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**Graham-type Trade Model under the  
Condition of Full Employment: Ricardian  
Trade Model with Link Commodities**

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# **Graham-type Trade Model under the Condition of Full Employment: Ricardian Trade Model with Link Commodities**

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## **Abstract**

This paper provides a Graham-type trade model, which is a multi-country multi-commodity Ricardian (one-factor) trade model and attaches great importance to commodities produced in common in more than one country, or link commodities: e.g. automobiles among Japan, USA, and Germany, IT products among Japan, Korea, and China, and beef among Brazil, Australia, and USA. The link commodities determine the relative wage rates of all the countries producing the same link commodities, thereby determining the relative prices of all commodities produced in these countries. Among the pattern of the international division of labor (IDL pattern) formed in the multi-country multi-commodity trade model, the linkage type IDL patterns linking all the countries directly and indirectly is overwhelmingly many, the limbo type IDL patterns having more than one disconnection accounts for a small part, especially one of them, the perfect specialization patterns (PSPs) having no link commodities are uncommon. When small changes in demand occur, quantity adjustments without price changes are conducted among countries linked directly and indirectly (the Graham case), and price adjustments among countries disconnected (the Mill case). Conventional trade theories have focused their attention on the PSP and have not incorporated the link commodities into their models. The link commodities, therefore the quantity adjustments or the Graham case matters. We need to free ourselves from undue emphasis on the PSPs.

**Keywords:** link commodities; multi-country multi-commodity; quantity adjustments;

F. D. Graham

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# **Graham-type Trade Model under the Condition of Full Employment: Ricardian Trade Model with Link Commodities**

Hideo Sato

## **1 Introduction**

Conventional trade theories have not sufficiently incorporated two important stylized facts, existence of intermediate goods traded internationally and existence of commodities produced in common in more than one country (we refer to these as commodities with **comparative average** in plain terms or **link commodities** in technical terms), into their models. According to WTO (2013), share of intermediate goods accounted for 51-55% of world non-fuel export in the years 2000-2011. In addition, among a huge number of items traded practically, we can see not a few commodities with comparative average, e.g. automobiles among Japan, USA, and Germany, IT products among Japan, Korea, and China, beef among Brazil, Australia, and USA, and so on. Nevertheless, most trade models have ignored these facts, especially the latter fact.

However, new stream of research into trade theory is changing this kind of situation. Shiozawa (2007, 2017a) and Oka (2017) presented a new trade model with intermediate goods and link commodities, and Sato (2017) showed the outline of a Graham-type trade model, which is a multi-country multi-commodity Ricardian (one-factor) trade model and attaches great importance to link commodities.

This paper provides the details of a Graham-type trade model. Although the model has two versions, a full employment version and an unemployment version, here, we deal with only the former. The remainder of this paper is organized as follows. Section 2 describes the essence of Graham's relatively unknown theory of international values, because our model is based on Graham's model and a modified version of the original. Section 3 explains the Graham-type trade model under the condition of full employment. Section 4 is concluding remarks.

Furthermore, we add three appendixes. First, we deal with the three country three commodity numerical example in Jones (1961), which clarified what pattern is the most efficient if the perfect international specializations is formed. We show that it is

uncommon for the perfect specializations to be practically formed. Second, we introduce the pioneering H. von Mangoldt's numerical trade model and compare it with Graham's model. The third is a supplement of subsection 3.2.4.

## 2 Graham's theory of international values

Frank D. Graham (1890-1949), a mainstream US economist, researched international values from the 1920s and published his major book *The Theory of International Values* in 1948<sup>1</sup>, the year before his accidental death. His research, however, has not been praised within mainstream economics and has been almost forgotten. The reason is that while the origin of mainstream trade theory is J. S. Mill's theory of reciprocal demand (Mill, 1848), Graham criticized Mill's theory thoroughly<sup>2</sup>. Also, non-mainstream economists, especially Marxian economists, who are critical of Mill's theory, have refused to accept Graham's theory as excellent and have ignored it entirely. The reason is that, while the labor theory of value is the most important foundation for Marxian economics, Graham regarded the theory as a stumbling block and refused it. However, Graham's theory was decisively important for the new trade theory. We explain the features of his theory in some detail.

