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Models of Structural Change and Kaldor's Facts: Critical Survey from the Cambridge Keynesian Perspective*

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Abstract

This study addresses the reconciliation of structural change with Kaldor's facts, which is a new research agenda in this area. The mainstream reconciliation strategy is that the facts are interpreted as a state at which the economy grows along the *generalised balanced growth path* and multi-sectoral models are transformed into the one-sectoral model that has the uniquely (saddle-path) stable steady state. We argue that the mainstream strategy is far from Kaldor's own thoughts and overlooks structural change in physical capital. The Cambridge Keynesian reconciliation based on Pasinetti's structural dynamics demonstrates that structural change inevitably accompanies changes in social institutions to maintain full employment, whereas the mainstream reconciliation is achieved entirely through the market mechanism.

JEL Classification: B24, E12, O14, O41

Keywords: Structural change; Kaldor's facts; Cambridge Keynesians; Pasinetti's structural economic dynamics; Adjustment through market

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1 Introduction

Since the advent of classical economics, the analysis of economic structures, which refers to the structures of prices, quantities, expenditure, and employment from the multi-industrial or multi-sectoral perspective, has been one of the central subjects in the principles of political economy. Smith (1979) argued for the *natural* process of economic development from a multi-industrial perspective. Ricardo (1951) constructed a growth model including the corn and gold industries. Marx (1967) constructed a schema of reproduction with two sectors. As is well known, even Walras (1984), one of the founders of neoclassical economics, constructed a general equilibrium model.

After aggregate models of economic growth such as Solow (1956) became popular following the Second World War, the attention paid to structural analyses in macroeconomics faded. Although some multi-sectoral/multi-industrial models à la Leontief (1941) and Neumann (1945) were used even after the Second World War, the focus was on the balanced growth path, as indicated by the turnpike theorem (Khan and Piazza, 2012; McKenzie, 2008). Only Goodwin (1949, 1974) and Pasinetti (1965, 1981, 1993) continued to focus on structural analysis (Kerr and Scazzieri, 2013).

As Arena (2017), Rogerson (2019), and Silva and Teixeira (2008) showed, however, mainstream economics has revived the attention paid to structural change since the 1990s. The growing attention on structural change is also verified by the fact that the term 'structural change' (Matsuyama, 2008) was added into the 2008 version of *The New Palgrave Dictionary of Economics* as well as the term 'structural economic dynamics' proposed by Pasinetti and Scazzieri (1987). Moreover, a handbook related to structural change was recently published (Monga and Lin, 2019).

Further, a new research subject related to structural change has emerged, namely, examining whether structural change can be reconciled with Kaldor's (1961) facts, which can be summarised as follows:

1. Per-capita output grows over time and its growth rate does not tend to diminish;
2. Physical capital per worker grows over time;
3. The rate of return on capital is nearly constant;
4. The ratio of physical capital to output is nearly constant;
5. The shares of labour and physical capital in national income are nearly constant; and
6. The growth rate of output per worker differs substantially across countries.

According to Barro and Sala-i-Martin (2004), all these facts except fact 3 seem to fit reasonably well with the long-run data for advanced countries, and fact 3 can be replaced with the fact that the rate of return on capital tends to decline *over some range* as an economy grows. Although fact 3 can be corrected slightly, the decline in the return is moderate. Herrendorf et al. (2019) also confirmed that Kaldor's facts continue to hold overall in that constant trends provide a reasonable first-order description of most of the data and that sizeable short- and medium-term fluctuations around the trends exist. These factors imply that Kaldor's facts remain eligible as an analytical point of reference with respect to the *long-run* economic growth of advanced countries, at least as a first approximation, and thus they indicate the facts that have empirical regularities—even at present.

Furthermore, some studies examine the possibility of reconciling structural change with not only Kaldor's but also Kuznets' (1973) facts. The latter facts indicate the structural changes in employment, consumption, and output in such a way that the shares of employment, consumption expenditure, and output shift from agriculture to manufacturing, and eventually to services, as income grows.[1] It has also been pointed out that the evolution of manufacturing is hump-shaped (e.g. Lin and Wang, 2019).

From a theoretical point of view, structural change occurs for demand-side or supply-side reasons, or a mixture of both.[2] The demand-side reason is represented by non-homothetic preferences. In other words, it is indicated by non-linear Engel curves. The supply-side reason implies that industrial or sectoral differences in the growth rates of total factor productivity (TFP) or/and in factor intensities are assumed.

Examples of mainstream studies paying attention to the former reason include Alonso-Carrera and Raurich (2015), Bonatti and Felice (2008), Falkinger (1994), Foellmi (2005), Foellmi and Zweimüller (2008), Hori et al. (2015), Kongsamut et al. (2001), Laitner (2000), and Matsuyama (2019).

Mainstream research focusing on the latter reason includes Acemoglu and Guerrieri (2008), Alvarez-Cuadrado et al. (2017), Bonatti and Felice (2008), and Ngai and Pissarides (2007). Furthermore, the following mainstream studies emphasise that structural change is caused by *both* reasons: Boppart (2014a, 2014b), Comin et al. (2020), Guilló et al. (2011), Meckl (2002), Muro (2017), and Święcki (2017).

In addition, we should mention the so-called Baumol (1967) disease. He found structural change in the allocation of labour and consumption in a two-sector pure labour model with sectoral differences in the growth rates of labour productivities when demand for the good produced by the sector with lower productivity growth is sufficiently price-inelastic and income-elastic. He also showed that the real growth rate converges to the growth rate of the lower labour productivity in a two-sector model. The sector with the lower productivity growth rate then becomes dominant in the long run.

In conducting the new research agenda, mainstream economists interpret Kaldor's facts with reference to the concept of balanced growth path. As shown in Sections 3–5, they investigate whether the model of structural change is consistent with balanced growth at *aggregate* levels with the sectoral reallocation of labour/capital by extending the concept of the balanced growth path (the *generalised balanced growth path* or GBGP) to deal with structural change. Moreover, mainstream economists tend to think of a multi-sectoral model as a *natural* extension of the one-sector growth model, such as the Ramsey (1928) and Solow (1956) growth models (Herrendorf et al., 2014). Therefore, such economists somehow

[1] Jorgenson and Timmer (2011) pointed out that Maddison (1980) also discovered the same facts as Kuznets (1973). In addition, they closely examined structural change in the service sector in advanced economies.

[2] Some studies indicate additional reasons such as the effects of international trade (e.g. Matsuyama, 2009; Uy et al., 2013; van Neuss, 2019), changes in sectoral interlinkages (e.g. van Neuss, 2019), and intersectoral labour wedges (e.g. Alonso-Carrera and Raurich, 2018; Święcki, 2017).

attempt to convert the multi-sectoral models of structural change into, at most, a two-dimensional differential system of equations, as in the optimal growth model of Ramsey. Such a conversion is achieved by combining various types of utility and production functions such as the Cobb–Douglas, constant elasticity of substitution (CES), and constant relative risk aversion (CRRA) functions.

In this study, we critically review mainstream models reconciling structural change with Kaldor's facts from the Cambridge Keynesian point of view.[3] The distinctive features of Cambridge Keynesians are given in Section 6. We exclusively focus on the mainstream theoretical strategy to reconcile structural change with Kaldor's facts and disregard how well the results obtained by mainstream models fit the data on structural change and economic growth.

Such purely theoretical attention to structural change and Kaldor's facts is relevant to Cambridge Keynesian economists. First, as already mentioned, structural change is nothing but the theoretical field of research that Pasinetti, one of the most influential Cambridge Keynesian economists, pioneered in the early 1960s and has extended since (Baranzini and Mirante, 2018). Although some studies such as Rogerson (2019) and Stijepic (2011) review the relationship between structural change and Kaldor's facts from mainstream perspectives, few works are written from the Cambridge Keynesian point of view.[4]

Second, Kaldor was one of the most influential and prominent figures among Cambridge Keynesian economists. Then, whether the mainstream understanding of Kaldor's facts is consistent with Kaldor's own thoughts can be examined. Concretely, whether the mainstream reduction of Kaldor's facts to the GBGP is an adequate approach is a theoretically important issue. Whether the consideration of the multi-sectoral model accompanying structural change as a natural extension of the one-sector growth model is an adequate treatment also has theoretical relevance.

Third, Cambridge Keynesian economists criticised the principle of marginal productivity and asserted the importance of heterogeneous and reproducible capital goods (Harcourt, 1972; Pasinetti, 1977). The factor of production termed capital consists of heterogeneous and reproducible commodities, as in Sraffa (1960), in the real world. To the best of our knowledge, capital is assumed to be a homogeneous factor of production in all mainstream multi-sectoral models reconciling structural change with Kaldor's facts. Therefore, whether the treatment of capital can be justified to analyse the relationship between structural change and Kaldor's facts is a worthwhile subject to address for Cambridge Keynesian economists.

The rest of this paper is organised as follows. Section 2 summarises the extended concept of the balanced growth path. In addition, we define structural change as the term is used in this study. Section 3 reviews representative mainstream models that reconcile the

[3] See, for example, Marcuzzo and Rosselli (2016) and Pasinetti (2007) with respect to the Cambridge Keynesian point of view.

[4] Arena (2017) referred to the above-described new agenda of the reconciliation of structural change with Kaldor's facts to clarify the relationships among the business cycle, economic growth, and structural change as the mainstream field of research and compare it with Pasinetti's contributions and 'evolutionary' approaches to structural change. Arena and Porta (2012) also mentioned it. This study aims to clarify the essential characteristics of the mainstream strategy of the reconciliation and evaluate them from the Cambridge Keynesian perspective.

structural change caused by the demand-side reason with Kaldor's facts. Section 4 reviews the mainstream models that reconcile the structural change caused by the supply-side reason with Kaldor's facts. Section 5 reviews the mainstream models that reconcile the structural change caused by both the demand-side and the supply-side reasons with Kaldor's facts. Section 6 argues that the mainstream strategy to reconcile structural change with Kaldor's facts considers multi-sectoral models to be natural extensions of the one-sector growth model, which has a uniquely (saddle-path) stable steady state. However, we assert that this is far from Kaldor's own thoughts and the Cambridge Keynesian perspective. We show that mainstream models overlook another important structural change: the change in the composition of physical capital. Furthermore, we argue that the mainstream reconciliation of structural change with Kaldor's facts depends on the perfect adjustment mechanism through markets. According to the Cambridge Keynesian perspective based on Pasinetti's (1981) structural economic dynamics, we assert that structural change inevitably requires social institutions to change to maintain full employment. The importance of paying attention to changes in social institutions is increasing. Section 7 concludes.

2 Definitions of the Extended Concept of the Balanced Growth Path and Structural Change

As stated in the previous section, mainstream economists consider Kaldor's facts in reference to balanced growth path; Kaldor's facts require the profit (or interest) rate and capital–output ratio to be constant despite growth in aggregate output and labour productivity. These results are obtained under standard neoclassical growth models if Harrod-neutral technical progress is assumed (Uzawa, 1961). On the contrary, structural change describes the phenomenon that the structures of prices, quantities, consumption expenditure, and employment can vary over time. In principle, therefore, it cannot be reconciled with the balanced growth path in the strict sense. This new research agenda is thus an important and interesting issue in theories of economic growth.

Mainstream economists extend the concept of the balanced growth path to make it possible for structural change to be reconciled with Kaldor's facts. This extended concept is the GBGP, the minimum requirement of which can be specified as follows:

Definition 1 *The GBGP is a path along which one or more variables grow at a constant rate.*

The GBGP does not require all the variables of the differential system of equations to grow at the same rate, unlike the balanced growth path; some variables can grow at different rates.

Suppose a three-dimensional differential system of equations: $x(t), y(t)$, and $z(t)$. Then, the GBGP can be exemplified by the state at which y grows at a constant rate but not x and z or y and z grow at an identical constant rate but not x (Stijepic, 2011).

