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The First Publication of Economic Manuscripts in
MEGA and New Aspects of Marx's Economic Theory:
Marx's Six-Sector Model and Its Theoretical
Implications

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Discussion Paper

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1. Background: Multisectoral Analysis in 1858

1.1. The world economic crisis and *Grundrisse*

The economic crisis of 1857, the first *world* economic crisis in the human history, is known as the historical event that drove the contemporary Marx into strenuous empirical research. He collected and systematically classified an amount of various economic data from all parts of the world. They were classified according to the countries and topics, then ordered chronologically. They cover countries as France, Italy, Spain, United Kingdom, Germany, Austria, United States, China, India, Egypt and Australia, and topics as financial market data (interest rates, share and security prices, bank balance, etc.), commodity market data (prices and sales of agricultural and industrial products etc.), and bankruptcies, unemployment, short-time working, wages, labour disputes etc. This voluminous research on the economic crisis of 1857 reached a total of 159 manuscript pages, which would correspond about 500 print pages, and were known as '*Books of Crisis*'. The *Books of Crisis* has just been published for the first time in MEGA, Part IV, Volume 14 (in the following: MEGA IV/14), which was edited by a team of German and Japanese editors including myself. In parallel with these *Books of Crisis* and based on them, Marx published research results on phenomena and causes of the crisis in seven articles of '*New York Daily Tribune*' (in the following: *NYDT*), which are now being edited for MEGA I/16.

As Mori (2017) examined in detail, Marx' *Grundrisse* of 1857-58, the so-called rough draft of *Capital*, can be seen as direct extension and generalization of research and analysis in his *Books of Crisis* and articles of *NYDT*. In the face of the world economic crisis, *Grundrisse* was written under the description plan whose goal is the explanation of 'world market and crises'. As well known, the explanation was planned to be made in the following five steps¹:

(1) *the general, abstract determinants which obtain in more or less all forms of society...*

¹ MEGA II/1.1, S. 43.

(2) the categories which make up the inner structure of bourgeois society and on which the fundamental classes rest. Capital, wage labour, landed property. Their interrelation. Town and country. The three great social classes. Exchange between them. Circulation. Credit system (private).

(3) Concentration of bourgeois society in the form of the state. Viewed in relation to itself. The 'unproductive' classes. Taxes. State debt. Public credit. The population. The colonies. Emigration.

(4) the international relation of production. International division of labour. International exchange. Export and import. Rate of exchange.

(5) the world market and crises.

The description of *Grundrisse* was obviously limited at most to the first and a part of the second step. However, various kinds of crisis literature were already critically reviewed there, especially in the second section 'Circulation process of capital', where J. R. MacCulloch, J. Mill and J.S. Say were harshly criticized for ignoring specific characteristics of the capitalistic production or identifying supply and demand so that they denied the general overproduction².

While criticizing the doctrines denying the possibility of general overproduction as 'stupidity'³ or 'childish'⁴, Marx tried to show precisely what he meant by his 'general overproduction', which must be different from the partial overproduction à la Say where 'at most, too much has been produced of one and too little of another'⁵. In his effort to define the general overproduction unambiguously, Marx thought up a sort of input-output table, certainly one of the oldest ones in the history of economic thought⁶. To be exact, this multisectoral analysis of Marx was motivated besides by his effort to define the general overproduction, also by his intention to thoroughly disprove Proudhon's doctrine on the profit according to which the profit arises because capitalists overcharge workers in selling their products.

1.2. Marx' first input-output table

Motivated in this way, Marx drew up the following table⁷:

² In the same sense, S. Hollander summarized this section of *Grundrisse* in his commentary on it: 'while there is no concerted discussion of the business cycle or of the structural relations that must be satisfied to allow the system to reproduce itself or to expand successfully, these issues were already on the agenda' (Hollander, 2008, p.289).

³ MEGA I/1.2, S.336.

⁴ MEGA I/1.2, S. 324.

⁵ MEGA I/1.2, S. 336.

⁶ So far as we know, Maurice Potron, a French Mathematician, was the first to calculate in fact input coefficients in 1913 prior to W. Leontief. See Mori (2008).

⁷ MEGA I/1.2, S.352.

Table 1 Marx' input-out table

	For labour	Raw material	Machinery	Surplus product	
A)Raw material manufacturer	20	40	20	20	=100
B)Ditto	20	40	20	20	=100
C)Machinery manufacturer	20	40	20	20	=100
E)Workers' necessaries	20	40	20	20	=100
D)Surplus producer	20	40	20	20	=100

This table has following properties:

- The economy consists of 5 production sectors (A, B, C, D, E): two sectors of raw material manufacture, machinery sector, wage-good ('workers' necessaries') sector, capitalists'-consumption-good ('surplus product') sector.
- Means of production for each sector consists of raw material and machinery.
- Workers and capitalists consume respectively products of E ('workers' necessaries') and products of D ('surplus products') exclusively.

One can read the table in the following manner. Each row of A, B, C, E and D represents price components of each sector's output; the row of A, for example, shows that the raw material of value of 100 is produced by paying wage (i.e. buying wage good) of 20, and using raw material of 40 and machinery of 20, and paying surplus value (i.e. buying surplus product) of 20. On the other hand, each column from 'for worker' to 'surplus product' represents the demand for each good; the column of 'for worker' shows that the wage good ('workers' necessaries') of value of 20 is sold to each of sectors A, B, C, E, D. In Marx' table, contrary to the usual input-output table, rows and columns represent respectively the buyers and sellers. The table represents an equilibrium in the sense that for each good the output equals to the real demand. Furthermore, it stands for an equilibrium state of simple reproduction because all surplus value is individually consumed by capitalists.

To translate Marx' table into the conventional form of input-output tables by reversing the row and column and separating households of workers and capitalists as final demand sectors from intermediate (industrial) sectors, we obtain the following transformed table of Marx. Note that we cannot separate sectors A and B because we cannot know how much each sector purchases from each other.

Table 2 Translation of Marx's input-output table

	A,B) raw material	C) machinery	E) wage good	D)capitalists' good	workers' household	capitalists' household	total demand
A,B) raw material	80	40	40	40	0	0	200
C) machinery	40	20	20	20	0	0	100
E)wage good	0	0	0	0	100	0	100 (d_E)
D)capitalists' good	0	0	0	0	0	100	100 (d_D)
wage	40	20	20	20			
surplus value	40	20	20	20			
product value	200	100	100 (x_E)	100 (x_D)			

Marx drew the following conclusions from his numerical example:

- 1) The proportion in which product value of each sector consists of wage, surplus value and the value of raw material and machinery on the one hand and the proportion in which the surplus value is divided into consumption and investment in additional inputs on the other hand determine the proportion of product value among sectors. The former proportion changes with development of productive forces⁸.
- 2) The general overproduction takes place if both sectors E und D produce too much, i.e. the product value of E and D is greater than the total demand for E and D respectively so that $x_E > d_E$ and $x_D > d_D$ ⁹.
- 3) While capitalist in E can realize his whole profit through the direct exchange with workers, capitalists in all the other sectors can realize their value only through the exchange among themselves¹⁰.