(1) *Graham was the first to present the existence of an equilibrium solution in a multi-country multi-commodity trade model.*

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<sup>1</sup> His related works are Graham (1923, 1932, 1948). The following explanation mainly relies on Graham (1948).

<sup>2</sup> Soon after the publication of Graham (1948), many book reviews and papers relating to Graham's theory were written, and most of them supported Mill's theory. Elliot (1950) insisted that Graham's two-country multi-commodity cases can be translated into offer curves by using the conception of A. Marshall's "representative bales" (Marshall, 1923, p.157). Metzler (1950) mentioned that Elliot was right in two-country cases, but not in three-or-more country cases, and Whitin (1953) wrote that Elliot was also right in three-or-more country cases. Before these comments, Viner (1937) had criticized Graham, supporting Mill and Marshall. Melvin (1969) also interpreted Graham's model on the base of offer curves. Almost attempts to understand Graham's trade theory within a framework of conventional trade theories did not succeed, because the theory transcended conventional theories.

We can sum up the fundamental structure of Graham's model as follows.

1. There are many countries and many commodities.
2. There are no intermediate goods and no profits. All commodities are for consumption.
3. For each country, constant opportunity costs, economic scales, and demand structures are given.
4. Full employment and trade equilibrium (or national expenditure equals national income in each country) are fulfilled.
5. There are no transport costs and no trade barriers.

Under these assumptions, the patterns of the international division of labor (hereafter, **IDL patterns**), international values, and each country's volumes of production, export, import, and consumption are determined uniquely.

Graham explains the above, while providing no mathematical treatment<sup>3</sup>, by using many numerical examples. In earlier trade theories, although there was an example that an equilibrium solution was derived in a two-country multi-commodity case<sup>4</sup>, in a multi-country multi-commodity case, some possible IDL patterns were only shown at best<sup>5</sup>. Indeed, Graham was the first to present the existence of an equilibrium solution in a multi-country multi-commodity (four-country three-commodity or 10-country 10-commodity) case<sup>6</sup>.

(2) *To explain domestic values and international values by the same logic, Graham*

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<sup>3</sup> McKenzie (1954a) presented a mathematical treatment for Graham's model and McKenzie (1954b) proved the existence and uniqueness of equilibrium in a multi-country multi-commodity one-factor trade model with some properties, of which subset is, McKenzie claimed, Graham's model.

<sup>4</sup> See von Mangoldt (1975) and **Appendix 2**.

<sup>5</sup> See Section 4, Chapter 8 of Viner (1937).

<sup>6</sup> Graham calculated equilibrium solutions of his numerical examples through a tedious process of trial and error. Although he requested mathematicians of great repute to provide a mathematical formula solving for the equilibrium of his numerical examples, they were unable to furnish any such formulas (Graham, 1948, p. 95). According to McKenzie (1999), one of them, von Neumann replied that no analytic solution was possible (p. 5). In fact, as shown later, there is no way to solve the problem at one stroke, and several complicated processes are needed in order to obtain the equilibrium solutions practically. As far as I know, it is Noguchi (1990) to provide an algorithm for solving for the first time. However, Noguchi's approach is limited to the linkage type (see 3.1.1) and considerably differs from our approach.

*expresses production techniques of commodities not by labor costs (inputted labor), but by opportunity costs.*

According to Graham, each country's production techniques differ in every sector. While the labor theory of value expresses the difference in these techniques by using the difference in labor input coefficients, he expresses it by using the difference in the opportunity cost of each commodity. Concretely, he designates a specific commodity as a benchmark commodity (the opportunity cost of this commodity is one) and expresses the production techniques of other commodities by the number of units producible by giving up production of one unit of the benchmark commodity. The opportunity costs are essentially constant<sup>7</sup>, as distinct from those of neoclassical trade theory which are increasing. Graham describes the reason for using opportunity costs as follows:

When we think in terms of opportunity cost it can be conclusively demonstrated that Ricardo, Mill, and the neo-classicists, were wholly wrong in supposing that the same rule which regulates the relative value of commodities in one country does not regulate the relative value of the commodities exchanged between two or more countries. (Graham 1948, p. 333)