In the context of the presented new research agenda, for example, the GBGP allows sectoral output shares to grow at different rates, whereas *aggregate consumption* and *aggregate capital* grow at a constant rate.

The mainstream strategy thinks of Kaldor's facts as the state at which the GBGP exists and the economy grows along the path. It can be shown that some variables such as the rate of interest, the aggregate capital-output ratio, and the share of capital income are kept constant when the economy grows along the GBGP.

Next, we define structural change in our study, since the term 'structure' has broad meanings in the fields of economic theory and policy (Monga and Lin, 2013). Too broad a definition is not suited to theoretical analyses. Therefore, we define it practically.

Definition 2 *Structural change represents the changes in the sectoral composition of relative prices, output, consumption (expenditure), and employment.*

Since the GBGP does not require all the variables to grow at the same rate, structural change (changes in sectoral composition) can be reconciled with the constant growth rate(s) of the aggregate variable(s). Definition 2 follows the one proposed by Pasinetti (1981), Pasinetti and Scazzieri (1987), and Scazzieri (2018), although some of the models reviewed in the forthcoming sections define it more narrowly.

3 Reconciliation of the Structural Change Caused by the Demand-side Reason with Kaldor's Facts

In this section, we examine the characteristic of mainstream multi-sectoral models that attempt to reconcile the structural change caused by the demand-side reason with Kaldor's facts. As a representative example, we review Kongsamut et al. (2001) and subsequently examine other examples of models in which the structural change caused by the demand-side reason is reconciled with Kaldor's facts.

3.1 Kongsamut et al. (2001)

There are three sectors: agriculture, manufacturing, and services. The output of each sector in period t is respectively denoted by $A(t) \in \left[\overline{A}, \infty\right)$, $M(t) \in \mathbb{R}_+$, and $S(t) \in \mathbb{R}_+$. All the sectors share the standard neoclassical production function, F, which is identical up to the constant of proportionality. It is assumed that only manufacturing goods can be consumed and invested and the remaining goods are just consumed. Since structural change is caused by the demand-side reason, the assumptions on technology are standard:

$$
\begin{gathered}
A(t) = B_A F\left(\phi^A(t) K(t), N^A(t) X(t)\right), \\
M(t) + \dot{K}(t) + \delta K(t) = B_M F\left(\phi^M(t) K(t), N^M(t) X(t)\right), \\
S(t) = B_S F\left(\phi^S(t) K(t), N^S(t) X(t)\right), \\
\phi^A(t) + \phi^M(t) + \phi^S(t) = 1, \\
N^A(t) + N^M(t) + N^S(t) = 1, \\
\dot{X}(t) = gX(t),
\end{gathered}
$$

where $N^i(t), \phi^i(t)$ denote the labour employed and share of capital employed in sector i in period t $(i = A, M, S)$, respectively. The total amount of labour is normalised to unity. $X(t)$ denotes Harrod-neutral technical progress, the rate of which is $g > 0$. δ and B_i are the depreciation rate and parameter denoting the technology level of sector i, respectively.

Since capital and labour are assumed to be freely mobile, the condition for optimal allocation is that the marginal rates of transformation are equal across the three sectors:

$$\frac{\phi^A(t)}{N^A(t)} = \frac{\phi^M(t)}{N^M(t)} = \frac{\phi^S(t)}{N^S(t)}.$$

Since the proportionality of production functions is assumed, the relative prices of agriculture and services to manufacturing are given as follows:

$$p_A = \frac{B_M}{B_A}, p_S = \frac{B_M}{B_S}.$$

It implies that there is no structural change in relative prices in the equilibrium.

Using the above formulation, the resource constraint for the whole economy is given as follows:

$$M(t) + \dot{K}(t) + \delta K(t) + p_A A(t) + p_S S(t) = B_M F(K(t), X(t)). \tag{1}$$

This transformation crucially depends on the assumption that the production functions are identical up to the constant of proportionality.

The demand-side factor is characterised by non-homothetic preferences called Stone–Geary preferences as follows:

$$U = \int_0^\infty \frac{c(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} \mathrm{d}t, \text{ where } c(t) \equiv \left(A(t) - \overline{A}\right)^\beta M(t)^\gamma \left(S(t) + \overline{S}\right)^\theta, \tag{2}$$

where $\sigma, \beta, \gamma, \theta, \rho$ (rate of time preference), $\bar{A}, \bar{S}$ are assumed to be strictly positive and $\beta + \gamma + \theta = 1$. The income elasticity of demand is less than 1 for agricultural goods, equal to 1 for manufacturing goods, and greater than 1 for services. According to Kongsamut et al. (2001), $\overline{A}$ and $\overline{S}$ can be interpreted as the level of subsistence consumption and home production of services, respectively.

The problem to solve here is to maximise (2) subject to (1). Thus, the equilibrium real rate of interest r is given by

$$r(t) = B_M f'(k(t)) - \delta, \tag{3}$$

where $k(t) \equiv K(t)/X(t), f(k(t)) \equiv F(k(t), 1)$. Moreover, the optimal allocation of consumption across sectors satisfies

$$\frac{p_A\left(A(t) - \overline{A}\right)}{\beta} = \frac{M(t)}{\gamma} \text{ and } \frac{p_S\left(S(t) + \overline{S}\right)}{\theta} = \frac{M(t)}{\gamma}. \tag{4}$$

(4) implies that both $A(t) - \overline{A}$ and $S(t) + \overline{S}$ are proportional to $M(t)$. Using (3) and (4), the optimal path for the consumption of manufacturing goods is given as

$$\frac{\dot{M}(t)}{M(t)} = \frac{r(t) - \rho}{\sigma}. \tag{5}$$

Since $\overline{A}, \overline{S}$ are positive, there is no balanced growth path in this model; even when the real rate of interest is constant, (4) and (5) imply that $A(t)$ and $S(t)$ do not grow at a constant rate. However, it is clear from (1) that the balanced growth path requires $A(t), M(t)$, and $S(t)$ to grow at rate g.

From (3) and (5), the steady-state value of k, if one exists, must satisfy

$$B_M f'(k) - \delta = \sigma g + \rho. \tag{6}$$

Now, suppose that $\overline{A}B_S = \overline{S}B_A$ holds. Then, (1) is rewritten as follows:

$$M(t) + \dot{K}(t) + \delta K(t) + p_A\left(A(t) - \overline{A}\right) + p_S\left(S(t) + \overline{S}\right) = B_M f(k(t)) X(t). \tag{7}$$

If $k(t)$ is kept constant (i.e. if the steady state exists), the right-hand side of (7) grows at the rate of g and $A(t) - \overline{A}$ and $S(t) + \overline{S}$ can grow at the same rate, as shown by (4).

Letting $m(t) \equiv M(t)/X(t)$, (5) can be rewritten as follows:

$$\dot{m}(t) = \frac{1}{\sigma}(r(t) - \rho - \sigma g) m(t). \tag{8}$$

Similarly, (7) is rewritten as follows:

$$\dot{k}(t) = B_M f(k(t)) - (\delta + g) k(t) - \frac{m(t)}{\gamma}. \tag{9}$$

(8) and (9) constitute the two-dimensional differential system of equations, which has the same properties as the Ramsey growth model. Then, we can prove the unique existence of the saddle-path stable equilibrium path converging towards the steady state (k^*, m^*). In the steady state, $K(t)$ and $M(t)$ grow at the rate of g. Then, we obtain the following proposition.

Proposition 3 *The GBGP exists if $\overline{A}B_S = \overline{S}B_A$ and the transversality condition is satisfied. The initial value of k consistent with the GBGP is given by (6).*

The properties of economic growth along the GBGP in the model can be summarised as follows: the rate of real interest is constant, while capital (manufacturing), agriculture, services, and aggregate output grow at the rate of g in the long run. Therefore, the capital–output ratio remains constant along the GBGP. This implies that Kaldor's facts can be obtained. Structural changes in production (consumption) and employment occur. In particular, the employment share in agriculture declines and that in services increases, whereas that in manufacturing remains constant. Although the output of each commodity grows over time, moreover, its share of each sector follows the same evolutions as the employment share, since the production functions are identical up to the constant of proportionally. The impacts of $\overline{A}$ and $\overline{S}$ fade over time and the economy converges to the GBGP. Indeed, as $A(t)$ and $S(t)$ become larger, the utility function is getting close to a homothetic utility function (the Cobb–Douglas utility function).

However, the structural changes in the model cannot continue forever and eventually cease along the GBGP. This is demonstrated by the fact that $\lim_{t\to\infty} \dot{N}^A(t) = \lim_{t\to\infty} \dot{N}^S(t) = 0$ and

$\lim_{t\to\infty} \dot{A}(t)/A(t) = \lim_{t\to\infty} \dot{S}(t)/S(t) = g$ ($\dot{M}(t)/M(t) = 0$ and $\dot{N}^M(t) = 0$ for all $t \geqq 0$). In this sense, we conclude that the model does not show the *persistent* coexistence of structural change and Kaldor's facts.

$\overline{A}B_S = \overline{S}B_A$ is termed the knife-edge condition. Under this condition, certain parameters or combinations of parameters are constrained to take on specific values for a viable equilibrium to exist (e.g. Turnovsky, 2002). According to Kongsamut et al. (2001), the knife-edge condition should be interpreted such that each agent has a positive endowment of services and a negative endowment of agricultural goods. The endowments in terms of relative prices are such that $p_S\overline{S} = p_A\overline{A}$. The knife-edge condition implies a specific equality between technology and the preference parameters, which is obviously restrictive. Indeed, Herrendorf et al. (2013) argued that the condition is not trivially consistent with final consumption expenditure data on the US economy since the relative price of services to goods has been increasing steadily since the Second World War, whereas $\overline{A}$ and $\overline{S}$ are constants.

3.2 Other examples of the reconciliation of the structural change caused by the demand-side reason with Kaldor's facts

Alonso-Carrera and Raurich (2015) also constructed a multi-sectoral model of structural change in which the GBGP exists by assuming the following utility function:

$$U = \int_0^\infty \left[\frac{\prod_{i=1}^m (c_i - \widetilde{c}_i)^{\theta_i(1-\sigma)}}{1-\sigma} \right] e^{-\rho t} \mathrm{d}t,$$

where c_i and $\widetilde{c}_i$ respectively denote the consumption and minimum consumption requirement of good i for $i = 1, \cdots, m$, among which only good m can be both consumed and invested and the remaining goods for $i = 1, \cdots, m-1$ are just consumed. $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution when $\widetilde{c}_i = 0$ for all i. The utility function is non-homothetic when $\widetilde{c}_i > 0$ for some i. $\theta_i \in (0,1)$ denotes the weights of the consumption goods in the utility function such that $\sum_{i=1}^m \theta_i = 1$. The assumptions on technology are characterised by the Cobb–Douglas production functions $Y_i(t) = [s_i(t) K(t)]^\alpha [A_i(t) u_i(t) L(t)]^{1-\alpha}$, where $\alpha \in (0,1)$, $s_i(t), u_i(t)$ denote the shares of capital and labour employed in sector i in period t and $A_i(t)$ represents the TFP of sector i, the growth rate of which is assumed to be identical across sectors: $\dot{A}_i(t)/A_i(t) = \gamma$ for $i = 1, \cdots, m$. There is no supply-side reason for structural change here.

Unlike in the standard Ramsey model, Alonso-Carrera and Raurich (2015) derived the three-dimensional differential system of equations with respect to $z(t) \equiv K(t)/A_m L(t)$, $e(t) \equiv E(t)/Y(t)$, $q(t) \equiv Q(t)/Y(t)$, where $Y(t) \equiv \sum_{i=1}^m p_i Y_i$, $E(t) \equiv \sum_{i=1}^m p_i c_i$, $Q(t) \equiv \sum_{i=1}^m p_i \widetilde{c}_i$. $Q(t)$ and $q(t)$ denote the aggregate value of the minimum consumption requirements and its intensity in period t, respectively. Using the conditions for profit maximisation, $Y(t)$ can be transformed into $Y(t) = A_m(t) L(t) z(t)^\alpha$.