⁸ MEGA II/1.1, S.352-4: 'At a given point in the development of the productive forces ... a fixed relation becomes established, in which the product is divided into one part – corresponding to raw material, machinery, necessary labour, surplus labour – and finally surplus labour divides into one part which goes to consumption and another which becomes capital again. This inner division, inherent in the concept of capital ... will appear in the exchange process as distribution among, say, 5 capitals' (MEGA II/1.1, S.353-4).

⁹ 'It is clear here that if D and E, where E represents all commodities consumed by the workers and D all those consumed by the capitalists, had produced too much– that is, too much relative to the proportion of the part of capital going to the worker, or too much relative to the part of capital consumable by the capitalists...then general overproduction would take place...' (MEGA II/1.1, S.353)

¹⁰ MEGA II/1.2, S.350, S.351-2; 'The case we have posited, where capital E realizes the whole of its profit in exchange with wages, is the most favourable– or expresses, rather, the only correct relation in which it is possible for capital to realize the surplus value created in production through exchange with the workers' consumption. But capitals A, B, C, D can realize their value in this case only

1.3. Limits of multisectoral analysis in *Grundrisse*

On these conclusions, we need to make some remarks and point out also some open questions: as for the first conclusion, Marx emphasized that the proportion of sectors depends in particular on the rate of surplus labour to necessary labour, the so-called the rate of exploitation (or the rate of surplus value), so that the former changes as the latter changes. Marx in fact tried to calculate (in vain) how the proportion of sectors changes as the composition of product value changes. However, according to Marx' input-output table, the proportion of sectors cannot be determined when the rate of surplus value once changes. This is because, as mentioned above, it is not known how much raw material from A the production of wage good and capitalists' consumption good each use, and how much from B. Therefore, Marx' table was not well defined yet to allow a 'comparative statics' with the rate of surplus as a varying parameter.

As regard the second point, Marx regarded the general overproduction as disproportion between supply and demand for consumption goods. Even if the demand for production goods is adequate for their supply, this adequacy is just an illusion ('Schein') in so far as the supply of consumption goods exceeds their demand¹¹, e.g. due to saving. This insight was already presented in the chapter of money¹², and the overproduction (shortage of demand) was there famously linked to the existence of money itself: 'The separation of exchange into purchase and sale makes it possible for me to buy without selling (stockpiling of commodities) or to sell without buying (accumulation of money)',¹³. However, a closer investigation about what concrete forms the saving (money accumulation) takes in the capitalistic production and what effects it has on the reproduction process was missing in *Grundrisse* and had to wait until the eighth manuscript of *Capital*, volume II 20 years later, where Marx tried, although incompletely, to explain the formation of money stock in the form of amortization fund and accumulation fund in the framework of multisectoral analysis.

Concerning the third conclusion, Marx' input-output table gives an effective counterexample to Proudhon's doctrine on profit according to which the profit arises because capitalists overcharge workers in selling their products. Obviously, sectors A, B, C, D do not directly sell anything to labours while their capitalists realize their profit through exchange among themselves. Then Marx

through exchange among one another, i.e. through the exchange of capitalists among themselves.' (MEGA II/1.1, S.351-2)

¹¹ MEGA II/1.1, S.333-4.

¹² 'Circulation, exchange within the merchant estate, and the final stage of circulation, exchange between the merchants and the consumers ... are determined by quite different laws and motives, and the greatest contradiction can develop between them. This separation alone can be the cause of trade crises' (MEGA II/1.1, S.83).

¹³ MEGA II/1.1, p.129.

went further to say that the exchange of sectors A, B, C, D with E is their *indirect* exchange with wage they pay to their workers¹⁴, and the profit cannot be realized by this ‘detour’ of wage, but only through the exchange among themselves¹⁵.

We have reconstructed and evaluated Marx’ multisectoral analysis in *Grundrisse*. Marx admitted: “This example may or may not be worked out in more detail. It does not actually belong here”¹⁶. Marx did work out his multisectoral analysis in fact in more detail 12 years later in his second manuscript of *Capital*, volume II of 1870. We can see there how Marx elaborated his input-output table and devoted himself to the above mentioned remaining questions. First, he redefined the sectors in a suitable way and carried out a comparative statics about equilibrium proportion by varying the rate of surplus. Second, he gave thorough consideration to the ‘detour’ of wage and formulate the ‘law of reflux’ while finding out how wage realizes on its ‘detour’ all value components including profit. Third, he made a bridge to the analysis of accumulation fund and amortization fund in the eighth manuscript of 1878 by suggesting, although implicitly, the possibility of money stock formation.

2. Marx’s 6-sector Production Model in 1870

2.1. Overview

It is known that Marx wrote in total eight manuscripts dedicated to the Volume II of *Capital*, of which only two manuscripts (Ms.II and Ms.VIII) were used by Engels for the edition of Part 3 of this volume¹⁷, i.e. the chapters on the so-called 'reproduction schemata'. Although Ms. VIII was fully utilized, the half of the relevant part of Ms. II, making up 36 manuscript pages, was totally ignored by Engels¹⁸ and was published after 140 years for the first time in MEGA II/11 .

In the omitted part of Ms. II, Marx presented a six-sector model of production: 1) means of workers' consumption (sector A3¹⁹), 2) means of production for A3 (sector A2), 3) means of production for

¹⁴ ‘E certainly exchanges for 1/5 of the product of A, B, C and D, i.e. 4/5 of their product; but this exchange is only a detour to get to the wages which A, B, C and D pay their own workers. ...this exchange with E is then only an indirect form of advancing the part of capital which represents necessary labour’ (MEGA II/1.2, S.351).

¹⁵ ‘They cannot therefore gain thereby. The gain comes from the realization of the remaining 4/5 of capital a, b, c, d, and this realization consists in that each receives the labour objectified in his product in another form through the exchange’ (MEGA II/1.2, S.351).

¹⁶ MEGA II/1.2, S.353.

¹⁷ MEGAI/12, Apparat, S.529-552, 887-934; MEW 23, S.12.