We also explain the other two given conditions. The economic scale of each country is expressed by the production volumes of the benchmark commodity which is realized when each country specializes in the commodity. Although full employment is supposed, the volumes of production factors and absolute productivity levels are not shown. Therefore, differentials in per capita income or wage rates among countries are not argued in the theory of international values directly and are treated as another problem<sup>8</sup>. The demand structures of each country are given by the expenditure coefficients (proportions of amounts expended on each commodity in total expenditure). The sum of the coefficients is one (i.e., all income is expended for

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<sup>7</sup> Graham refers to the case of variable opportunity costs too, and indicates that the number of commodities produced in common in more than one country would grow under the increasing opportunity costs (Graham 1948, pp. 146-151).

<sup>8</sup> Graham is not indifferent to the problem. For example, he writes that national prosperity (per capita income or wage rate) is a function of two variables, per capita physical productivity and the terms of trade, and the former is more important (p. 50, pp. 212-213, p. 233). He also refers to money wages (p. 261, p. 307).

consumption) in every country.

(3) *International values are determined by the opportunity costs in each country and link commodities.*

International values or the world relative prices of commodities are determined not by reciprocal national demand, but by the opportunity costs in each country just like domestic values. What is important in this determination is the existence of commodities produced in common in more than one country, termed *link commodities* (p. 254, p. 332)<sup>9</sup>. This link commodities link the opportunity costs of countries that produce the same link commodities, meaning that the relative prices of all the commodities produced in these countries are determined uniquely. In principle, every country has at least one link commodity, suggesting numerous link commodities in the world at large. As a result, a body of link commodities links the opportunity costs of all countries and thus determines the international values of all the commodities in the world. The link commodities are, in turn, determined by the interaction among the opportunity costs, economic scales, and demand structures in each country. According to Graham, the link commodities were the missing link of the classical theory of value.

(4) *In the face of changes in demand, international values are highly stable.*

International values, formed once, are highly stable in the face of changes in demand. Such changes are adjusted through changes in production volumes and export-import volumes without price changes. If drastic changes in demand occur, prices might change slightly. In this case, the price changes are necessarily accompanied by changes in the IDL pattern. Newly formed international values are also based on the linkage of the opportunity costs in each country.

However, depending on the three given conditions of 3 in the above (1), the linkage of opportunity costs might be disconnected. Graham calls such a state of disconnection *limbo*, and regards this state as highly improbable<sup>10</sup>. In the limbo case, a small change

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<sup>9</sup> Graham also called them the common products (p. 69, p. 257) or the common commodities (pp. 253-4). Considering the concept of the terms, however, “link commodities” are the most suitable.

<sup>10</sup> According to Graham, in limbo cases, small changes in demand cause changes in commodity prices and also commodity prices are unstable and indeterminate (p. 35). Although Graham almost ignored the limbo cases, as pointed out by McKenzie (1954a) and shown in subsection 3.2, the possibility of limbo cases should not be

in demand brings about an immediate change in international values. Exemplified by using a two-country two-commodity model, the limbo case is a situation in which each country specializes in a commodity with a comparative advantage, an ordinary case used in textbooks. According to him, however, a situation in which one country produces two commodities and the other country produces either commodity with a comparative advantage has a far higher probability. Then, international values are determined by the opportunity cost of the former country and reciprocal demand plays no part.

### **3 A Graham-type model under the condition of full employment**

#### ***3.1 Derivation of the equilibrium solution in the M-country N-commodity model***

##### **3.1.1 Model Setting and Definition of Terms**

At first, we set a general Graham-type model that assumes full employment as follows.