Assume that the transversality condition is satisfied. The three-dimensional differential system of equations of the model can be summarised as follows:

$$\frac{\dot{z}}{z} = f_1\left(e, z\right), \frac{\dot{q}}{q} = f_2\left(e, z\right), \text{ and } \frac{\dot{e}}{e} = f_3\left(e, z, q\right).$$

Here, the initial values of both z and q are given and they are chosen independently of $A_m\left(0\right)$. Unlike the standard Ramsey growth model, the steady state, if it exists, is defined using one control variable e and two state variables z, q.

The above three-dimensional differential system of equations can be considered to be the generalisation of Kongsamut et al. (2001). This is because the knife-edge condition of Kongsamut et al. (2001) is equivalent to assuming $Q = 0$ (or $q = 0$). By assuming $q = 0$, we can reduce the dimensionality of the steady state from the model. In other words, the knife-edge condition selects a particular equilibrium path of the two-dimensional manifold. Then, we can prove the unique existence of the GBGP, which is saddle-path stable, whenever $q = 0$. This means that along the GBGP, aggregate capital, output, and consumption expenditure grow at the rate of γ, and the rate of interest is also kept constant since z is kept constant. Then, Kaldor's facts are obtained. It is also shown that Kuznets' facts are obtained in the model with three sectors.

Since Alonso-Carrera and Raurich (2015) can be considered to be the generalisation of Kongsamut et al. (2001), they have similar implications. First, structural change eventually ceases along the GBGP. Moreover, the implausibility of the knife-edge condition, as pointed out by Herrendorf et al. (2013) with respect to Kongsamut et al. (2001), also applies to Alonso-Carrera and Raurich (2015).

Furthermore, Li et al. (2019) extended Kongsamut et al. (2001) by introducing Romer's (1990) endogenous technological change into the three-sector model. The structure of the model is equivalent to Kongsamut et al. (2001), but an intermediate sector and a research sector are introduced in addition to the final goods sector. The final goods (agriculture, manufacturing, and services) are produced using labour, human capital, and all the types of intermediate goods designed by the knowledge. The types of intermediate goods are determined by the knowledge stock created by the research sector. The number of types of intermediate goods in period t is expressed by the knowledge stock Γ_t. The research sector creates new knowledge using human capital and the existing knowledge Γ_t, and the linear production function of the knowledge is assumed, as is usual in endogenous growth models. The utility function assumed by Li et al. (2019) is slightly different from that of Kongsamut et al. (2001); $c\left(t\right) = \left(A\left(t\right) - \overline{A}\right)^{\beta}\left(M\left(t\right) + \overline{M}\right)^{\gamma}\left(S\left(t\right) + \overline{S}\right)^{\theta}$, where $\overline{A}, \overline{M}, \overline{S} > 0$ (the first denotes the subsistence consumption of agriculture and the second and third the home production of manufacturing and services, respectively). A term denoting human capital is included in the consumer's budget constraint in Li et al. (2019).

Since it is assumed that manufacturing can be both consumed and invested, we can transform the multi-sectoral model into a type of one-sector optimal growth model using a similar procedure to that of Kongsamut et al. (2001). Then, we can prove the unique existence of the GBGP whenever the knife-edge condition $\frac{\overline{A}}{B_A} = \frac{\overline{M}}{B_M} + \frac{\overline{S}}{B_S}$, where B_i denotes that the technology parameter of sector $i = A, M, S$, is satisfied. The growth rate of aggregate variables along the GBGP is endogenously determined by the total stock of human capital, rate of time preference, and technology parameters. Moreover, Li et al. (2019) investigated the effect of human capital on structural change but did not consider the plausibility of the knife-edge condition.

Foellmi (2005) and Foellmi and Zweimüller (2008) presented multi-sectoral models that reconcile the structural change caused by the demand-side reason with Kaldor's facts. Therefore, the technology assumptions are neoclassical and identical across all the sectors in the models; capital and consumption goods are produced by the same neoclassical production function as consumption goods: $F\left[K\left(i,t\right), A\left(t\right)L\left(i,t\right)\right]$, where $K\left(i,t\right)$ and $A\left(t\right)L\left(i,t\right)$ denote the amount of capital and efficiency unit of labour employed in sector i in period t, respectively. $A\left(t\right)$ is the stock of labour-augmenting technical knowledge, which increases at an exogenous rate of $g>0$. Since all the sectors have the same production functions, each sector produces at the same capital per-efficiency unit of labour in the equilibrium ($k\left(i,t\right)\equiv\frac{K(i,t)}{A(t)L(i,t)}=k$ for all i). Thus, the marginal costs and hence the prices can be normalised to unity without loss of generality. This means that structural change in prices does not occur in the models. However, the model allows for the emergence of new goods.

The characteristic of the model is to introduce a 'hierarchy' of wants using the following utility function:

$$\begin{aligned} u\left(t\right) &= \int_t^\infty \frac{\upsilon\left(\tau\right)^{1-\sigma}}{1-\sigma}e^{-\rho(\tau-t)}\mathrm{d}\tau, \\ \text{where } \upsilon\left(\tau\right) &= \frac{1}{2}\int_0^\infty i^{-\xi}\left[s^2-\left(s-c\left(i,\tau\right)\right)^2\right]\mathrm{d}i. \end{aligned}$$

i and $i^{-\xi}$ are the index of consumption goods and hierarchy function, respectively, and $s>0$, $\xi\in(0,1)$. Goods with lower i have higher weights than goods with higher i. It is shown that consumption demand for a particular good, derived by the above utility function, depends on the relative position in the hierarchy of wants; goods at a lower position in the hierarchy, which are given relatively higher priority, are consumed in higher quantity. The income elasticity of consumption demand for good i is given by $\frac{\xi(i/N(t))^\xi}{1-(i/N(t))^\xi}$, where $N\left(t\right)$ denotes the number of consumption goods in period t. It suffices for our purpose to assume that $N\left(t\right)$ exogenously increases, although Foellmi and Zweimüller (2008) considered R&D activity as well. Since the number of goods increases over time, the relative position of good i in the hierarchy of wants, shown by $\frac{i}{N(t)}$, declines over time. This means that the income elasticity of demand for good i monotonically declines as new goods emerge over time and consumption demand for the good finally reaches saturation. As a result, non-linear Engel curves are obtained.

Aggregate consumption expenditure can be defined as $E\left(t\right)=\int_0^N p_jc_j\mathrm{d}j$, where p_j, c_j denote the price and consumption of good j. Remember that the prices are normalised. Let aggregate expenditure in the efficiency unit be $e\left(t\right)=\frac{E(t)}{A(t)}$. Then, the model is summarised by the two-dimensional differential system of equations with respect to $e\left(t\right)$ and $k\left(t\right)$. The method used to solve this model is basically the same as the Ramsey optimal growth model.

Then, the unique existence of the saddle-path steady state (i.e. the GBGP) can be proven. Along the GBGP, aggregate consumption expenditure and capital in the efficiency units are constant at the steady state. This implies that aggregate output, consumption, and capital grow at a constant and identical rate and that the rate of interest is kept constant. Thus, Kaldor's facts are obtained. Reducing the generally multi-sectoral model into a model with three sectors (agriculture, manufacturing, and services), furthermore, it is shown that the evolution of the employment share of manufacturing is hump-shaped and thus Kuznets' facts are also obtained.

Particular attention should be paid to the fact that the model shows the persistent coexistence of structural change and Kaldor's facts. This is because the composition of consumption demand changes along the GBGP. In other words, structural change in sectoral consumption occurs along the GBGP due to the emergence of new goods, although aggregate consumption grows at a constant rate. The success of proving the persistent coexistence of structural change and Kaldor's facts crucially depends on the form of hierarchy function $i^{-\xi}$. Thanks to this, the income elasticity of demand for various goods changes over time, as already mentioned, whereas the indirect utility function is equivalent to CRRA in the one-sector model from the viewpoint of a single individual (Foellmi, 2005, p. 21).

4 Reconciliation of the Structural Change Caused by the Supply-side Reason with Kaldor's Facts

In this section, we first take Ngai and Pissarides (2007) as a representative example of multi-sectoral models that reconcile the structural change caused by the supply-side reason with Kaldor's facts. Subsequently, we review other multi-sectoral models that address the reconciliation of the structural change caused by the supply-side reason with Kaldor's facts.

4.1 Ngai and Pissarides (2007)

There are m sectors, among which $m-1$ sectors ($i = 1, \cdots, m-1$) produce pure consumption goods and the last one ($i = m$) produces a good that can be both consumed and invested. Moreover, it is assumed that the labour force grows at the exogenous rate of $n > 0$.

The household's preferences are represented by the following utility function:

$$U = \int_0^\infty e^{-\rho t} \upsilon \left[c_1(t), \cdots, c_m(t) \right] \mathrm{d}t, \text{ where} \tag{10}$$

$$\upsilon \left[c_1(t), \cdots, c_m(t) \right] \equiv \frac{\phi(\cdot)^{1-\theta} - 1}{1-\theta}; \phi(\cdot) \equiv \left(\sum_{i=1}^{m} \omega_i c_i(t)^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)},$$

and $c_i(t) \geqq 0$ denote the per-capita consumption of good i in period t. Moreover, $\theta, \varepsilon, \omega_i > 0$, and $\sum_{i=1}^{m} \omega_i = 1$ are satisfied. If $\theta = 1$, then $\upsilon \left[c_1(t), \cdots, c_m(t) \right] = \ln \phi(\cdot)$, and if $\varepsilon = 1$, then $\ln \phi(\cdot) = \sum_{i=1}^{m} \omega_i \ln c_i(t)$. These are standard assumptions on preferences; the demand functions have constant price elasticity $-\varepsilon$ and unit income elasticity.

On the contrary, the production function of each sector is distinctively formulated as follows:

$$c_i(t) = A_i(t) F(n_i(t) k_i(t), n_i(t)), \text{ for } i = 1, \cdots, m-1,$$
$$\dot{k}(t) = A_m(t) F(n_m(t) k_m(t), n_m(t)) - c_m(t) - (\delta + n) k(t),$$

where $n_i(t), k_i(t), k(t) \geqq 0$ denote the employment share, capital–labour ratio in sector i, and aggregate capital–labour ratio in period t, respectively. F is the standard neoclassical production function, which is common to all sectors, and $A_i(t)$ $(i = 1, \cdots, m)$ denotes Hicks-neutral technical progress such that $\dot{A}_i(t)/A_i(t) = \gamma_i$ is assumed: $A_i(t)$ is TFP. $\gamma_i \neq \gamma_j$ if $i \neq j$ is the supply-side reason for structural change. The free mobility of both factors is assumed. Moreover, the following constraints are satisfied:

$$\sum_{i=1}^{m} n_i(t) = 1, \quad \sum_{i=1}^{m} n_i(t) k_i(t) = k(t). \tag{11}$$

The optimal allocation condition requires that the marginal rates of substitution are equal to the marginal rates of transformation, which implies the following:

$$\frac{\upsilon_i(t)}{\upsilon_m(t)} = \frac{A_m(t) F_{m1}}{A_i(t) F_{i1}} = \frac{A_m(t) F_{m2}}{A_i(t) F_{i2}}, \text{ for } i = 1, \cdots, m-1, \tag{12}$$

where $\upsilon_i(t) \equiv \partial \upsilon / \partial c_i$, and F_{ij} denotes the partial derivatives of the F of sector i with respect to the jth variable $(j = 1, 2)$. From the properties of the production functions that we assume, conditions (11) and (12) imply

$$k_i(t) = k(t) \text{ for } \forall i, \text{ and } \frac{p_i(t)}{p_m(t)} = \frac{\upsilon_i(t)}{\upsilon_m(t)} = \frac{A_m(t)}{A_i(t)} \text{ for } i = 1, \cdots, m-1. \tag{13}$$

The optimal condition for the representative consumer yields

$$-\frac{\dot{\upsilon}_m(t)}{\upsilon_m(t)} = A_m(t) F_k - (\delta + n + \rho), \tag{14}$$

where $F_k \equiv \frac{\partial F}{\partial k}$.