¹⁸ In Ms. II, the text related to Part 3 of Volume II of the edited *Capital* begins on manuscript page 130 (MEGAI/11,S.340) and ends on manuscript page 202 (MEGA II/11, S.522). The pages 167-202 (MEGA II/11, S.443-522) was omitted by Engels.

¹⁹ Note that Marx used in his original somewhat awkward notations $II\alpha$, $II\alpha\alpha$, Ia , $II\beta$, $II\beta\beta$ and Ib , which we substitute in this paper with A1, A2, A3, B1, B2 and B3 respectively.

A2 (sector A1), 4) means of capitalists' consumption (sector B3), 5) means of production for B3 (sector B2), and 6) means of production for B2 (sector B1). And simple reproduction was assumed. By means of this multisectoral model, he investigated, above all, following two main issues.

First²⁰, Marx tried to find out equilibrium conditions for the quantity system of the 6-sector model. Indeed, he succeeded in formulating conditions necessary for the quantity equilibrium in three equations. Furthermore, he carried out some comparative statics in order to examine the shift of equilibrium output, employment etc. by varying a parameter of labour share, i.e. the 'rate of surplus value (M/V)'.

Marx's investigation of the second issue²¹ leads the readers, because of its intensity and persistence, to suppose a large amount of his effort dedicated to it. He illustrated the 'law' of monetary reflux using numerical examples in the 6-sector model, i.e. he traced each possible route of monetary circuit in which money advanced by capitalists returns to its starting point after realizing various components of output of the 6-sector economy. Particularly, he concentrated his attention on the reflux of wage paid by capitalists to workers in each sector.

As just mentioned, Marx explained the monetary circuit as the result of the reflux of wage. Wage advanced by capitalists to workers in each sector returns to its starting points after realizing various components of output (not only the commodity labour power but also variable-capital part, constant-capital part and surplus value of social products). However, some rest (Überschuß)²² remains which cannot be realized by circulating money initially advanced as wage payments. In order to realize this rest, it is necessary for capitalists to advance additional money. In this context, this explanation can be seen to provide us with possibilities to re-evaluate Marx's own contribution especially to the theory of monetary circuit along the line of the so-called 'Franco-Italian circuit school'. The crucial role that e.g. A. Graziani (1990, 1998, 2003), E. J. Nell (2004) and others attach to wage payments in monetary circulation was *de facto* anticipated by Marx although the bank, one of their three main agents of the economy besides the classes of capitalists and workers, fades into the background in Marx's model. Furthermore, Marx's model of monetary circuit has a suitable framework to which the money flow computation of I-O analysis by Leontief and Brody (1993) can be effectively applied. Since Mori (2009) already elaborated on the second issue in detail, we are now going to concentrate on a closer examination of the first issue.

²⁰ MEGAI/11, S.481-503.

²¹ MEGAI/11, S.443-481.

²² MEGA II/11,S.491.

2.2. Equilibrium conditions

The subject of Part 3 of Volume II of *Capital*, well known as ‘reproduction schemata’, was dealt with in Ms. II as the third ‘chapter’ after the two chapters on “circuit of capital” and “turnover of capital”. According to Marx’s own table of contents written on the title page of Ms. II, this “third chapter” was structured as follows²³:

The real conditions of the processes of circulation and reproduction

- 1) Variable capital, constant capital and surplus value from a social point of view
 - A) Reproduction on a simple scale
 - a) Described without the mediating monetary circulation
 - b) Description with the mediating monetary circulation
 - B) Reproduction on an extended scale. Accumulation.
 - a) Described without monetary circulation
 - b) Description with the mediating monetary circulation
- 2) (no title)

After the description of A) a) and b) based on 2-sector reproduction schemata, Marx changed, in a continuation of A) b), to 6-sector schemata. This part of Ms. II was neglected by Engels for the edition of *Capital*, Volume II. The subjects of B) a) and the following were never written in Ms. II and to be left to the later Ms. VIII.

As mentioned above, the first issue to investigate in the omitted part of Ms. II consists in the induction of equilibrium conditions for quantity system of the 6-sector production model. This model has following properties:

- The economy consists of 6 production sectors (A1, A2, A3, B1, B2, B3)²⁴, household (individual consumption) of workers and that of capitalists. Each sector produces one distinct good (i.e. single production), which is given the same notation as the sector.
- Workers and capitalists consume respectively A3 and B3 exclusively.
- Means of production of A3 is A2; that of A2 is both A2 itself and A1; and that of A1 is A1 itself. Similarly, means of production of B3 is B2; that of B2 is both B2 itself and B1; and that of B1 are B1 itself.
- The output value of each sector consists of value of means of production and value added, and value added consists of wage and surplus value.
- The income of workers and capitalists come respectively from wage and surplus value exclusively.

²³ MEGAI/11,S.3-4.

²⁴ See Footnote 19.

Both classes do not save.

-Prices are proportional to labour values.

Historically, we can find a similar structure also in multisectoral models by G. Mathur and A. Lowe, where the economy is composed of three sectors; heavy equipment, machinery and corn in Mathur (1965) while primary equipment, secondary equipment and consumer good in Lowe (1976). The most characteristic feature of Marx’s model, however, consists in a symmetrical structure between A-sectors and B-sectors, which we will show as basic and non-basic sectors respectively. More relevant in this respect is Shibata (1939) who divided the economy into four sectors according to “labourers’ consumers’ goods” and “labourers’ producers’ goods” on the one hand, “capitalists’ consumers’ goods” and “capitalists’ producers’ goods”²⁵.

For the 6-sector model characterized above, Marx devised the following numerical example (Example 1: see Table 3).

Table 3 Marx’s 6-sector model (Example 1)

	A1	A2	A3	B1	B2	B3	workers	capitalists	total demand
A1	200	100	0	0	0	0	0	0	300
A2	0	100	200	0	0	0	0	0	300
A3	0	0	0	0	0	0	300	0	300
B1	0	0	0	200	100	0	0	0	300
B2	0	0	0	0	100	200	0	0	300
B3	0	0	0	0	0	0	0	300	300
wage	50	50	50	50	50	50			300
surplus value	50	50	50	50	50	50			300
output	300	300	300	300	300	300	300	300	

One can ‘read’ the table in the same manner as a usual input-output table. Each column from A1 to B3 represents price components of each sector’s output; the column of A3, for example, shows that the good A3 of value of 300 is produced by using A2 of 200 and paying wage of 50 and surplus value of 50. Each row from A1 to B3 represents the demand for each good; the row of A2 e.g. shows that the good A2 of value 100 and 200 is sold respectively to the sector producing A2 and A3. The table represents an equilibrium in the sense that for each good the output equals to the real demand.

