while the **PECP** does not hold in general, because Definition A.1 satisfies **LVE** on $\mathcal{E}_{vN}$, **PECP** does hold on this restricted domain of economies.

C Proof of Proposition 3

To see that Definition 3 satisfies **PECP**, we check condition (2) of Proposition 2. Remember that $\alpha^{\nu}_{\max} = \tau^{c^{\nu}}\alpha^{p,w,r}$ holds for Definition 3. Let $\alpha^{\nu}_{\pi} \equiv \frac{\Lambda^{\nu}}{\alpha^{p,w,r}_{l}}\alpha^{p,w,r}$. As $\tau^{c^{\nu}} = \frac{pc^{\nu}}{p\widehat{\alpha}^{p,w,r}}$; $pc^{\nu} = w\Lambda^{\nu}$ for any propertyless employed agent ν in equilibrium; and $p\widehat{\alpha}^{p,w,r} - w\alpha^{p,w,r}_{l} > 0$ holds by $r > 0$, it follows that $\frac{\Lambda^{\nu}}{\alpha^{p,w,r}_{l}} > \frac{pc^{\nu}}{p\widehat{\alpha}^{p,w,r}} = \tau^{c^{\nu}}$ for any propertyless employed agent ν. Therefore, Definition 3 passes condition (2) of Proposition 2.

To see why the other definitions do not pass the test, consider the following example.

Example 1: Let $n = 2$. Assume there are two production processes, and let

$$A \equiv \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1.5 \\ 1 & 0.5 \end{bmatrix},\ B \equiv \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} 2 & 3.5 \\ 2 & 1.5 \end{bmatrix},\ L \equiv (L_1, L_2) = (1,1).$$

As $B - A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $L = (1,1)$, process 2 is superior to process 1: using one unit of labour, it produces 2 units of commodity 1 and one unit of commodity 2 as net outputs, while process 1 only produces one unit of each commodity.

Assume that (i) every agent needs to consume a bundle $c^* = (1,1)'$ in order to supply one unit of labour; (ii) $N = 2$; and (iii) agent 1 is propertyless ($\omega^1 = \mathbf{0}$) while agent 2 is a capitalist who holds and invests $\omega = \omega^2 = (1.5, 0.5)$ and employs the propertyless agent in order to activate a profit maximising production technique.

Let $\alpha^* \equiv (-1, -(1.5, 0.5), (3.5, 1.5))$. Consider (p^*, w^*) with $p^*_1 = 0.5 = p^*_2$ and $w^* = 1$. It is immediate to show that $((p^*, w^*, r^*), \alpha^*)$ constitute an equilibrium, in which $r^* = 0.5$, and the propertyless agent earns one unit of wage revenue and purchases c^* while the capitalist earns 0.5 unit of profit revenue and consumes one unit of commodity 1. In this economy,

$$l.v.(c^*) = 1 = l.v.(c^*; p^*, w^*, r^*) \leqq \max_{c:\ p^*c = w^*} l.v.(c; p^*, w^*, r^*)$$

holds, which implies that the propertyless employed agent is not exploited in terms of Definitions 1-2, though $r^* > 0$. Therefore, Definitions 1-2 do not satisfy **PECP**. ■

D Proof of Proposition 4

In terms of notations, let $P(\alpha_l = \ell) \equiv \{\alpha \in P \mid \alpha_l = \ell\}$ and $\widehat{P}(\alpha_l = \ell) \equiv \{c \in \mathbb{R}^n \mid \exists \alpha \in P : \alpha_l = \ell \ \& \ c \leqq \widehat{\alpha}\}$. Also, let $\partial \widehat{P}(\alpha_l = \ell) \equiv \left\{c \in \widehat{P}(\alpha_l = \ell) \mid \nexists c' \in \widehat{P}(\alpha_l = \ell) : c' > c\right\}$. Then, let $int\ \widehat{P}(\alpha_l = \ell) \equiv \widehat{P}(\alpha_l = \ell) \setminus \partial \widehat{P}(\alpha_l = \ell)$.

(1)⇒(2): Take any $E = \langle \mathcal{N}, P, \boldsymbol{\omega}, \mathbf{1} \rangle \in \mathcal{E}$ and any equilibrium $\left(p, w, r; \boldsymbol{x}; (\alpha^\nu, \beta^\nu, \gamma^\nu)_{\nu \in \mathcal{N}}\right) \in \mathcal{S}_E$. Define

$$\overline{c} \equiv \arg \max_{c \in \partial \widehat{P}(\alpha_l = 1) \cap \mathbb{R}^n_+} pc$$

and

$$\underline{c} \equiv \arg \min_{c \in \partial \widehat{P}(\alpha_l = 1) \cap \mathbb{R}^n_+} pc.$$