Given utility function (10), (13) yields

$$\frac{p_i(t) c_i(t)}{p_m(t) c_m(t)} = \left(\frac{\omega_i}{\omega_m}\right)^{\varepsilon} \left(\frac{A_m(t)}{A_i(t)}\right)^{1-\varepsilon} \equiv x_i(t), \text{ for } i = 1, \cdots, m-1. \tag{15}$$

$x_i(t)$ is a variable denoting the ratio of consumption expenditure on good i to that on manufacturing good in period t.

Let us define aggregate consumption expenditure and output per-capita in terms of manufacturing as follows: $c(t) \equiv \sum_{i=1}^{m} \frac{p_i(t)}{p_m(t)} c_i(t)$, $y(t) \equiv \sum_{i=1}^{m} \frac{p_i(t)}{p_m(t)} A_i(t) F(n_i(t) k_i(t), n_i(t))$, which can be rewritten using (15) and the production functions:

$$c(t) = c_m(t) X(t), \quad y(t) = A_m(t) F(k(t), 1),$$

where $X(t) \equiv \sum_{i=1}^{m} x_i(t)$.

(13) and (15) imply that structural changes in relative prices and consumption expenditure occur since $\gamma_i \neq \gamma_j$ if $i \neq j$. Furthermore, we obtain

$$\begin{aligned}
\frac{\dot{n}_i(t)}{n_i(t)} &= \frac{\mathrm{d}(c/y)/\mathrm{d}t}{c/y} + (1-\varepsilon)\left(\overline{\gamma}(t) - \gamma_i\right), \text{ for } i = 1, \cdots, m-1, \\
\frac{\dot{n}_m(t)}{n_m(t)} &= \left[\frac{\mathrm{d}(c/y)/\mathrm{d}t}{c/y} + (1-\varepsilon)\left(\overline{\gamma}(t) - \gamma_m\right)\right] \times \frac{(c/y)(x_m/X)}{n_m(t)} \\
&\quad + \left(\frac{-\mathrm{d}(c/y)/\mathrm{d}t}{1-c/y}\right)\left(\frac{1-c/y}{n_m(t)}\right),
\end{aligned}$$

where $\overline{\gamma}(t) \equiv \sum_{i=1}^{m} \left(\frac{x_i(t)}{X(t)}\right) \gamma_i$, which is a weighted average of the sectoral TFP growth rates, with the weight given by each good's consumption share. These show that structural change in the employment share occurs if $\varepsilon \neq 1$ and c/y is kept constant since $\gamma_i \neq \gamma_j$ if $i \neq j$ is assumed. Note that $\frac{\mathrm{d}\overline{\gamma}(t)}{\mathrm{d}t} \lesseqgtr 0$ if and only if $\varepsilon \lesseqgtr 1$ (see Lemma A3 in Ngai and Pissarides, 2007). Furthermore, if c/y is kept constant and $\varepsilon < 1$ (i.e. consumption demand is price inelastic), employment shifts from the sector with the highest TFP growth rate to that with the lowest TFP growth rate, and the converse is true if $\varepsilon > 1$.

Here, let us specify the production functions in a Cobb–Douglas form: $F(n_i(t)k_i(t), n_i(t)) \equiv (n_i(t)k_i(t))^{\alpha} n_i(t)^{1-\alpha} = k_i(t)^{\alpha} n_i(t)$ for $\alpha \in (0,1)$. Note that α is a common parameter to all sectors; this implies that the factor intensities are equal in all the sectors. Then, we can obtain

$$\begin{aligned}
\dot{k}(t) &= A_m(t)k(t)^{\alpha} n_m(t) - c_m(t) - (\delta+n)k(t) \\
&= A_m(t)k(t)^{\alpha} - c(t) - (\delta+n)k(t).
\end{aligned} \tag{16}$$

From (14), we have

$$\theta\frac{\dot{c}(t)}{c(t)} = (\theta-1)\left(\gamma_m - \overline{\gamma}(t)\right) + \alpha A_m(t)k(t)^{\alpha-1} - (\delta+n+\rho). \tag{17}$$

(16) and (17) constitute the two-dimensional differential system of equations determining the motions of the variables in the model. Then, we obtain the following proposition:

Proposition 4 *Given any initial $k(0) > 0$, the necessary and sufficient condition for the existence of the GBGP is given by*

$$\theta = 1, \varepsilon \neq 1, \text{ and } \exists i \in \{i = 1, \cdots, m-1 \,|\gamma_i \neq \gamma_m\}.$$

In Proposition 4, the multi-sectoral model is transformed into the one-sector model, and then the existence of the GBGP (i.e. the saddle-path stable steady state $\left(\widetilde{k}^*, \widetilde{c}^*\right)$, where

$\widetilde{k}^* = kA_m(t)^{\frac{-1}{1-\alpha}}$ and $\widetilde{c}^* = c(t)A_m(t)^{\frac{-1}{1-\alpha}}$) is proven. Along the GBGP, therefore, both $k(t)$ and $c(t)$ grow at the rate of $\frac{\gamma_m}{1-\alpha}$, and thus aggregate output $y(t)$ also grows at the same rate. As in standard models where balanced growth path exists, this means that aggregate consumption must be a constant proportion of aggregate output along the GBGP. Given (10), this can hold either when consumption is independent of the rate of interest or when the rate of interest is constant. Since the rate of interest is determined by the marginal productivity of capital in this model, the constant rate of interest is inconsistent with structural change. Therefore, consumption must be independent of the rate of interest, which implies the logarithmic utility function. Then, $\theta = 1$ is required to prove the existence of the GBGP in the model.

Moreover, the employment shares of sector m and the sector whose TFP growth rate is the lowest (highest) in the case of $\varepsilon < 1$ ($\varepsilon > 1$) converge to positive values and those of the remaining sectors converge to zero in the long run along the GBGP.[5] It is intuitive that the employment share of sector m converges to a positive value since capital is produced at a constant rate along the GBGP. The positive convergence in the employment share of the sector with the lowest TFP growth rate in the case of $\varepsilon < 1$, which is more empirically relevant than $\varepsilon > 1$, depends on the same mechanism as that causing the Baumol (1967) disease. In other words, capital except that allocated to sector m tends to be absorbed by the sector with the lowest TFP growth rate along the GBGP when $\varepsilon < 1$.

From the production functions, however, the output growth rate of sector $i = 1, \cdots, m-1$ along the GBGP is given by $\varepsilon\gamma_i + \alpha\frac{\dot{k}_i}{k_i} + \frac{\dot{n}_i}{n_i} = \varepsilon\gamma_i + \alpha g_m + (1-\varepsilon)\overline{\gamma}(t)$ (note $\frac{d(c/y)/dt}{c/y} = 0$ along the GBGP), where g_m denotes the growth rate of capital (sector m). Therefore, structural changes in relative prices, output, and consumption persistently occur along the GBGP, even though sectors vanish in the employment share in the long run. Therefore, Ngai and Pissarides (2007) found the persistent coexistence of structural change and Kaldor's facts.

4.2 Other examples of the reconciliation of the structural change caused by the supply-side reason with Kaldor's facts

Acemoglu and Guerrieri (2008) presented a model that reconciles the structural change caused by the supply-side reason with Kaldor's facts. Their model pays particular attention to the resource reallocation during the growth process and cannot analyse the structural change in consumption expenditure caused by income growth, since it has only one consumption good.

Suppose an economy with three sectors, one of which produces a consumption good and two of which produce different intermediate goods. The consumption good is produced using the intermediate goods following the CES production function, without employing the direct labour, and can be both consumed and invested. The budget constraint is thus given by $\dot{K}(t) + \delta K(t) + c(t)L(t) \leqq Y(t)$, where $c(t), L(t), Y(t)$ denote per-capita consumption, population, and the output of the final good. $\dot{L}(t)/L(t) = n$ is assumed. The intermediate goods are produced using capital and labour and the production functions take the Cobb–Douglas form with sectoral differences in factor intensities and TFP growth rates.

[5] Ngai and Pissarides (2007) indicated that the employment shares of the remaining sectors are either hump-shaped or monotonically declining. Therefore, the model can account for Kuznets' facts as well.

The technology assumptions are summarised as follows:

$$Y(t) = \left[\lambda Y_1(t)^{(\varepsilon-1)/\varepsilon} + (1-\lambda) Y_2(t)^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)}, \tag{18}$$

where $\lambda \in (0,1)$, and Y, Y_1, Y_2 respectively denote the amount of consumption good and those of the two intermediate goods produced by such Cobb–Douglas functions as $Y_i(t) = M_i(t) L_i(t)^{\alpha_i} K_i(t)^{1-\alpha_i}$ for $i = 1, 2$ with $\dot{M}_i(t)/M_i(t) = \gamma_i$ and $\alpha_1 > \alpha_2$ (sector 1 is more labour-intensive). The factor intensities differ in contrast to Ngai and Pissarides (2007). The resource constraints are given as follows: $K_1(t) + K_2(t) \leqq K(t)$ and $L_1(t) + L_2(t) \leqq L(t)$. Concerning the preferences, the CRRA utility function is assumed. This is maximised subject to the budget constraint.

Here, let us concentrate on the case of $\varepsilon < 1$, which has more empirical relevance. Since it is assumed that capital and labour are homogeneous and the production functions are specified in the Cobb–Douglas form, the multi-sectoral model can be transformed into a type of one-sector Ramsey optimal growth model, but it consists of the three-dimensional differential system of equations that has one control variable ($\widetilde{c}(t) \equiv \frac{c(t)}{M_1(t)^{1/\alpha_1}}$) and two state variables ($\widetilde{k}(t) \equiv \frac{K(t)}{L(t)M_1(t)^{1/\alpha_1}}$ and $\kappa(t) \equiv \frac{K_1(t)}{K(t)}$). This is because an equation determining the sectoral allocation of capital must be added. Assuming the knife-edge condition, however, the three-dimensional differential system of equations can be reduced to the two-dimensional system, as in Alonso-Carrera and Raurich (2015). Then, the unique existence of the (locally) saddle-path stable steady state $\left(\widetilde{k}^*, \widetilde{c}^*\right)$, given $\widetilde{k}(0)$ and $\kappa(0) > 0$, can be proven if the transversality condition is satisfied.

Along the GBGP, therefore, per-capita consumption $c(t)$ grows at the rate of γ_1/α_1 (the augmented rate of technical progress), output $Y(t)$ and capital $K(t)$ grow at the rate of $n + \gamma_1/\alpha_1$, and the rate of interest is kept constant. Thanks to the knife-edge condition, sector 1, which is more labour-intensive, is the asymptotically dominant sector along the GBGP, meaning that it determines the long-run growth rate of the economy. In other words, the shares of both capital and labour allocated to sector 1 converge to unity in the long run along the GBGP (i.e. $\lim_{t\to\infty} \frac{K_1(t)}{K(t)} = \lim_{t\to\infty} \frac{L_1(t)}{L(t)} = 1$), since the augmented rate of technical progress of sector 2 is assumed to be higher than that of sector 1 (i.e. the relative output of sector 1 is zero in the long run along the GBGP: $\lim_{t\to\infty} \frac{Y_1(t)}{Y_2(t)} = 0$). The asymptotic dominance of sector 1 is nothing but the result of Baumol (1967).[6] Moreover, the growth rates of sectors 1 and 2 differ along the GBGP and this difference persists even in the long run. We can conclude that structural change can be reconciled with Kaldor's facts in the model.