²⁵ On the similarity between Marx’s and Shibata’s construction, see Mori (2007).

Furthermore, it stands for an equilibrium state of simple reproduction because all surplus value is individually consumed by capitalists.

On the basis of the above example, Marx traced every commodity transaction among production sectors, workers' and capitalists' household to induce general conditions necessary for an equilibrium of supply and demand. He formulated them in the following three formulae:

$$C_{A3} = (V_{A1} + M_{A1}) + (V_{A2} + M_{A2}) \quad ^{26} \quad (1)$$

$$C_{B3} = (V_{B1} + M_{B1}) + (V_{B2} + M_{B2}) \quad ^{27} \quad (2)$$

$$M_{A1} + M_{A2} + M_{A3} = V_{B1} + V_{B2} + V_{B3} \quad ^{28} \quad (3)$$

where C_j , V_j , M_j are respectively constant capital, variable capital and surplus value of sector j ($j = A1, A2, A3, B1, B2, B3$). First, constant capital of sector A3 equals to the sum of wage and surplus value of both sectors A1 and A2. Second, constant capital of sector B3 equals to the sum of wage and surplus value of both sectors B1 and B2. Third, the surplus value of sectors A1, A2 and A3 (we call them together 'A-sectors') equals to the variable capital of sectors B1, B2 and B3 ('B-sectors'). In both equations (1) and (2), one can see the well-known condition about Marxian 2-sector model reappear, i.e. that constant capital of consumption-good sector equals to the sum of wage and surplus value of means-of-production sector. Furthermore, the third condition anticipates the relationship between subsectors of necessary consumption good and luxury which was to be considered in the later Ms. VIII.

2.3. Rate of surplus value and comparative statics

After formulating the equilibrium conditions, he carried out some comparative statics in order to examine the shift of equilibrium output, employment etc. by varying a parameter of labour share, i.e. the 'rate of surplus value' (M/V). While the rate of surplus value is unity in all sectors in Example 1, it varied to 1/2 in Example 2 (Table 4), 5/7 in Example 3 (Table 5) and 7/5 in Example 4 (Table 6), where input coefficients and the total number of employed workers remain constant.

²⁶ MEGAI/11,S.483,493,495.

²⁷ MEGAI/11,S.483,488,490,494.

²⁸ MEGAI/11,S.488-91,493,495.

Table 4 Marx's 6-sector model (Example 2) ($M/V = 1/2$)²⁹

	A1	A2	A3	B1	B2	B3	workers	capitalists	total demand
A1	$266 \frac{2}{3}$	$133 \frac{1}{3}$	0	0	0	0	0	0	400
A2	0	$133 \frac{1}{3}$	$266 \frac{2}{3}$	0	0	0	0	0	400
A3	0	0	0	0	0	0	400	0	400
B1	0	0	0	$133 \frac{1}{3}$	$66 \frac{2}{3}$	0	0	0	200
B2	0	0	0	0	$66 \frac{2}{3}$	$133 \frac{1}{3}$	0	0	200
B3	0	0	0	0	0	0	0	200	200
wage	$88 \frac{8}{9}$	$88 \frac{8}{9}$	$88 \frac{8}{9}$	$44 \frac{4}{9}$	$44 \frac{4}{9}$	$44 \frac{4}{9}$	0	0	400
surplus value	$44 \frac{4}{9}$	$44 \frac{4}{9}$	$44 \frac{4}{9}$	$22 \frac{2}{9}$	$22 \frac{2}{9}$	$22 \frac{2}{9}$	0	0	200
output	400	400	400	200	200	200	400	200	

Table 5 Marx's 6-sector model (Example 3) ($M/V = 5/7$)³⁰

	A1	A2	A3	B1	B2	B3	workers	capitalists	total demand
A1	$233 \frac{1}{3}$	$116 \frac{2}{3}$	0	0	0	0	0	0	350
A2	0	$116 \frac{2}{3}$	$233 \frac{1}{3}$	0	0	0	0	0	350
A3	0	0	0	0	0	0	350	0	350
B1	0	0	0	$166 \frac{2}{3}$	$83 \frac{1}{3}$	0	0	0	250
B2	0	0	0	0	$83 \frac{1}{3}$	$166 \frac{2}{3}$	0	0	250
B3	0	0	0	0	0	0	0	250	250
wage	$68 \frac{1}{18}$	$68 \frac{1}{18}$	$68 \frac{1}{18}$	48 $\frac{11}{18}$	48 $\frac{11}{18}$	48 $\frac{11}{18}$	0	0	350
surplus value	48 $\frac{11}{18}$	48 $\frac{11}{18}$	48 $\frac{11}{18}$	34 $\frac{13}{18}$	34 $\frac{13}{18}$	34 $\frac{13}{18}$	0	0	250
output	350	350	350	250	250	250	350	250	

²⁹ MEGAI/11,S.481-4,494.³⁰ MEGAI/11,S.484-8,494.

Table 6 Marx's 6-sector model (Example 4) ($M/V = 7/5$)³¹

	A1	A2	A3	B1	B2	B3	workers	capitalists	total demand
A1	$166 \frac{2}{3}$	$83 \frac{1}{3}$	0	0	0	0	0	0	250
A2	0	$83 \frac{1}{3}$	$166 \frac{2}{3}$	0	0	0	0	0	250
A3	0	0	0	0	0	0	250	0	250
B1	0	0	0	$233 \frac{1}{3}$	$116 \frac{2}{3}$	0	0	0	350
B2	0	0	0	0	$116 \frac{2}{3}$	$233 \frac{1}{3}$	0	0	350
B3	0	0	0	0	0	0	0	350	350
wage	34 $\frac{13}{18}$	34 $\frac{13}{18}$	34 $\frac{13}{18}$	48 $\frac{11}{18}$	48 $\frac{11}{18}$	48 $\frac{11}{18}$	0	0	250
surplus value	48 $\frac{11}{18}$	48 $\frac{11}{18}$	48 $\frac{11}{18}$	$68 \frac{1}{18}$	$68 \frac{1}{18}$	$68 \frac{1}{18}$	0	0	350
output	250	250	250	350	350	350	250	350	

The conclusion Marx drew from these calculations is that the output, employment and means of production in A-sectors must increase as the rate of surplus value decreases. Theoretical implications of these comparative statics, however, cannot be seen far-reaching because, first, the calculations are nothing more than arithmetic exercises for adapting the equilibrium conditions and their results are almost self-evident. Second, an assumption of his comparative statics, i.e. that all sectors have a common labour share can sustain its meaning only in case of prices proportional to labour values (i.e. in case of a uniform capital composition across sectors).