1. There are M countries and N commodities. Here, M and N are integers more than 2 and N is larger than M.
2. There are no intermediate goods and no profits. All commodities are for consumption.
3. Full employment and trade equilibrium (or national expenditure equals national income in each country) are fulfilled.
4. There are no transport costs and no trade barriers.
5. There are no international movements of labor and domestic wage rates are equal in all sectors.
6. For each country, production techniques expressed by constant labor input coefficients, volumes of usable labor, and demand structures expressed by expenditure coefficients are given. Although we don't absolutely need the information about the labor input coefficients of some sectors (e. g. the car

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ignored.

industries in developing countries or the crude oil extraction industries in non-oil-producing countries) in which a probability of having a comparative advantage is almost zero, we give all sector's data for convenience of explanation and assume that the degrees of comparative advantage between two countries selected arbitrarily differ in every sector. The sum of the expenditure coefficients is one in every country.

In the full employment version of the Graham-type model, under these assumptions, the IDL patterns, international values or the world relative prices, wage rates in each country, and each country's volumes of production, export, import, and consumption are determined uniquely. Before explaining logic of the determination and a way to derive the equilibrium solution, we will define some terms.

At first, the *equilibrium solution* consists of identification of the IDL pattern, each commodity's equilibrium production volumes in each country, each country's equilibrium wage rates, and the equilibrium international values. By obtaining the equilibrium solution, we can calculate the each commodity's volumes of export, import and consumption in each country.

Given the international division of labor, some sectors in each country continue the production activity and other sectors cease it. We call the former *active points* and the latter *non-active points*. The IDL patterns have to be *reasonable*. Here, "reasonable" means a situation in which both the "production costs of active points = commodity prices" and "production costs of non-active points > commodity prices" are fulfilled.

Since the purpose of economic models is to understand the real world, let us watch the actual situation of the international division of labor. Then, we can see a scene that there does not exist in ordinary trade models, that is, a fact that there are commodities produced in common in more than one country: Cars produced in Japan, USA., and Germany, IT products in China, Korea, and Japan, textile products in some developing countries, and so on. We call such commodities *link commodities* after Graham. The link commodities determine the relative wage rates of all the countries producing the same link commodities, thereby determining the relative prices of all commodities produced in these countries. As a same commodity has an identical price, the relative labor productivities (inverses of the labor input coefficients) of the link commodities

are precisely the relative wage rates<sup>11</sup>, and the relative prices are obtained by multiplying the relative wage rates by constant labor input coefficients.

When countries have the same link commodities, we say, these countries are directly linked. Countries, however, can also be indirectly linked. Suppose that there are countries *A*, *B*, and *C*, and that *A* and *B* produce a link commodity in common, further that *B* and *C* produce another link commodity in common. In this case, all the three countries are linked: *A* and *C* are not directly but indirectly (by the medium of *B*) linked. Hereafter, the term “link” means not only “link directly” but also “link indirectly” and we use the term “linkage” to express “the state of being linked”.

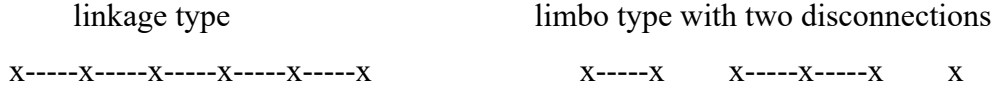
The IDL patterns can be classified into two types. One is when all the countries are linked through link commodities. We refer to this as *the linkage type*. In this type, the relative wage rates of all countries and the relative prices of all commodities can be expressed by labor input coefficients according to each IDL pattern. In other words, once the patterns are determined, all the relative wage rates and commodity prices (hereafter, *the wage rates/prices*) are determined by the patterns themselves, or there is a one-to-one correspondence between the IDL patterns and the wage rates/prices.

The second type is called *the limbo type*. In this type, the linkage of countries is not perfect, and one or more disconnections of the linkage occur. Therefore, determining all the wage rates/prices only by the IDL patterns is not possible. Theoretically, the disconnection can occur in the range from 1 to  $M-1$ . When there are  $M-1$  disconnections, perfect specialization patterns (hereafter, *PSPs*) are formed, which have no link commodities and are ordinary cases in a textbook explanation of a two-country two-commodity trade model: each country specializes in a commodity with a comparative advantage. An ordinary case in a two-country two-commodity model is an extreme case in a multi-country multi-commodity model. We need to pay attention that, except this extreme case, there are link commodities also in the limbo type and that the link commodities perform the above-mentioned functions.