Despite the emergence of the dominant sector, structural change in sectoral output persistently occurs along the GBGP; sector 2 always grows faster than sector 1. When there is capital deepening and both capital and labour are allocated to the two sectors in a constant proportion, sector 2 can grow faster than sector 1 because it is the more capital-intensive. As a result, the price of good 2 declines, which leads to a reallocation; however, this reallocation

[6] According to Acemoglu and Guerrieri (2008), the sectors growing faster tend to have higher capital intensity—at least in the United States. Therefore, the assumed knife-edge condition seems plausible.

never offsets the greater increase in sector 2 because of $\alpha_1 > \alpha_2$. Then, this model accounts for the persistent coexistence of structural change and Kaldor's facts.

Alvarez-Cuadrado et al. (2017) extended Acemoglu and Guerrieri (2008) by introducing a new supply-side reason (sectoral differences in the elasticities of substitution between capital and labour). If the aggregate capital–labour ratio and wage–interest ratio increase, the sector with the higher elasticity of substitution is in a better position to substitute capital for labour. The sectoral differences cause structural change. According to Alvarez-Cuadrado et al. (2017), the significance of the introduction of sectoral differences in the elasticities of substitution is, first, that they have been confirmed by many empirical studies. Second, empirical evidence confirms the sectoral differences in the growth rates of the capital–labour ratios and in the evolution of the factor income shares.

Then, the following production functions of the two intermediate goods are assumed:

$$Y_i(t) = \left[(1-\alpha_i)\left(A_i(t)\,L_i(t)\right)^{(\sigma_i-1)/\sigma_i} + \alpha_i K_i(t)^{(\sigma_i-1)/\sigma_i}\right]^{\sigma_i/(\sigma_i-1)},$$

where $\alpha_i \in (0,1)$,$\sigma_i \in [0,\infty)$, and $\dot{A}_i(t)/A_i(t) = \gamma_i > 0$ for $i = 1,2$. Without loss of generality, $\sigma_2 > \sigma_1$ can be assumed. Sector 2 is the more flexible sector in that it is easier to substitute in sector 2 than in sector 1. The production function of the consumption good is assumed to be the same as (18). The model can be reduced to Ngai and Pissarides (2007), where structural change is caused by the sectoral differences in the TFP growth rates if $\varepsilon \neq 1, \alpha_1 = \alpha_2, \sigma_1 = \sigma_2, \gamma_1 \neq \gamma_2$, while it can be reduced to Acemoglu and Guerrieri (2008), where structural change is caused by the sectoral differences in the factor intensities if $\varepsilon \neq 1, \sigma_1 = \sigma_2, \gamma_1 = \gamma_2, \alpha_1 \neq \alpha_2$.

For tractability, Alvarez-Cuadrado et al. (2017) paid attention only to the case of $\varepsilon = 1, \alpha_1 = \alpha_2 = \alpha, \sigma_2 > \sigma_1 = 1, \gamma_1 = \gamma_2 = \gamma$ with respect to the dynamics of the model. In other words, the consumption good and intermediate good 1 are produced by the Cobb–Douglas production function and intermediate good 2 is produced by the CES production function; moreover, no sectoral differences in the factor intensities and TFP growth rates exist. Hence, the Baumol disease does not emerge. In this case, any result derived from the model is attributable to the sectoral differences in substitution.

To avoid unnecessary complication and concentrate on the analysis of the supply-side reasons, the model does not formulate the consumer's optimisation problem (the introduction of the problem into the model is straightforward). Since the constant saving rate υ is assumed to be exogenously given, as in the Solow model, the motion of capital accumulation follows $\dot{K}(t) = \upsilon Y(t) - \delta K(t)$.

Combining the motion of capital accumulation with the static optimisation conditions, we can thus obtain the non-linear differential equation with respect to $\kappa \equiv \frac{K_1(t)}{K(t)}$: $\dot{\kappa}(t) = h(\kappa(t))$ with the property of $\frac{\mathrm{d}h}{\mathrm{d}\kappa} < 0$. Therefore, the unique existence and *local* stability of the steady state can be proven, just as in the Solow model. This means the unique existence of the *locally* stable GBGP, along which $Y(t), Y_1(t), Y_2(t), K(t), K_1(t), K_2(t)$ grow at the rate of $n+\gamma$ and $L_1(t), L_2(t)$ grow at the rate of n, while the rate of interest is kept constant. Therefore, Kaldor's facts are obtained.

Structural change ends once the economy reaches the GBGP. It occurs only during the transition towards the GBGP. When the economy starts from K lower than its steady-state value, K grows faster than AL, and thus the sector with the higher elasticity of substitution absorbs more capital and releases labour. This is because that sector tends to substitute the cheaper factor (capital) for the more expensive one (labour) as capital is accumulated and the wage–interest ratio rises. Therefore, the sectoral capital–labour ratios and factor income shares evolve differently in the two sectors. Hence, structural change occurs. However, the coexistence of structural change and Kaldor's facts is not shown in the model. If structural change occurs, then Kaldor's facts cannot be obtained; by contrast, if structural change ceases, then Kaldor's facts are obtained.

5 Reconciliation of the Structural Change Caused by Both Reasons with Kaldor's Facts

In this section, we review mainstream multi-sectoral models which reconcile structural change caused by both the demand-side and the supply-side reasons with Kaldor's facts. Boppart (2014a), Herrendorf et al. (2020), Comin et al. (2020), Guilló et al. (2011), and Meckl (2002) are recent examples of such models.

5.1 Boppart (2014a)

This model has two consumption goods: good (G) and service (S). It is assumed that the household is indexed by $i \in [0,1]$, each of which consists of $N(t)$ identical members, where $N(t) = \exp[nt]$, $n > 0$. Each member of household i is endowed with $l_i \in (\bar{l}, \infty), \bar{l} > 0$ units of labour and labour is supplied inelastically in every period. Therefore, the aggregate labour supply is defined by $L(t) \equiv N(t) \int_0^1 l_i \mathrm{d}i$, the growth rate of which is given by n. Household i, which is indexed by $i \in [0,1]$, has the following intertemporal preferences:

$$U_i = \int_0^\infty \exp[-(\rho - n)t]\, v(p_G(t), p_S(t), e_i(t))\, \mathrm{d}t, \text{ where}$$
$$v(p_G(t), p_S(t), e_i(t)) = \frac{1}{\varepsilon}\left(\frac{e_i(t)}{p_S(t)}\right)^{\varepsilon} - \frac{\eta}{\zeta}\left(\frac{p_G(t)}{p_S(t)}\right)^{\zeta} - \frac{1}{\varepsilon} + \frac{\eta}{\zeta}, \tag{19}$$

where ρ is the rate of time preference, and $\rho > n > 0$ is assumed. (19) is the indirect instantaneous utility function, where $0 \leqq \varepsilon \leqq \zeta < 1$ and $\eta > 0$ are assumed. ε is the parameter affecting the degree of non-homotheticity of the preferences; if $\varepsilon = 0$, the preferences are homothetic. ζ is the parameter affecting the degree of substitutability between G and S. If $\varepsilon = \zeta = 0$, the preferences are reduced to the Cobb–Douglas form. As shown later, η is the parameter affecting the expenditure share on G; if $\eta = 0$, the model is reduced to a one-sector model and the preferences are reduced to the CRRA form. $p_G(t), p_S(t), e_i(t)$ are the price of goods, services, and nominal per-capita expenditure of household i, respectively.

(19) is a preference termed *price-independent generalised linearity*. This makes the aggregation of households' expenditure trivial since the aggregate expenditure share coincides with that of a *representative* household whose expenditure is the same share as that of the aggregate economy. Moreover, it ensures that the representative expenditure is independent of prices (Muellbauer, 1976).

The technology assumptions are similar to those of Ngai and Pissarides (2007):[7]

$$\begin{aligned} Y_j(t) &= A_j(t) F[K_j(t), \exp(gt) L_j(t)], \text{ for } j = G, S, \\ Y_I(t) &= F[K_I(t), \exp(gt) L_I(t)], \end{aligned}$$

where index I denotes an investment good and F is the identical neoclassical production function. The TFP growth rates differ: $\frac{\dot{A}_j(t)}{A_j(t)} \equiv \gamma_j \geqq 0$ for $j = G, S$. The price of the investment good is adopted as the numéraire in each period. As in (13), $\frac{\dot{p}_G}{p_G} - \frac{\dot{p}_S}{p_S} = \gamma_S - \gamma_G$ holds, which means that structural change in prices occurs. In addition, the sectoral factor intensities in terms of the efficiency unit ($k_j(t) \equiv \frac{K_j(t)}{\exp(gt)L_j(t)}$) are uniform in the equilibrium: $k_G(t) = k_S(t) = k_I(t) = k(t)$, where $k(t) \equiv \frac{K(t)}{\exp(gt)L(t)} = \frac{K_G(t)+K_S(t)+K_I(t)}{\exp(gt)L(t)}$ and $L(t) = L_G(t) + L_S(t) + L_I(t)$.

In each period, household i maximises (19) subject to the budget constraint: $e_i(t) = p_G(t) x_G^i(t) + p_S(t) x_S^i(t)$, where $x_G^i(t), x_S^i(t)$ denote the per-capita consumption of goods and services, respectively.[8] Then, their aggregate consumption can be respectively defined as follows: $X_j(t) \equiv N(t) \int_0^1 x_j^i(t) \mathrm{d}i$ for $j = G, S$. Aggregate consumption expenditure can be defined as $E(t) \equiv \int_0^1 e^i(t) \mathrm{d}i = p_G X_G(t) + p_S(t) X_S(t)$. To analyse structural change, define the aggregate expenditure share of good as $\varphi_G(t) \equiv \frac{p_G X_G(t)}{E(t)}$.

Letting $e(t) \equiv \frac{E(t)}{\exp(gt)L(t)}$, capital accumulation is determined by the differential equation, as in Ngai and Pissarides (2007):

$$\dot{k}(t) = f[k(t)] - (\delta + n + g) k(t) - e(t), \tag{20}$$

where $f[k(t)] = F(k(t), 1)$. In addition, we can obtain the differential equation with respect to $e(t)$: corresponding to the Euler equation:

$$\frac{\dot{e}(t)}{e(t)} = \frac{r(t) - \rho + \varepsilon\gamma_S}{1 - \varepsilon} - g, \tag{21}$$

where $r(t) = f'[k(t)] - \delta$ is satisfied by the firms' optimisation.

(20) and (21) constitute the two-dimensional differential system of equations. The method used to solve this system is the same as for the Ramsey optimal growth model. Therefore, we can prove the following proposition:

[7] Boppart (2014a) used the AK production function in the investment good sector to avoid the analysis of transition dynamics. However, we use the standard neoclassical production function in the sector, as shown by Boppart (2014b), to understand the mainstream strategy to reconcile structural change with Kaldor's facts.

[8] As a result, the following consumption demand functions are obtained:

$$x_G^i(t) = \eta \frac{e_i(t)}{p_G(t)} \left(\frac{p_S(t)}{e_i(t)}\right)^{\varepsilon} \left(\frac{p_G(t)}{p_S(t)}\right)^{\gamma} \text{ and } x_S^i(t) = \frac{e_i(t)}{p_S(t)} \left[1 - \eta \left(\frac{p_S(t)}{e_i(t)}\right)^{\varepsilon} \left(\frac{p_G(t)}{p_S(t)}\right)^{\gamma}\right].$$

The Engel curves are non-linear if $\varepsilon < 1$. Moreover, the elasticity of substitution between goods and services is less than unity for all the households in each period thanks to the assumption of $0 \leqq \varepsilon \leqq \gamma < 1$.

Proposition 5 *The two-dimensional differential systems of equations (20) and (21) have the uniquely saddle-path stable steady state* (k^*, e^*)*, given* $k(0) > 0$*, if the transversality conditions,* $\rho - n > \varepsilon\widetilde{g}$*, and* $f'(k^*) > \delta - n - \widetilde{g}$ *are satisfied.*

Proposition 5 indicates the existence of the GBGP, along which the aggregate capital, consumption expenditure, and output grow at the constant rate of $g + n$. Moreover, the capital–labour ratio $(\frac{K_j(t)}{L_j(t)})$ grows at the rate of g in all the sectors, whereas the rates of interest and saving are kept constant along the GBGP. Then, Kaldor's facts are obtained.