Much more interesting about Marx's comparative statics, however, is the fact that he raised some complementary research questions which are anything but trivial and he never posed anywhere else. We take up particularly two of such important research questions here. The first is how to carry out a comparative statics by taking the capital composition (C/V) as a varying parameter instead of the rate of surplus value as before. He asked not only how the quantity equilibrium would change but also what would be a new equilibrium price after varying the capital composition. He was namely well aware that the prices could not remain proportional to labour values when the sectors have different capital compositions. As we will show in the next section, this question confronted Marx with a serious theoretical problem.

Another important research question Marx posed complementarily to the comparative statics with regard to varying rates of surplus value was how a dynamic process of transition from one

³¹ MEGAI/11,S.492-4.

equilibrium to another is possible. Marx was namely confronted with the problem of “traverse” triggered by changing wage rates.

In each of the following two sections, we will focus respectively on the first and second research question, i.e. the comparative statics with regard to the capital composition and the dynamic process of transition of equilibria.

3. The Problem of Non-Basic System in Marx’s 6-Sector Model

3.1. Definition of basic and non-basic products

According to Sraffa (1960), if “a commodity enters (no matter whether directly or indirectly) into the production of *all* commodities”, it is *basic*, and “those that do not, *non-basic* products” (Sraffa 1960, 8, parenthesis and italics in original). This definition can be in principle adapted to Marx’s model although the concept must be extended to ‘augmented’ inputs in that basic products are used directly or indirectly in all sectors as means of production *or* wage good. Non-basic products are used neither directly nor indirectly in at least one sector.

Consider any single production, where each sector produces one distinct good (i.e. there is no joint production). The definition of basic and non-basic products in the augmented sense can be formally reformulated as follows.

We first introduce the following symbols:

- input coefficient of good i for sector j : $a_{ij} \in \mathbf{R}_+$

- input coefficient matrix: $A := (a_{ij}) \in M(n \times n, \mathbf{R}_+)$

- labour input coefficient for sector j : $l_j \in \mathbf{R}_+$

- vector of labour input coefficients: $l := (l_1, \dots, l_n) \in \mathbf{R}_+^n$

- vector of wage goods (wage basket) per labour unit: $d \in \mathbf{R}_+^n$

- augmented input coefficient matrix: $B = (b_{ij}) := A + dl \in M(n \times n, \mathbf{R}_+)$

- activity level (or output) of sector j : $x_j \in \mathbf{R}_+$

- activity (or output) vector: $x' := (x_1, \dots, x_n) \in \mathbf{R}_+^n$. Note that the prime applied to a matrix or a vector denotes, as usual, their transposition.

- price of good i : $p_i \in \mathbf{R}_+$
- price vector: $p := (p_1, \dots, p_n) \in \mathbf{R}_+^n$
- general rate of profit: $r \in \mathbf{R}$

Definition 1.

Consider then the matrix $\sum_{t=1}^n B^t$. A good i is basic if and only if the i -th row of this matrix is positive. Goods that are not basic are non-basic.

An alternative equivalent definition can be given in the following manner. If B is indecomposable, all n goods are basic. If B is decomposable, it can be transformed into the following form by suitable simultaneous substitutions of rows and columns.

$$B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1n_0} \\ 0 & B_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & B_{n_0 n_0} \end{pmatrix} \quad (4)$$

where $B_{11}, \dots, B_{n_0 n_0}$ ($n_0 \leq n$) denote either a square null-matrix or a non-negative indecomposable square matrix. Let I_i be the set of columns whose component is included in B_{ii} , i.e. $I_i := \{j = 1, \dots, n \mid b_{jj} \in B_{ii}\}$. Obviously, all goods in I_2, \dots, I_{n_0} are non-basic. If B_{11} is a null matrix,

there is no basic good. Otherwise, assume that for each $j = 1, \dots, n_0$, $\begin{pmatrix} B_{1j} \\ \vdots \\ B_{j-1j} \end{pmatrix} \gneq 0$ if B_{jj} is

indecomposable, and $\iota \begin{pmatrix} B_{1j} \\ \vdots \\ B_{j-1j} \end{pmatrix} > 0$ ($\iota := (1, \dots, 1)$) if B_{jj} is a null-matrix. If and only if this

assumption is true, any good in I_1 is basic³². Note that we use inequality signs for vectors and

³² To prove the equivalence of both definitions, use the property of a non-negative indecomposable $n \times n$ matrix M : $\sum_{t=1}^n M^t > 0$.

matrices in this paper in the manner that $X > Y$, $X \succeq Y$ and $X \geq Y$ denote that $X - Y$ is positive, semi-positive and non-negative, respectively.

3.2. The problem of non-basic system

We assume in the following that there are both basic and non-basic goods. Non-basic goods can be *always* eliminated from the balanced growth output vector (or standard system) as Sraffa suggested³³. On the other hand, they can be *sometimes* included in the balanced growth path depending on their input coefficients. It can be shown that a balanced growth output includes a non-basic good in I_i ($i > 1$) if the Frobenius root of B_{ii} is larger than that of $B_{11}, \dots, B_{i-1,i-1}$, i.e. $\lambda_i > \lambda_h$ for all $h < i$, where λ_j denotes the Frobenius root of B_{jj} ³⁴. In this case, the Frobenius root of B_{ii} determines the growth rate, which is now equal to $g_i := (1/\lambda_i) - 1$, lower than the previous rate of $g_1 := (1/\lambda_1) - 1$. Then, we can take as a balanced-growth path the vector $x' := (x'_1, \dots, x'_i, \dots, x'_{n_0})$, where x is partitioned into n_0 sub-vectors whose length corresponds to the dimension of $B_{11}, \dots, B_{n_0 n_0}$, with x_i being a positive eigencolumn of B_{ii} , and $x_j = (\lambda_i I - B_{jj})^{-1} \sum_{h=j+1}^i B_{jh} x_h$ for $j < i$ and $x_j = 0$ for $j > i$.

It is known that the above mentioned possibility of a balanced growth path including a non-basic good (we call it “non-basic system” in this paper) caused a theoretical problem. If there is a group of non-basic goods I_i ($i > 1$) such that the Frobenius root of B_{ii} is not less than that of B_{11} , i.e. $\lambda_i \geq \lambda_1$, which case is implied by the “non-basic” system, then one of the following two alternatives must occur. Either basics’ prices are zero while non-basics’ prices in I_i are positive with a (possibly) lower rate of profit $r_i = (1/\lambda_i) - 1$. Or basics’ prices are positive while some prices in I_i are negative with a higher rate of profit $r_1 = (1/\lambda_1) - 1$. This is the problem we mean by the problem of non-basic system, which we will be dealing with, i.e. choice between two evils: either zero prices for all basics or negative prices for some non-basics.