Note that if there exists $\alpha^{\overline{c}} \in P(\alpha_l = 1) \cap P(p, w, r)$ such that $\widehat{\alpha}^{\overline{c}} \geqq \overline{c}$, then $p\overline{c} \leqq rp\underline{\alpha}^{\max} + w$ holds. For any $\alpha^{\overline{c}} \in P(\alpha_l = 1)$ with $\widehat{\alpha}^{\overline{c}} \geqq \overline{c}$, if $\alpha^{\overline{c}} \notin P(p, w, r)$ and $p\overline{c} > rp\underline{\alpha}^{\max} + w$, then there exists $c \in \partial \widehat{P}(\alpha_l = 1) \cap \mathbb{R}^n_+$ such that $pc = rp\underline{\alpha}^{\max} + w$ by the convexity of $\widehat{P}(\alpha_l = 1) \cap \mathbb{R}^n_+$. Let $\overline{c}_\pi \in \partial \widehat{P}(\alpha_l = 1) \cap \mathbb{R}^n_+$ be such that $p\overline{c}_\pi = \min\{p\overline{c}, rp\underline{\alpha}^{\max} + w\}$. Likewise, we can find $\underline{c}_\pi \in \partial \widehat{P}(\alpha_l = 1) \cap \mathbb{R}^n_+$ such that $p\underline{c}_\pi = \max\left\{p\underline{c}, rp\underline{\alpha}^{\min} + w\right\}$. Then, by definition, $rp\underline{\alpha}^{\max} + w \geqq p\overline{c}_\pi \geqq p\underline{c}_\pi \geqq rp\underline{\alpha}^{\min} + w$ holds.

Let $H^+(p, \overline{c}_\pi) \equiv \left\{c \in \mathbb{R}^n_+ \mid pc > p\overline{c}_\pi\right\}$ and $H^-(p, \underline{c}_\pi) \equiv \left\{c \in \mathbb{R}^n_+ \mid pc < p\underline{c}_\pi\right\}$.

Consider $\nu \in \mathcal{N}$. Let $pc^\nu_{\min} > p\overline{c}_\pi$. There are two possibilities. First, let $p\overline{c}_\pi = p\overline{c} > rp\underline{\alpha}^{\max} + w$. Note that, by definition of $\overline{c}$, it follows:

$$\left(\widehat{P}(\alpha_l = 1) \cap \mathbb{R}^n_+\right) \cap H^+(p, \overline{c}_\pi) = \varnothing.$$

This implies that, as $pc^\nu_{\min} > p\overline{c}_\pi$, $c^\nu_{\min} \in H^+(p, \overline{c}_\pi) \setminus \widehat{P}(\alpha_l = 1)$ holds, so that $\alpha^\nu_{l\,\min} > 1$, which implies $\nu \in \mathcal{N}^{ter}$. Second, let $p\overline{c}_\pi = rp\underline{\alpha}^{\max} + w \leqq p\overline{c}$. Then, $pc^\nu_{\min} > p\overline{c}_\pi$ implies that $\nu \in C^H$ by Lemma 1 and **LE**, which implies by **CECP** that $\nu \in \mathcal{N}^{ter}$.

Let $pc^\nu_{\max} < p\underline{c}_\pi$. Likewise, there are two possibilities. First, let $p\underline{c}_\pi = p\underline{c} > rp\underline{\alpha}^{\min} + w$. Note that, by definition of $\underline{c}$, it follows:

$$\left(int\ \widehat{P}(\alpha_l = 1) \cap \mathbb{R}^n_+\right) \supseteq H^-(p, \underline{c}_\pi).$$

Then, as $pc^\nu_{\max} < p\underline{c}_\pi$, $c^\nu_{\max} \in H^-(p, \underline{c}_\pi) \subseteq int\ \widehat{P}(\alpha_l = 1) \cap \mathbb{R}^n_+$ holds, so that $\alpha^\nu_{l\,\max} < 1$, which implies $\nu \in \mathcal{N}^{ted}$. Second, let $p\underline{c}_\pi = rp\underline{\alpha}^{\min} + w \geqq p\underline{c}$. Then, $pc^\nu_{\max} < p\underline{c}_\pi$ implies that $\nu \in C^S \cup C^P$ by Lemma 1 and **LE**, which implies by **CECP** that $\nu \in \mathcal{N}^{ted}$.

In summary, condition (2) holds true.