Since the steady state value of k is constant along the GBGP, each sector grows at the constantly specific rate. It is also proven that $\frac{\dot{\varphi}_G(t)}{\varphi_G(t)} \leqq 0$ holds along the GBGP, implying that the expenditure share of the good is decreasing. This implies that a structural change in expenditure occurs along the GBGP because the relative price of the good changes along the GBGP and $\varepsilon \leqq \zeta < 1$ is assumed. $\frac{\dot{\varphi}_G(t)}{\varphi_G(t)} \leqq 0$ immediately implies that $\lim_{t\to\infty} \varphi_G(t) = 0$ holds along the GBGP.

By the same logic as in Ngai and Pissarides (2007), however, structural changes in relative prices, output, and consumption persistently occur along the GBGP; the structural changes in output and consumption are verified by the consumption demand functions in footnote 8. Therefore, the persistent coexistence of structural change and Kaldor's facts is ensured.

The novelty of the model is that preference (19) enables us to address the heterogeneity of households. Although the evolution of the macroeconomic variables can be indicated by the *representative* household, each household's behaviour is also understood. It is shown that poorer households, represented by those with lower $e_i(t)$, tend to spend a larger proportion of their income on the good than richer ones.

5.2 Other examples of the reconciliation of the structural change caused by both reasons with Kaldor's facts

In addition to that of Boppart (2014a, 2014b), other multi-sectoral models reconcile structural change with Kaldor's facts.[9]

[9] Here, let us refer to Echevarria (1995, 1997, 2000). Although her model did not focus on Kaldor's facts, it presented an innovative model to address structural change and economic growth, which promoted the emergence of the new research agenda described above. The economy she supposed has three goods; agriculture (indexed by sector 1), manufacturing (sector 2), and services (sector 3). Manufacturing can be both consumed and invested and the remaining goods are pure consumption goods. The preferences are assumed as follows: $U = \sum_{t=0}^{\infty} \rho^t \sum_{j=1}^{3} \left(\alpha_j \ln C_j(t) - \eta_j C_j(t)^{-\beta_j}\right)$, where $\sum_{j=1}^{3} \alpha_j = 1, \alpha_j > 0, \rho \in (0,1), \beta_j > 0, \eta_j \geqq 0$. If at least one of η_j is strictly positive,term $\eta_j C_j(t)^{-\beta_j}$ indicates the demand-side reason for structural change. The evolution of consumption of each good is similar to that obtained by the Stone–Geary type of preferences used in Kongsamut et al. (2001). The advantage of the preferences by Echevarria is that it can avoid some unpleasant features of the Stone–Geary preferences: it cannot be defined for $A(t) < \overline{A}$ in (2). Hence, the interior solution to the static optimisation problem always exists in Echevarria's specification. Moreover, she assumed Cobb–Douglas technologies with sectoral differences in both TFP growth rates and factor intensities. These are the supply-side reasons for structural change. She proved the existence of the GBGP if $\eta_j = 0$ for $j = 1, 2, 3$. In other words, capital in the three sectors, aggregate capital, investment, and consumption of manufacturing grow at the same constant rate. Moreover, consumptions of agriculture

Herrendorf et al. (2020) also constructed a multi-sectoral model of the structural change caused by both reasons. Their model has the same structure as that of Boppart (2014a), except the assumption of the production of an investment good. In other words, the preferences are assumed to take the price-independent generalised linearity form. The characteristic of the model lies on the supply side. In the model, a representative household consumes a good and a service, which are produced by inputting capital and labour using Cobb–Douglas production functions with identical factor intensities but different TFP growth rates. The production functions are defined in terms of the value-added. Although it is often assumed that only the good can be both consumed and invested, the model assumes that the investment good is produced by combining the good and service, the aggregator of which is given by the CES form with exogenously investment-specific technical change. Thanks to this assumption, aggregate output (i.e. the sum of investment and consumption) in the equilibrium can be expressed in the Cobb–Douglas form, with the investment-specific TFP growth rate endogenously determined by the composition of investment input, which in turn depends on the TFP growth rates of the production of the good and service.

Making a set of assumptions on the parameters, the unique existence of the GBGP can be proven, along which the aggregate capital, output, consumption expenditure, and investment grow at the same constant rate determined by the investment-specific TFP growth rate and factor intensity, Moreover, the rate of interest and capital–output ratio are kept constant. Therefore, Kaldor's facts are obtained. Furthermore, structural change in relative prices and consumption expenditure persistently occurs along the GBGP, as in Boppart (2014a). Then, service is the dominant sector in the long run.

Comin et al. (2020) assumed pure reproducible capital (not consumable) and pure consumption goods, all of which are produced by capital and labour under Cobb–Douglas production functions. The TFP growth rates and factor intensities are assumed to differ by sector, which are the supply-side reasons driving structural change. Since a pure capital good is assumed, capital accumulation is determined by a single equation: $Y_m(t) = A_m(t) K_m(t)^{\alpha_m} L_m(t)^{1-\alpha_m} = K(t+1) - (1-\delta) K(t)$, where the subscript m denotes the sector producing the pure capital good.

The preferences in Comin et al. (2020) generalise those in Comin et al. (2018). Here, we review the preferences in the latter version, in which some functional forms are specified. The preferences are given over a bundle of consumption goods $\boldsymbol{C}(t) \equiv (C_1(t), \cdots, C_N(t))$ such that C, an index of real income measuring consumer utility, is implicitly defined through the constraint:

$$\sum_{i=1}^{N} \left(\Omega_i C^{\varepsilon_i}\right)^{\frac{1}{\sigma}} C_i^{\frac{\sigma-1}{\sigma}} = 1, \tag{22}$$

where $\Omega_i > 0, \varepsilon_i > 0$, $\sigma < 1$, and $\varepsilon_i > 0$ for $i = 1, \cdots, N$. The sectoral difference ε_i

and services grow at the different constant rates with each other (see Appendix in Echevarria, 1997). Then, Kaldor's facts are obtained if $\eta_j = 0$ for $j = 1, 2, 3$. Furthermore, she showed that the economy with $\eta_j > 0$ for $j = 1, 2, 3$ *asymptotically* converges to the GBGP. This is because income (and thus consumption) increases, $\eta_j C_j(t)^{-\beta_j}$ becomes negligible. The utility function is getting close to the Cobb–Douglas form for high levels of consumptions. However, the model indicates the persistent coexistence of structural change and Kaldor's facts, due to the same logic of Ngai and Pissarides (2007).

controls for the relative income elasticity of demand and leads the demand functions to be non-homothetic. Let $E = \sum_{i=1}^{N} p_i C_i$ be consumption expenditure. Solving the expenditure minimisation problem subject to (22) yields $C_i = \Omega_i \left(\frac{p_i}{E}\right)^{-\sigma} C^{\varepsilon_i}$. The properties of the *non-homothetic CES* in (22) are that non-homothetic features do not systematically vanish as income (and thus utility) rises (i.e. $\frac{\partial \ln(C_i/C_j)}{\partial \ln C} = \varepsilon_i - \varepsilon_j$, for $\forall i, j = 1, \cdots, N$) and that the elasticity of substitution between different goods is constant (i.e. $\frac{\partial \ln(C_i/C_j)}{\partial \ln(p_i/p_j)} = \sigma$, for $\forall i, j = 1, \cdots, N$).

Each household inelastically supplies one unit of labour and is endowed with a homogeneous asset $A(0)$ in the initial period. The utility function is defined by the index C and assumed to have the CRRA form:

$$U = \sum_{t=0}^{\infty} \rho^t \left(\frac{C(t)^{1-\theta} - 1}{1-\theta} \right).$$

Maximising the utility function subject to the budget constraint yields the Euler condition. Capital accumulation follows the equation shown above. Therefore, we obtain the two-dimensional differential system of equations with respect to $C(t)$ and $K(t)$.

Further imposing the assumptions and conditions, the unique existence of the GBGP for any initial value of capital stock is proven. It is shown that along the GBGP, i) the real rate of interest is constant; ii) aggregate consumption expenditure, total nominal output, and capital stock grow at the same constant rate; and iii) the dominant sector in the long run emerges. Then, Kaldor's facts are obtained.

Although the dominant sector surviving in the long run is determined solely by the TFP growth rate in Ngai and Pissarides (2007), it is also determined by the income elasticity of demand ε_i and sectoral factor intensity α_i in this model.

6 Structural Change and Kaldor's Facts from Cambridge Keynesian Perspectives

So far, we have extensively reviewed the mainstream multi-sectoral models that attempt to reconcile structural change with Kaldor's facts. Although there are some exceptions,[10] we can

[10] For example, Muro (2017) presented a growth model with three factors and three goods and attempted to reconcile structural change with Kaldor's facts. The author proved the unique existence of the GBGP and (sufficient) conditions for saddle-path stability by transforming the three-good model into a type of two-sector optimal growth model à la Uzawa (1964). The structural change was caused by both non-homothetic preferences and sectoral differences in factor intensities. Second, Hori et al. (2015) constructed an endogenous growth model with two sectors. Structural change is driven by the demand-side reason in the model, which assumes that utility depends on not only the level of consumption but also the reference levels of consumption represented by the stock of external habits. External habits cause endogenous growth, and these make the Engel curves non-linear. We obtain the three-dimensional differential system of equations. Although the model obtains the various types of equilibria, depending on the values of the parameters, it shows the possibility of the existence of the local saddle-path stable steady state. Third, Alonso-Carrera and Raurich (2018) built a two-sector model reconciling structural change with Kaldor's facts. In their model, both

confirm the mainstream strategy of the reconciliation: holding Kaldor's facts means the state at which the aggregate variables such as capital, output, and consumption expenditure grow at constant rates, and the multi-sectoral models are transformed into the one-sector growth model that has a unique (saddle-path) stable steady state. This transformation guarantees the existence of the GBGP.

Herrendorf et al. (2014) characterised mainstream multi-sectoral models of structural change as 'a natural extension of the one-sector growth model that incorporates structural transformation'. The dynamics of the one-sector growth model transformed from multi-sectoral models can be described by, at most, a two-dimensional differential system of equations.

As confirmed in Section 1, Kaldor's facts can be considered to be empirical regularities of *long-run* economic growth, at least in advanced economies. To the best of our knowledge, there is no research arguing that the facts have already lost its relevance to the analysis of economic growth. The problem to address here is whether the mainstream strategy is consistent with Kaldor's own thoughts. In this section, we evaluate the mainstream strategy from the Cambridge Keynesian perspective (Pasinetti, 2007), in which Kaldor's own thoughts are included. We first argue that the mainstream strategy is entirely inconsistent with Kaldor's own thoughts. Second, we indicate that the mainstream strategy overlooks another important structural change seen from the Cambridge Keynesian perspective. To allow the alternative strategy to reconcile structural change with Kaldor's facts, moreover, we criticise such a mainstream characteristic of the reconciliation that crucially relies on a perfect adjustment mechanism through markets.

The Cambridge Keynesian group was formed after the Second World War and its founders were the pupils of Keynes, such as Kahn, J. Robinson, Sraffa, Kaldor, and others. According to Pasinetti (2007, pp. 219–237), the features of Cambridge Keynesians can be summarised as follows:

1. *Reality (not simply abstract rationality) as the starting point of economic theory*;
2. *Economic logic with internal consistency (not only formal rigour)*;
3. *Malthus and the Classics (not Walras and the Marginalists)*;
4. *Non-ergodic (in place of stationary, timeless) economic systems*;
5. *Causality vs. interdependence*;
6. *Macroeconomics before microeconomics*;
7. *Disequilibrium and instability (not equilibrium) as the normal state of industrial economies*;

reasons for structural change are included: the minimum consumption requirement for agriculture and sectoral differences in both the TFP growth rates and the factor intensities. It takes into account the additional reason to affect structural change: sectoral differences in wages. It is argued that the additional reason is required to account for the relationship between the sectoral evolution of output and that of employment in the United States. They obtain the four-dimensional differential system of equations with three state variables. The unique existence of the saddle-path stable steady state is proved.