As mentioned above, the problem of non-basic system must always occur if non-basics’ input coefficients for their own use, B_{ii} , are sufficiently large. This will be typically the case when the non-basics concerned are not just luxuries but means of production which reproduce themselves quite costly. The problem was recognized by several authors besides or even before Sraffa (1960, 90-91, Appendix B). Gautam Mathur (1965) treated as a non-basic good a heavy equipment (e.g. dam) in its accumulation process which is being expanded before it can be employed as a means of production for other (machinery) industries in future.

³³ Sraffa (1960), 25.

³⁴ This is a *sufficient* condition for the possibility of a eigencolumn (balanced growth output vector) including non-basics in I_i . A *necessary* condition would only need the weak inequality, i.e. $\lambda_i \geq \lambda_h$ for all $h < i$.

“The heavy equipment in this state of affairs is thus a *non-basic good*, i.e., it does not enter directly or indirectly into the production of all goods ... All that the heavy equipment does is to reproduce itself as fast as it can by feeding on the surplus produced by the rest of the economy” (Mathur 1965, 115-116, italics in original).

Indeed, the input coefficients Mathur exemplified for an accumulation process of the heavy equipment exhibit the above mentioned problem of non-basic system³⁵. In the consequence,

“no pricing system can be discovered which would give an equal profit rate for each process in these (basic and non-basic) subeconomies”. Therefore, “the H-making process (heavy equipment industry) would be a loss-process and has to be subsidised. Because the dependent processes can exist only after subsidies are given, the prices can be calculated only after subsidies. After the subsidy is given, positive prices can be found which equalise the rate of profit to the rate of growth. ... Thus in perfect competition this subeconomy would not exist”³⁶.

More than 30 years before Sraffa (1960), a mathematician Robert Remak (1929) was well aware of the problem of non-basic system. Adapted to the assumptions and notations in this section, his rather abstract algebraic analysis can be summarized in the following way. First, he assumed an input coefficient matrix whose quantity (column) eigenvector is the aggregation vector ι and the associated Frobenius eigenvalue is unity, i.e., $B\iota = \iota$. Then, for this input matrix B , he proved that there exists a non-negative price eigenvector p such that $p = pB$, and examined how zeros are located in p . Defining I_{n_0} as “the highest group (höchste Gruppe)”, he concluded that all prices except the highest group must be zero³⁷. Obviously, this implies that the prices of all basic goods in I_1 must be zero. This result is true because in Remak’s matrix B , we have $1 = \lambda_{n_0} \geq \lambda_j$ for all $j < n_0$.

The problem of non-basic system can be further traced back to Marx. In 1870, Marx attempted in his Manuscript II of Capital Volume II to calculate production prices in a six-sector production model whose setting exhibits the problem of non-basic goods.

Since Marx regards not only physical means of production but also wage goods as inputs based on which the production price is calculated, we have to use an augmented input coefficient matrix

³⁵ Strictly speaking, Mathur’s model deviates from the above explanation of the problem because he uses a joint-production model instead of the single production based on which the problem of non-basic system was analyzed. However, the problem of non-basic system occurs there as well.

³⁶ Mathur 1965, 123-124, parentheses added.

³⁷ Remak 1929, 730.

including wage goods to determine the production price. The (augmented) input coefficient matrix (simply “input matrix” in the following if the distinction from a usual input coefficient matrix is not necessary) implied in his six-sector model can be calculated from Table 3 in the following way:

$$B = (b_{ij}) = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (5)$$

b_{ij} denotes the input of good i to produce one unit of good j ($i, j = 1, \dots, 6$), where good 1, 2, 3, 4, 5 and 6 stand for A1, A2, A3, B1, B2 and B3, respectively.

As we can easily see from (4) and (5),

$$I_1 = \{A1, A2, A3\}, \quad B_{11} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{pmatrix} \text{ and its Frobenius root is } \lambda_1 = 5/6$$

$$I_2 = \{B1\} \quad B_{22} = \left(\frac{2}{3}\right) \text{ and its Frobenius root is } \lambda_2 = 2/3$$

$$I_3 = \{B2\} \quad B_{33} = \left(\frac{1}{3}\right) \text{ and its Frobenius root is } \lambda_3 = 1/3$$

$$I_4 = \{B3\} \quad B_{44} = (0) \text{ and its Frobenius root is } \lambda_4 = 0$$

The basic good is defined as a good that is used directly or indirectly in every sector’s production process. In Marx’s model, A1, A2 and A3 are basic goods because A1 goes into the production of A2, and A2 goes into A3, while A3, i.e. workers’ wage good, goes into all six production processes. Therefore, A1, A2 and A3 enter all six production processes, A3 directly while A1 and A2 indirectly. All goods that are not basic are called non-basic goods; in Marx model, B1, B2 and B3 are all non-basic. Obviously, B3 is the “highest group” according to Remak’s definition.

Furthermore, since we have $\lambda_1 > \lambda_j$ for all $j > 1$, all goods can have positive production prices, say $p_j = 100$ for all j and $r = 1/5 = 20\%$ as Marx’s price setting based on labour values yields. We know,

however, such is not always the case. A characteristic feature of the non-basic system is, as we already saw, that basic goods cannot have positive prices if $\lambda_1 \leq \lambda_j$ for some $j > 1$. And this will be exactly the case in the following example. Marx namely asked himself the following research question. He suggested that the value composition (C/V) can differ between A-sectors and B-sectors, e.g. 4 in A-sectors and 10 in B-sectors, and “Let’s see temporarily how the things turn out to be under the condition of the general profit rate”³⁸. He used to set the prices so far equal to labour values because the value composition was the same in all sectors. Now he asks what happens with production prices representing a general profit rate if different value compositions between A-sectors and B-sectors are assumed. This was to be a tricky question for himself.

Indeed, he changed the value composition in B-sectors from 4 to 10 leaving A-sectors as before. Then, the new equilibrium in value terms will be the following.