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<sup>11</sup> Suppose that countries *A* and *B* produce a same link commodity (e.g. commodity 1). Then, the commodity's price ( $p_1$ ) is expressed as the product of wage rates ( $w_A$ ,  $w_B$ ) and labor input coefficients ( $a_{A1}$ ,  $a_{B1}$ ), or  $p_1 = w_A * a_{A1} = w_B * a_{B1}$ . Therefore,  $w_B/w_A = a_{A1}/a_{B1}$ .

Figure 1 illustrates these two types in a six-country case. Six countries (expressed by x) are all linked in the linkage type, whereas in the limbo type, the linkage is disconnected in two places and six countries are divided into three groups, within which more than one country is linked unless the groups consist of a single country.



**Fig. 1** An example of the two types of the IDL patterns

### 3.1.2 Derivation of the equilibrium solution in the linkage type

Here, we explain a way to derive equilibrium solution in the linkage type practically. The process has three phases as follows: searching for and identifying reasonable IDL patterns, setting up simultaneous equations according to each IDL pattern and solving them mathematically, and selecting an economically meaningful solution set. We describe details in order.

First, we have to search for and identify reasonable IDL patterns classified into the linkage type. Whether an IDL pattern is reasonable is determined only by the labor input coefficients and judgement of reasonableness is, though laborious, simple. As, in the linkage type, all the wage rates/prices are already known according to the IDL patterns, we only have to compare the commodity prices with the production costs of non-active points, which are obtained by multiplying their labor input coefficients by the each country's wage rate. The number of reasonable patterns of the linkage type is  $(M+N-2)!/\{(M-1)!(N-1)!\}$  in an M-country N-commodity case<sup>12</sup>. If M and N are large, because of a large number of IDL patterns to be judged<sup>13</sup>, it is difficult even to identify the patterns. Including the rest of the process, the support of computer program would be needed in order to calculate actually.

<sup>12</sup> Based on Shiozawa (2012, p. 50), the number of reasonable IDL patterns with  $l$  disconnections is  $(M+N-l-2)!/\{(M-l-1)!(N-l-1)!\}$ . By substituting zero for  $l$  in this expression, the number of linkage type IDL patterns is obtained since the linkage type has no disconnection.

<sup>13</sup> The number is  $\{M^{(N-1)}\} \{N^{(M-1)}\}$  according to Shiozawa's direct suggestion.

Second, for all the reasonable patterns, we identify the wage rates/prices and formulate simultaneous equations, which are composed of  $M$  equations expressing full employment conditions of each country and  $N$  equations expressing supply-demand balance for each commodity. However, independent equations are  $M+N-1$ , because one of  $N$  equations is not independent owing to Walras' law. Unknowns are production volumes in each active point, the number of which is  $M+N-1$  in the linkage type (McKenzie, 1954a, p. 175). Why  $M+N-1$ ? There must be  $N$  active points to ensure that all commodities are produced. In this situation, the countries are entirely unconnected. Therefore,  $M-1$  additional active points are needed to link all of them. Thus, as the equations and unknowns are equal in number, we can solve all the sets of equations mathematically. However, whether the solutions obtained mathematically are meaningful economically is another problem, leading to the next process.

Third, since the production volumes must be positive economically, we have to select a set having all positive solutions from the  $(M+N-2)!/\{(M-1)!(N-1)!\}$  sets of solutions. If there is such a set, the solutions of this set are the equilibrium solutions required. The IDL pattern, production volumes, and the wage rates/prices are determined. The consumption volumes and export-import volumes in each country are also able to be calculated easily.