8. *Need of finding an appropriate analytical framework for dealing with technical change and economic growth*;
9. *A strong, deeply felt social concern.*

According to him, these features are the most important of Cambridge Keynesians, as they are not always shared by all members and not exactly found in their works.

6.1 Mainstream interpretation of Kaldor's facts

First, we must confirm Kaldor's statement: 'none of these "facts" can be plausibly "explained" by the theoretical constructions of neo-classical theory' (Kaldor, 1961, p. 179). However, mainstream economists assert that Uzawa (1961) proved that the neoclassical growth model with Harrod-neutral technical progress can account for Kaldor's facts.[11]

Although the basic idea of Kaldor (1961) had already been presented at the conference on the theory of capital held on Corfu in 1958, it is inconceivable that Kaldor did not know about Uzawa's paper when completing his paper because Kaldor was the chair of the editorial committee of the *Review of Economic Studies* when the paper was published in that journal. In fact, Kaldor was one of the most distinguished figures of the journal, verified by the fact that he served as chair from Vol. 9, No. 1 (1941) to Vol. 28, No. 3 (1961). Therefore, we conjecture that Kaldor must have known Uzawa's paper, but considered that it failed to explain the facts.

Kaldor (1961) noted the inherent logical difficulties of defining capital using the neoclassical production function and criticised the smooth substitutability between capital and labour as an unrealistic assumption. Instead, he assumed strict complementarity between capital and labour that, according to him, has more affinity with the classical economics of Ricardo and Marx as well as Neumann (1945) model (Lutz and Hague, 1961, pp. 289–403; Kaldor, 1975). Moreover, he asserted that marginal productivity has no relevance in determining the share of factor income.

In addition, Kaldor (1957) had already pointed out that the 'constancies' of the capital–output ratio, profit rate, and profit share are observed in many advanced economies, stating that 'existing theories are unable to account for such constancies except in terms of particular hypotheses (unsupported by any independent evidence), such as the unitary-elasticity of substitution between Capital and Labour, or more recently, constancy of the degree of monopoly or the "neutrality" of technical progress'. The multi-sectoral models reviewed in the preceding sections are transformed using the CES and Cobb–Douglas functions. Moreover, they assume Harrod-neutral technical progress somewhere. Otherwise, the steady state does not exist in the neoclassical growth model, as Uzawa (1961) showed. However, Kaldor explicitly stated that he was trying to get away from the rigid idea that if the capital–output ratio remained constant, this was caused by a peculiarity of technical progress—by its 'neutrality'

[11] Jones and Romer (2010) argued that there is no longer any interesting debate about the properties of Kaldor's first five facts that a model must contain to explain them; only fact 6 continues to have analytical relevance today. Moreover, they asserted that ideas, institutions, populations, and human capital are important to the growth of modern advanced countries.

(Lutz and Hague, 1961, p. 370). Thus, it is impossible to consider that Kaldor was satisfied with the neoclassical explanation of the facts based on Uzawa (1961).

Furthermore, the Cambridge Keynesian feature 1, namely *Reality (not merely abstract rationality) as the starting point of any economic theory*, means that 'any theory needs to be based on factual evidence, to be evaluated right from the start and not only to be empirical tested at the end' (Pasinetti, 2007, p. 220). According to him, Keynes expressed the feature by stressing that his analysis refers to a 'monetary theory of production' (Keynes, 1973, pp. 253–255) and Kaldor did so by proposing 'stylised facts'. Kaldor used the term to mean 'empirical regularities that are sufficiently general and persistent as to be able to capture the corresponding objective features of reality' (Pasinetti, 2007, p. 220) but never intended the facts to be used to justify the balanced growth achieved by forms of production functions with a smooth substitution between the factors of production and Harrod-neutral technical progress (or the coincidence that capital-saving and labour-saving technical progress happen to precisely offset one another, as Kaldor (1957, p. 593) said). Therefore, the mainstream strategy is also inconsistent with Cambridge Keynesian thought.

Additionally, it is unclear how Kaldor considered the relationship between structural change and the facts. However, he recognised that economic growth entails structural change. He observed that aggregate capital–output ratios have great stability in the long run in many developed countries, stating that 'this stability in over-all ratios concealed much variation over time in the capital-output ratios of individual industries and particular sectors of the economy'. The coexistence of a stable aggregate ratio and variable sectoral capital–output ratios was considered to be 'very puzzling' (Lutz and Hague, 1961, pp. 339–340).

Furthermore, Kaldor (1984) constructed a two-sector growth model featuring agriculture (corn) and manufacturing (steel). The interrelation between these two sectors is the same as in Marx's (1967) schema of reproduction in that corn is the wage good and steel is a pure capital good used in both sectors. He showed that the equilibrium growth rate of the two sectors is equal. However, Kaldor (1984, p. 45) added, 'In practice, the growth of industry is likely to be greater than the growth of agriculture because the income elasticity of demand for manufactured goods is higher than the income elasticity of demand for agricultural goods'. The demand-side reason for structural change is indicated here.

6.2 Mainstream treatment of structural change

Feature 1 implies a further inconsistency with Cambridge Keynesians. Mainstream economics only cares about how well the data fit empirical tests or model predictions when the models assume the Cobb–Douglas or/and CES production function. On the basis of the Cambridge Keynesian perspective, however, theoretical research that only cares about the empirical test or model predictions is insufficient

First, the mainstream strategy of the reconciliation overlooks another important structural change: changes in the sectoral composition of physical capital. The questions at the starting point of model building include 'what is capital' and 'of what does capital consist'. The *realistic* answer is that a large part of capital consists of a bundle of heterogeneous and reproducible commodities.

Remember that all the mainstream multi-sectoral models reviewed so far assume that capital is reproducible but consists of a single commodity. In other words, the mainstream models consider capital to be homogeneous.[12] Homogeneous capital is an important assumption for mainstream economics. If capital consists of a bundle of heterogeneous and reproducible commodities, one cannot aggregate such commodities without them being multiplied by prices. If heterogeneous capital goods exist, the aggregate value is defined as $K(r) \equiv \sum_{j=1}^{n} p_j(r) K_j(r)$, where $K_j(r)$ denotes the amount of commodity j used as the capital input, which is produced by the neoclassical production function $F_j(K_{1j},, \cdots, K_{nj}, L_j)$ with $F_j \neq F_i$ if $j \neq i$. As is well known (see Burmeister, 1980, pp. 122–123), $\frac{\mathrm{d}K}{\mathrm{d}r} < 0$ does not necessarily hold. In other words, the general monotonically decreasing relation between capital intensity and the interest rate no longer holds unlike in the one-sector model in which single homogeneous capital exists.

This gives rise to difficulties proving the unique existence of the steady-state rate of interest (profit) and convergence of the economic system to the steady state. We can obtain the condition for the existence of the index of aggregate capital K and aggregate production function $F(K)$ satisfying all the properties of a well-behaved neoclassical production function around the steady state, as Burmeister (1980, pp. 131–134) showed, although it is restrictive. More importantly, the aggregate production function is only useful for comparing alternative steady-state equilibria and cannot be used to study more general dynamic paths (Burmeister, 2000).

Mainstream economics tends to focus only on the heterogeneity of consumption goods and exclude that of capital goods in the dynamic analysis because the more capital goods are included in the model, the higher the dimension of the differential system of equations that must be analysed. In general, the number of consumption (capital) goods corresponds to the control (state) variable. As the number of state variables increases, the stability analysis (in particular, the proof of global stability) becomes harder and more complicated, as shown by Alonso-Carrera and Raurich (2015) and Acemoglu and Guerrieri (2008) than when only one state variable exists (Stijepic, 2011, p. 104).

In other words, mainstream economists avoid assuming heterogeneous capital goods for purely technical reasons. Moreover, the Ramsey optimal growth model, into which many mainstream multi-sectoral models of structural change are transformed, had been regarded as a normative one but has recently been used to describe actual phenomena observed in the market. This seems to reflect the mainstream confidence of adjustment mechanism through markets, reinforced from the 1980s, as Pasinetti (2018) argued. The optimal growth path obtained by the Ramsey model, to which optimal control theory is applied, coincides with the decentralised market equilibrium (Barro and Sala-i-Martin, 2004). How stringent conditions are required for the existence of the market equilibrium is seldom examined seriously. The mainstream strategy is chosen from the viewpoint of purely *abstract rationality*, which is clearly against Cambridge Keynesian feature 1.

[12] Herrendorf et al. (2015) argued that the structural change in the agriculture, manufacturing, and services sectors in the United States after the Second World War can be sufficiently captured by Cobb–Douglas production functions with sector-specific TFP growth rates. To estimate the production functions, however, exogenous exponent capital is assumed. That is, the existence of heterogeneous and reproducible capital is not taken into consideration.

The Cambridge Keynesian features reflect that industrial production, not exchange, is the dominant activity. The emergence of classical economics corresponds to this, and thus classical economists adopted the *production paradigm* (Pasinetti, 1977, 1981, 1986, 2007). The industry, which does not need to be confined to only manufacturing, is a dynamic concept, meaning that every component of the economic system continues to change forever (Pasinetti, 1981, 1993, 2007). Solow (2014) stated that multi-sectoral and one-sector growth models are complements, not rivals. Pasinetti objected to his opinion and said:

> It [structural dynamic approach to growth] goes much beyond complementarity, to such an extent that, when we consider the passage of time, one-sector approach becomes incompatible with structural analysis. Essentially, the two approaches embody two different *visions* of the industrial world. The *vision* behind structural dynamics originates from the consideration of permanently *evolving* economic system. The *vision* behind the aggregate model of traditional growth model embodies a static, or at most a stationary, view of the economic system, $\cdots$ (Pasinetti, 2014, p. 284)

The statement implies that Pasinetti denies the mainstream perception that multi-sectoral models are a natural extension of the one-sector model.

Are the properties of the one-sector model desirable? Burmeister and Turnovsky (1972) stated so because this model exhibits capital deepening when the steady state having a higher rate of interest (profit) is compared with that having a lower rate. Here, capital deepening can be defined as the increase in the physical capital–labour ratio. In the *regular* economy (Burmeister and Turnovsky, 1972; Burmeister, 1980), lower steady-state rates of interest (profit) are always associated with higher steady-state consumption. This *neoclassical parable* has 'considerable heuristic values in giving insights into the fundamentals of interest theory' (Samuelson, 1962, p. 193). This parable must not be violated in mainstream economics. Furthermore, as Burmeister (1980) and Hahn (1966) clarified, there is the inconvenience for mainstream economics that the stability of equilibria is difficult to obtain in models with heterogeneous capital goods.

If capital consists of a bundle of heterogeneous and reproducible commodities, capital deepening cannot be defined physically, as already pointed out. Therefore, the parable holding in the one-sector growth model cannot be necessarily generalised to multi-sectoral models with heterogeneous capital goods. Since phenomena inconsistent with the parable is undesirable, mainstream economists call them 'paradoxical', 'perverse', 'exceptional', and 'anomalous' (Pasinetti, 1966, 2000).

In the production paradigm, capital is not acceptable as the primary and homogeneous factor of production. There is *no* intrinsic reason to believe that the parable holding in the one-sector growth model is desirable, given that capital in reality consists of a bundle of heterogeneous and reproducible commodities such as equipment, machinery, buildings, facilities, and vehicles (see also Hagemann and Scazzieri, 2009, with respect to capital structure and economic dynamics). The importance of capital as a bundle of heterogeneous and reproducible commodities was recognised by classical economists and Marx. Economic growth in capitalist economies, as Marx (1965, p. 622) clearly understood, is typically characterised by the reduction in the mass of labour in proportion to the mass of the means of production moved by labour. The fact that some means of production are composed of heterogeneous and reproducible commodities (i.e. past labour) was also clearly understood by Marx, so that labour productivity rises in the growth process.