Table 7 Marx’s 6-sector model (Example 5) (C/V=4 in A, 10 in B)

	A1	A2	A3	B1	B2	B3	workers	capitalists	total demand
A1	200	100	0	0	0	0	0	0	300
A2	0	100	200	0	0	0	0	0	300
A3	0	0	0	0	0	0	300	0	300
B1	0	0	0	625	125	0	0	0	750
B2	0	0	0	0	500	250	0	0	750
B3	0	0	0	0	0	0	0	300	300
wage	50	50	50	62.5	62.5	25	0	0	300
surplus value	50	50	50	62.5	62.5	25	0	0	300
output	300	300	300	750	750	300	300	300	

And the augmented input coefficient matrix corresponding to this new state will be

³⁸ MEGA II/11, 494-495.

$$B = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \\ 0 & 0 & 0 & \frac{5}{6} & \frac{5}{12} & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{12} & \frac{5}{6} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Note that we have now $\lambda_1 = \lambda_2 = 5/6$ and, therefore, this new state exhibits the problem of non-basic system. In fact, we can easily verify that any non-negative solution must have zero prices for all three A-sectors, i.e. the non-negative solution must be $p_1=p_2=p_3=0$ and $p_1=p_2=p_3=c$, where c is any non-negative number. And the profit rate is uniquely $r = 1/11$.

In this way, Marx set a numerical example that exhibits the problem of non-basic system, i.e. a choice between zero price for all basics and negative price for some non-basics. Once we have this problem of non-basic system, there will be two possibilities to treat the problem and keep all prices positive. First, non-basic sectors, i.e. B-sectors, should be closed. If every producer moves from B-sectors to A-sectors, A-sectors can regain positive prices, and the profit rate will return from 1/11 ($\approx 9\%$) to 20%. This is the only possibility the free market economy would allow. The second possibility is just what Mathur (1965) suggested: non-basic sectors, i.e. B-sectors, should be maintained by receiving subsidies from the government. The profit rate for A-sectors only is 20% while it is 9% in the whole economy including B-sectors. To balance this inequality of profit, B-sectors must be given a subsidy, e.g. 75 to B1 and B2 each, and 30 to B3, leaving each price at 100 and the profit rate at 20% as before.

4. The Problem of Traverse

As already mentioned, Marx posed another research question concerning a dynamic process of traverse from one equilibrium to another. He assumed that after wage increase by 1/3, the rate of surplus value decreases from the original level of 1 to 1/2. As shown in Table 4, he could calculate without any problem a new equilibrium state, where each of A-sectors must expand its output from 300 to 400 while each B-sector must shrink from 300 to 200. Here, not only the proportion between A-sectors and B-sectors is changed, but this change must be also accompanied by a growth in production and employment in A-sectors. Then, Marx began to consider how to start such a growth process from the stationary state, or how to move from the old equilibrium (300) to the new one (400).

Marx was clear what occurs after such wage increase.

“A large part of products [A3] ... consist of foodstuffs which must be produced (at least in their raw material) beforehand for an annual consumption. The demand for them would increase very much. The profit made in [A3] etc. would rise so far very much. In short, capital and workforce would be drawn from [B3] etc. to [A3] etc”³⁹.

He was, however, well aware that “the mechanism of the bourgeois society brings with it that such changes ... are accompanied by circumstances which *paralyze* their effect and break the changes themselves”⁴⁰. The circumstance Marx meant by the paralysis can be interpreted as follows: Despite the high profit of A3, “the transfer to [A3] is not so fast as the dismissal under [B3]” so that an ensuing unemployment lowers the wage and annuls in the end the initial effect of wage increase. Due to such paralysis, he concluded with “difficulty and relative impossibility of the change on this way”⁴¹.

Marx, however, did not elaborate in more detail on how such paralysis must occur and why the expected expansion of A-sectors does not take place. A century later, a comparable study on the problem of traverse was resumed by Lowe (1974)⁴². There, he identified four phases of the efficient (speediest) course of the expanding traverse⁴³.

1. Partial liberation of existing capacity
2. Augmentation of output of primary equipment (which corresponds A1 in our definition)
3. Augmentation of output of secondary equipment (A2)
4. Augmentation of consumer-good output (A3).

The first phase, in particular, means that we must release means of production from sector A2 and employ them in sector A1 to expand A1 as fast as possible. After analyzing these four phases of the efficient traverse, Lowe came to the same conclusion as Marx⁴⁴. It is, therefore, not surprising that

³⁹ MEGA II/11, 501-502. Sector indices in square parentheses were changed from the original in order to be consistent with our definition.

⁴⁰ MEGA II/11, 501, italics added.

⁴¹ MEGA II/11, 502.

⁴² Another pioneering work on the problem of traverse was carried out by Hicks (1973) almost simultaneously. See Mori (2017).

⁴³ Lowe 1974, 109-114.

⁴⁴ It must be noted that Lowe treated unlike Marx a traverse between *steady growth* equilibria (instead of stationary state equilibria) and a traverse due to a sudden increase of labour supply (instead of a wage increase). Therefore, the situation of the traverse is symmetrically different between both authors: Lowe assumed wage fall and decreasing demand for consumption goods

both use the same word “paralyze/paralysis”: The motorial and behavioral conditions in the free market system are “goal-inadequate” and “diverting action in the wrong direction if not blocking it altogether”. “[I]ts consequences are not just under- or overbuilding but a total *paralysis* of the adjustment process”⁴⁵.

Let us now reconstruct Marx’s traverse problem in order to make sense of his “paralysis”. First, we define the *efficient* traverse as the path of each sector’s output which moves the most speedily from the old to the new equilibrium. Formally, the efficient traverse in A-sectors⁴⁶ is defined by the solution of the following minimizing problem.

Problem 1.

$$\arg \min_{x(t) \geq 0} T!$$

s.t.

$$x(t) \geq \begin{pmatrix} 2/3 & 1/3 & 0 \\ 0 & 1/3 & 2/3 \\ 2/9 & 2/9 & 2/9 \end{pmatrix} (x(t) + \dot{x}(t))$$

$$x(0) \leq \begin{pmatrix} 300 \\ 300 \\ 300 \end{pmatrix}, \quad x(T) \geq \begin{pmatrix} 400 \\ 400 \\ 400 \end{pmatrix}$$

where $x(t) := (x_1(t), x_2(t), x_3(t))'$ and $x_i(t)$ denotes the output of sector i at time t .