(2)⇒(1): Take any $E = \langle \mathcal{N}, P, \boldsymbol{\omega}, \mathbf{1} \rangle \in \mathcal{E}$ and any equilibrium $\left(p, w, r; \boldsymbol{x}; (\alpha^\nu, \beta^\nu, \gamma^\nu)_{\nu \in \mathcal{N}}\right) \in \mathcal{S}_E$. Then, consider $\nu \in \mathcal{N}$. Let $\nu \in C^H$. Then, by **LE** and Lemma 1, $pc^\nu_{\min} > rp\underline{\alpha}^{\max} + w \geqq p\overline{c}_\pi$. Thus, by condition (2), $\nu \in \mathcal{N}^{ter}$. Let $\nu \in C^S \cup C^P$. Then, by **LE** and Lemma 1, $pc^\nu_{\max} < rp\underline{\alpha}^{\min} + w \leqq p\underline{c}_\pi$. Thus, by condition (2), $\nu \in \mathcal{N}^{ted}$. Therefore, **CECP** holds for the given UE definition. ■

E Proof of Proposition 5

To see that Definitions 1-2 do not satisfy **CECP**, it is sufficient to review the argument in Example 1, where agent 1 is propertyless and employed but is not exploited in terms of Definitions 1-2. As agent 1 belongs to C^P, this implies that **CECP** is violated.

To see that Definition 3 satisfies **CECP** let $\overline{c}_\pi = \frac{\widehat{\alpha}^{p,w,r}}{N} = \underline{c}_\pi$. It is then immediate to verify that condition (2) of Proposition 4 is satisfied. ■

F Proof of Theorem 1

The argument closely follows the logic of the proof of Theorem 2 in Yoshihara [45].

Proof. Take any UE definition satisfying **LVE**. Take any economy $E = \langle \mathcal{N}, P_{(A,B,L)}, \boldsymbol{\omega}, \boldsymbol{l} \rangle \in \mathcal{E}_{vN}$ and let $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$ be an equilibrium. Then, for each propertyless employed agent $\nu \in \mathcal{W}$, there exist $c^\nu_{\max} \in \mathbb{R}^2_+$ and $\alpha^\nu_{\max} \in \partial P_{(A,B,L)}$ satisfying $\widehat{\alpha}^\nu_{\max} \geqq c^\nu_{\max}$ with $\widehat{\alpha}^\nu_{\max} \not> c^\nu_{\max}$, such that $pc^\nu_{\max} = w\Lambda^\nu$ holds, and $\nu \in \mathcal{N}^{ted} \Leftrightarrow \alpha^\nu_{l\max} < \Lambda^\nu$. Then, there exist $\left(A^{(\nu)}, B^{(\nu)}, L^{(\nu)}\right) \in \mathcal{T}_{(A,B,L)}$ and $\boldsymbol{y} \geq \mathbf{0}$ such that $\alpha^\nu_{\max} = \left(-L^{(\nu)}\boldsymbol{y}, -A^{(\nu)}\boldsymbol{y}, B^{(\nu)}\boldsymbol{y}\right)$.

Then, at the equilibrium $(p, w, r; \boldsymbol{x}) \in \mathcal{S}_E$, it follows that:

$$\begin{aligned} pB^{(p,w,r)} &= (1+r)\,pA^{(p,w,r)} + wL^{(p,w,r)} \\ pB^{(ij)} &\leqq (1+r)\,pA^{(ij)} + wL^{(ij)} \text{ for any } \left(A^{(ij)}, B^{(ij)}, L^{(ij)}\right) \in \mathcal{T}_{(A,B,L)}, \end{aligned}$$

where the former equations are equivalent to:

$$p = rpA^{(p,w,r)}\left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} + wv^{(p,w,r)} > \mathbf{0}.$$

Then, by multiplying $c^\nu_{\max}$ from the right in both sides of the above equations, we have:

$$pc^\nu_{\max} = rpA^{(p,w,r)}\left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} c^\nu_{\max} + wv^{(p,w,r)}c^\nu_{\max}.$$

As $pc^\nu_{\max} = w\Lambda^\nu$ holds, the above equations can be rewritten by:

$$\begin{aligned} w\Lambda^\nu &= rpA^{(p,w,r)}\left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} c^\nu_{\max} + wv^{(p,w,r)}c^\nu_{\max} \\ &\geqq rpA^{(p,w,r)}\left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} c^\nu_{\max} + w\alpha^\nu_{l\max}, \end{aligned}$$

where the last inequalities follow from **LVE**.

Note that $\left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} \geq \mathbf{0}$ by all-productiveness of $\left(A^{(p,w,r)}, B^{(p,w,r)}, L^{(p,w,r)}\right)$ and $pA^{(p,w,r)} > \mathbf{0}$ by $p > \mathbf{0}$, and thus

$$rpA^{(p,w,r)}\left(B^{(p,w,r)} - A^{(p,w,r)}\right)^{-1} c^\nu_{\max} > 0 \Leftrightarrow r > 0.$$

Hence, it follows that

$$r > 0 \Leftrightarrow \Lambda^{\nu} > \alpha^{\nu}_{l\,\max}$$

for any propertyless employed agent $\nu \in \mathcal{N}$. This implies that **PECP** holds for this definition of exploitation. ■