Indeed, much research insists on the importance of the heterogeneity of capital goods in economic growth. First, the composition of heterogeneous capital changes as the economy grows. For example, Nomura (2004, p. 155) showed that the proportion of construction to total capital stock declined by about 13% in real terms in Japan from 1960 to 2000 and that the average growth rate of construction (5.8%) is lower than that of total capital stock (6.8%) during the same period. On the contrary, the proportions of general instrumentation and electric machinery tend to increase and their average growth rates are much higher than that of total capital stock.[13] These results imply that the composition of physical capital changes as income grows.

Mutreja (2014) asserted that the relation between the composition of physical capital and income differences has lacked scholarly attention while the relation between the capital–output ratio and income differences has been closely analysed. She demonstrated that the composition of physical capital is systematically related to income, which, according to her, is an important factor in explaining income differences across countries.[14] She also showed that cross-country differences in equipment capital are much larger than those in structure capital; the equipment capital–output ratio is a factor of approximately 7 between rich and poor, while the structure capital–output ratio is a factor of only 3. These results should be considered carefully when we pay attention to fact 6, although Jones and Romer (2010) asserted the effects of ideas, institutions, and human capital.

Moreover, the cross-country dispersion in the equipment capital–output ratio has also increased over time, while the dispersion in the structure capital–output ratio has declined. Moreover, standard growth accounting has attributed a larger proportion of income differences to the TFP differences in the models that excluded heterogeneous capital goods. Mutreja's (2014) results imply that when a country's income is rising, the composition of physical capital changes in such a way that the proportion of equipment capital to aggregate capital increases. These results strongly support the importance of the existence of heterogeneous capital goods in the analysis of structural change.[15]

The importance of structural change in physical capital is reinforced by the finding that

[13] As Herrendorf et al. (2014) pointed out, the change in capital structure considerably differs depending on whether it is measured in real or nominal terms. In general, the proportions are more stable when measured in nominal terms.

[14] Caselli (2005) also showed the effect of the capital composition on income differences.

[15] Mutreja et al. (2018) also constructed a multi-sectoral and multi-country Ricardian model with economic growth to investigate the effect of the international trade of capital goods through two channels (capital formation and TFP) on economic development. There are consumption goods, intermediate goods, capital goods (machinery and equipment), and structure (other manufacturing) in the economies. Intermediate and capital goods are tradable. It was shown that importing capital goods from developed countries allows developing ones to access more efficient technologies for capital goods. This reduces the relative price of capital goods in developing countries and increases their investment rates as well as the steady-state capital–output ratio. Combining this with the comparative advantage raising the TFP relatively, the income differences between developed and developing countries are reduced.

skilled labour is more complementary to *equipment* capital than unskilled labour (e.g. Krusell et al., 2000). This implies that unskilled labour is easily substituted as the growth in the stock of equipment capital increases. Therefore, as the share of equipment capital increases, as Mutreja (2014) pointed out, the inequality of labour income between skilled and unskilled labour grows. This phenomenon is termed *capital–skill complementarity* (Hornstein et al. 2005; Griliches, 1969). Although Krusell et al. (2000) did not treat the heterogeneity of physical capital in their model, they found that the stock of equipment capital measured in efficiency units grew at about twice the rate of both capital structures and consumption in the post-Second World War period, with its growth rate accelerating since the late 1970s. Hence, the importance of analysing the change in the composition of physical capital is increasing.

In addition, the importance of the structure of physical capital in economic growth and income distribution is supported by Piketty (2014, p. 22), although some of the conceptual and methodological problems with his definition of capital were pointed out by Garbellini (2020), who showed the evolution of the prices and quantities of various capital goods (e.g. construction, housing, equipment, and metal products and machinery). Models omitting heterogeneous capital goods might worsen the analysis of capitalist economies. Moreover, Herrendorf et al. (2014, p. 901) confessed that the mainstream multi-sectoral models that have the GBGP are overly stringent. The validity of some assumptions and conditions made on the parameters are seldom examined fully. Hence, it is a pity that the mainstream avoidance to include the existence of heterogeneous capital goods into their models comes from the above-mentioned technical (mathematical) reasons.

Second, we pay attention to the mainstream characteristic that the reconciliation is achieved entirely by the adjustment mechanism through markets. Mainstream multi-sectoral models assume perfect competition and no rigidities in markets. The adjustment mechanism through markets means that changes in (factor) prices guarantee that demand and supply are equal in the markets. Mainstream economics believes that Kaldor's facts are obtained as a result of the adjustment mechanism working perfectly. The reliance on the market mechanism has become stronger over time, especially since the collapse of socialist economies. Now, such a reliance is the common belief of mainstream economics. Indeed, the mainstream strategy is consistent solely with the institution of perfect competition, which does not exist in any economies in the real world.

Pasinetti's structural economic dynamics include both the demand-side and the supply-side reasons behind structural change, and heterogeneous and reproducible capital goods are also taken into consideration. The novelty of Pasinetti's structural economic dynamics is the capability of addressing the 'institutional problem' as well as structural change, thanks to the separation theorem (Pasinetti, 2007). Here, the institution means a set of procedures, rules of individual and social behaviour, regulations, and administrative bodies. The institutional problem of a society is to 'construct(ing) its institutions, adopting and modifying them as time goes on, perfecting them and ... even discarding some of them, while inventing new ones' (Pasinetti, 2007, p. 306). Furthermore, the institutional problem is solved to search for and lead to the *natural* positions within relevant and reasonably acceptable span of time (Pasinetti, 1993, p. 117). Put another way, it is solved to obtain a set of the natural magnitudes indicated below and fulfill the effective demand condition (Pasinetti, 1981, 1993). Obviously, fulfilling the condition is one of the policy objectives of almost all governments.

The first stage of investigation in the theorem is to construct the natural economic system, which is an abstract economic system defined independently of any institution as well as the cultural, geographical, and historical circumstances. The natural economic system represents a logically ideal state of economies. Hence, it comprises a set of the natural magnitudes, namely natural prices, natural quantities, natural rates of profit, natural wage rate, and natural rate of interest. If we mention the *ideal* level of employment, which might be called the natural level of employment, it must be the level of full employment. Then, the natural economic system possesses the normative property.

To analyse economies under the specific institution, we must depart from the natural economic system. As Pasinetti (1981, pp. 151–152) argued, the natural rates of profit are inconsistent with the capitalist economic system, since the competition leads them to equalise and consumption of capital owners must be positive in the system. Thus, he argued that such rates should be replaced with the equilibrium profit rate given by the Cambridge equation (Pasinetti, 1974, Chaps. 5 and 6): $\pi_e = \frac{1}{s_c}(g + r^*)$, where s_c, r^* denote capitalists' saving rate and the weighted average growth rate of per-capita demand, respectively.[16]

According to our understanding, the essential features of the separation theorem is

> the theoretical schemes erected at the first stage of investigations cannot be closed. They must contain a sufficient number of degrees of freedom to allow the insertion of whatever type of rules of behaviour that may then emerge from carrying out the second (the more practically oriented, more down-to-earth) stage of investigation (Pasinetti, 2007, pp. 276–277).

In other words, the solution of the natural economic system must be *indeterminate*, meaning that the natural magnitudes above are obtained as the functions with respect to certain parameters. In the second stage of investigation, the model is closed by determining these parameters. In other words, solving the institutional problem is nothing to determine the parameters (Kurose, 2018).[17] Such determination is open to all disciplines, not only economics. Therefore, we no longer need to cling to rigid formulations such as maximisation and minimisation.

The important implication derived from Pasinetti's structural economic dynamics and the separation theorem is that the institutions *must* be changed to maintain full employment. The institutional problem is not a once-and-for-all condition, and must be solved to maintain the effective demand condition over time since the fulfilment does not entail an automatic self-adjusting process in structural economic dynamics.

[16] The equilibrium rate of profit is obtained under the assumption of $\frac{P_c}{S_c} = \frac{P_w}{S_w}$, where P, S denote the amount of profit and saving, respectively, and subscripts c, w denote the capitalist and worker. Under what conditions it holds in structural economic dynamics must be scrutinised.

[17] In this sense, the separation theorem positively attaches a significance to the indeterminacy of equilibria, although it is in general interpreted as indicator of model misspecification and thus detested in mainstream economic theories. The indeterminacy of equilibria has a significant implication for neo-Ricardian economists. This is because it enables them to assert that income distribution cannot be determined solely by the demand-supply mechanism. Furthermore, they argue that some outside and non-economic factors, such as the collective bargaining by labour union, are necessary to determine the income distribution and relative prices. See Kurose (2018) and Kurose and Yoshihara (2019) concerning this point.

Since per-capita demand growth eventually reaches saturation and productivity growth rates are generally increase, the effective demand condition is unlikely to be satisfied. This is represented by Cambridge Keynesian feature 7: Disequilibrium and instability. Hence, the coordination problem necessarily occurs. One of the solution to the problem is given by the institutional changes.

It would be relevant to consider the possibility of the effects of the institutional changes on reconciling structural change with Kaldor's facts. The income distribution, which is related to fact 5, is not entirely determined by the market forces. It is obviously affected by the institutional factors in Pasinetti's sense. In particular, the labour market regulations greatly have an effect on the income distributed to workers, and the labour market institutions have been changed in many countries (e.g. Boeri, 2011; MacLeod, 2011).[18]

As Arena (2017, pp. 116–117) argued, it is difficult for Pasinetti's structural dynamic model to analyse the standard mathematical stability of equilibrium, because no adjustment mechanism is built into the model. However, this is not necessarily a disadvantage of Pasinetti's structural economic dynamics. Instead of the analysis of the stability, the separation theorem has the potential to conduct institutional analysis, which is impossible for mainstream economists. The relationship between the reconciliation and the institutional changes remains an open question.

7 Concluding Remarks

Kaldor's facts are considered to be 'empirical regularities that are sufficiently general and persistent'. In this study, we review mainstream multi-sectoral models in which structural change is reconciled with Kaldor's facts. The mainstream strategy of reconciliation is that holding the facts is regarded as the state at which the economy grows along the GBGP and the multi-sectoral model is transformed into the one-sector growth model (e.g. the Ramsey and Solow models) that has the uniquely (saddle-path) stable state. In this sense, mainstream economists consider multi-sectoral models to be a natural extension of the one-sector growth model.

We argue that the mainstream strategy is far from Kaldor's own thoughts and the Cambridge Keynesian perspective. Moreover, the mainstream analysis of structural change overlooks another important structural change, namely, changes in the sectoral composition of physical capital, because all mainstream multi-sectoral models assume homogeneous capital. As is shown by Mutreja (2014), structural change in physical capital is systematically related to income. Therefore, structural change in physical capital is an important characteristic of economic growth that should not be omitted from theoretical studies.

Moreover, the attempt to reconcile structural change with Kaldor's facts from the Cambridge Keynesian perspective is expected to pave the way for institutional analysis to be required for maintaining full employment. Such analysis can be feasible only by adopting

[18] In this context, parameters μ and ν appearing in the effective demand condition in Pasinetti (1981, 1993), which respectively denote the proportion of the active to the total population and the ratio of working hours to the total number of hours forming the unit of time considered, are of great significance. The fulfillment of the condition over time requires for the parameters to change in Pasinetti's structural economic dynamics.

the Cambridge Keynesian perspective since the mainstream strategy assumes a perfect adjustment mechanism. Adopting the Cambridge Keynesian perspective to reconcile structural change with Kaldor's facts is also an attempt to connect economic theory with institutions. As Pasinetti (2020) argued, this attempt ought to be paid more attention in economics.

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