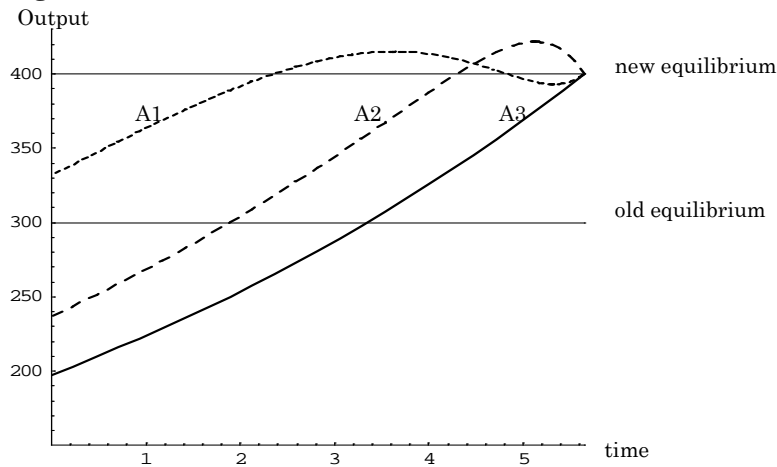
Assuming that time is measured continuously, and there is no storage to be carried over (i.e. the output must be used immediately or disposed), the efficient traverse can be determined uniquely (see Appendix) and depicted in Figure 1.

while Marx assumed wage hike and increasing demand for consumption goods. Despite this difference, they reached the same result that the efficient traverse is impossible.

⁴⁵ Lowe 1976, 151, 161. Italics added.

⁴⁶ The efficient traverse in B-sectors can be treated in the analogous way, which elaboration we omit in this paper.

Figure 1.



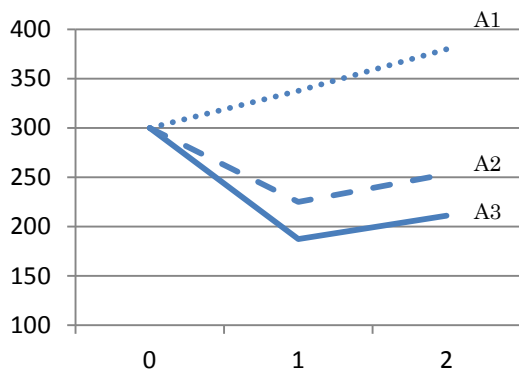
As we can see from Figure 1, the duration of the efficient traverse is about 5.65, and its path can be characterized by following remarkable features.

1. Sector A1 starts from a higher point while sectors A2 and A3 must instead start from a lower point than the old level (300).
2. A1 is the first to reach the new level (400), A2 is the second, and A3 the last.
3. The growth of A1 and A2 temporarily overshoots the new level to keep sustaining the growth of respectively A2 and A3 to the end.
4. A3 follows a balanced growth path (with the rate of growth $g=1/8$) while A1 and A2 are decelerating the growth.

Comparing with Lowe's efficient traverse, we can see that the first feature here corresponds to the first phase of Lowe's traverse, and the second feature represents the order of augmentations of the three sectors characterizing the second, third and fourth phases⁴⁷. Note that Lowe's first phase is invisible in the continuous-time model in Figure 1 because the "liberation" takes place there instantly at time 0. If we magnify the neighborhood of time 0 by measuring time discretely (see Figure 2), we can see the first phase (from time 0 to time 1) more clearly, where sector A1 is growing rapidly at cost of sector A2 (while sector A3 shrinks by leaving their capital idle).

⁴⁷ Lowe admitted that the three phases may not necessarily proceed successively (or "horizontally") but parallel (or "vertically") as shown in Figure 1.

Figure 2.



Lowe regarded this process of “liberation” in the first phase as “paradoxical” in the sense that “in order ultimately to increase the output of consumer goods [A3], such output must, to begin with, be reduced”⁴⁸. It might be possible to make sense of Marx’s “paralysis” by referring to Lowe’s “paradox”: Sector A3, which must be reduced at first in order to finally expand effectively to the goal, enjoys instead increasing demand and, therefore, high profit so that it draws capital and workers from other sectors. Obviously, if sector A3 expands, sector A2 as provider of their means of production must expand as well. Then, the liberation of resources from A2 to A1, i.e. the first requirement of the efficient traverse, fails.

5. Conclusion

In his comparative statics with regard to the capital composition, Marx must have been confronted with an aporia because such a situation as zero or negative prices is impossible according to Marx’s theory of production price. His formula of profit rate, $r = M/(C+V)$, yields namely positive prices in all these cases. It is symptomatic of this aporia that Marx could not manage to solve the prices and interrupted this tricky example by changing assumptions and finally saying: “This must be investigated later (Dieß später zu untersuchen)”⁴⁹. We know that the announced investigation never happened.

On the other hand, the research problem on the dynamic process of traverse is a good proof for Marx’s awareness that in general the dynamic analysis is a theoretically different question from the comparative statics and a much trickier one because such changes are “paralyzed” by “hindrances that are immanent in the mechanism of capitalistic production”⁵⁰. And we hope to have been able to

⁴⁸ Lowe 1976, 110.

⁴⁹ MEGA II/11, 495.

⁵⁰ MEGA II/11, 501-503.

show that Marx de facto anticipated there the problem Lowe tackled with more than one hundred years later.

These interesting theoretical episodes in his *Capital* manuscript which might have lead readers to question the validity of Marx's price theory on the one hand, and to open new perspectives to further develop his multisectoral analysis on the other hand, was made invisible in the *Capital* edition after Engels eliminated this part of manuscript in his edition of *Capital* Volume II. As already stated, this 6-sectoral analysis by Marx as a whole was completely ignored by Engels when he edited *Capital* Volume II. Can Engels's omission of this part of Marx's manuscript be justified? The consideration of this paper would tend to suggest denying the question.

It is true that most of Marx's essential results reappear either in Engels's edition of "Reproduction Schemes", or they might be no more than trivial arithmetic exercises which would not deserve being published. However, some research questions Marx posed for his own further investigation suggest profound theoretical problems although Marx obviously could never give any answer to them. If they had appeared in the *Capital* edition, it would have been at least clear to the readers that Marx was confronted with critical and significant problems in economic theories. It could be possible that they would have exerted an influence on the course of posthumous controversies on his economic theories.

Appendix: The solution to Problem 1

The solution of Problem 1 can be found by solving the following differential equations.

$$\begin{cases} x_3(t) = x_3(0)e^{t/8} \\ \dot{x}_2(t) = 2\left(x_2(t) - \frac{9}{8}x_3(t)\right) \\ \dot{x}_1(t) = \frac{1}{2}(x_1(t) - x_2(t) - \dot{x}_2(t)) \\ x_1(0) + \frac{1}{2}x_2(0) = 300 \\ x_1(T) = x_2(T) = x_3(T) = 400 \end{cases}$$

The solution can be determined uniquely as follows:

$$\begin{aligned} x_1(t) &\approx -23.71628e^{t/2} + 0.00099e^{2t} + 355.28684e^{t/8} \\ x_2(t) &\approx -0.00099e^{2t} + 236.85789e^{t/8} \\ x_3(t) &\approx 197.38158e^{t/8} \\ T &\approx 5.65061 \end{aligned}$$

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