We now give an example of the above-mentioned in the case of a three-country four-commodity. There are the three countries of  $A$ ,  $B$ , and  $C$  and the four commodities of 1, 2, 3, and 4. We define  $a_{ij}$ ,  $b_{ij}$ ,  $L_i$ ,  $p_j$ , and  $w_i$  as commodity  $j$ 's labor input coefficient in country  $i$ , commodity  $j$ 's expenditure coefficient in country  $i$ , volumes of usable labor in country  $i$ , commodity  $j$ 's price, and wage rate of country  $i$ , respectively. The numéraire is commodity 1. Commodity  $j$ 's production volumes in country  $i$  are expressed by  $x_{ij}$ . Consumption volumes are expressed as  $w_i L_i b_{ij} / p_j$  and export-import volumes are difference between production volumes and consumption volumes in each country. In a three-country four-commodity case, the production volumes of six active points are unknowns and those of non-active points are zero. For example, in the IDL pattern that country  $A$  produces commodities 1 and 2, country  $B$  commodities 2 and 3, and country  $C$  commodities 3 and 4, the wage rates/prices and simultaneous equations are expressed as follows. Although we have to rewrite these in the case of other patterns, this is easy and would be sufficient for exemplification.

Prices and wage rates:

$$p_1=1$$

$$p_2=a_{A2}/a_{A1}$$

$$p_3=(a_{B3}/a_{B2}) * p_2=(a_{B3}/a_{B2}) * (a_{A2}/a_{A1})$$

$$p_4=(a_{C4}/a_{C3}) * p_3=(a_{C4}/a_{C3}) * (a_{B3}/a_{B2}) * (a_{A2}/a_{A1})$$

$$w_A=1/a_{A1}$$

$$w_B=(a_{A2}/a_{B2}) * w_A=a_{A2}/(a_{B2} * a_{A1})$$

$$w_C=(a_{B3}/a_{C3}) * w_B=(a_{B3} * a_{A2}) / (a_{C3} * a_{B2} * a_{A1})$$

Conditions of full employment:

$$a_{A1} * x_{A1} + a_{A2} * x_{A2} = L_A$$

$$a_{B2} * x_{B2} + a_{B3} * x_{B3} = L_B$$

$$a_{C3} * x_{C3} + a_{C4} * x_{C4} = L_C$$

Conditions of supply-demand balance (only three of the four are independent):

$$x_{A1} * p_1 = w_A * L_A * b_{A1} + w_B * L_B * b_{B1} + w_C * L_C * b_{C1}$$

$$x_{A2} * p_2 + x_{B2} * p_2 = w_A * L_A * b_{A2} + w_B * L_B * b_{B2} + w_C * L_C * b_{C2}$$

$$x_{B3} * p_3 + x_{C3} * p_3 = w_A * L_A * b_{A3} + w_B * L_B * b_{B3} + w_C * L_C * b_{C3}$$

$$x_{C4} * p_4 = w_A * L_A * b_{A4} + w_B * L_B * b_{B4} + w_C * L_C * b_{C4}$$

As the wage rates/prices are expressed only by the labor input coefficients, we can confirm that, once the IDL pattern is determined, the wage rates/prices are determined only by the condition of production techniques. Although there are no explicit expressions, by multiplying the both sides of full employment expressions by wage rates, and also by deforming the left-hand sides adequately: by replacing products of labor input coefficients and wage rates with commodity prices, and furthermore by multiplying the right-hand sides by the summation expressions of expenditure coefficients or  $b_{i1} + b_{i2} + b_{i3} + b_{i4}$  ( $=1$ ), we can also confirm that national income (summation of production volumes multiplied by prices) equals national expenditure (summation of amounts expended on each commodity)<sup>14</sup> or trade equilibrium is

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<sup>14</sup> For example, about country A:  $p_1 * x_{A1} + p_2 * x_{A2} = w_A * L_A * (b_{A1} + b_{A2} + b_{A3} + b_{A4})$ .

fulfilled. If the solutions of the above equations are all positive, these are the equilibrium solutions. When the solutions include zero or negative production volumes, we have to solve other sets of equations.

### 3.1.3 Derivation of the equilibrium solution in the limbo type

If there is no set that all the solutions are positive, it means that one or more disconnections of the linkage occur. Then, we must expand the search range to find the equilibrium solution to the IDL patterns of the limbo type. The process is as follows.

First, the same as the linkage type, we identify the reasonable patterns of the limbo type. The number of the patterns are very large:  $\sum(M+N-l-2)!/\{(M-l-1)!(N-l-1)!\}$  ( $l=1, 2, \dots, M-1$ )<sup>15</sup>. Also in the case of the limbo type, whether the pattern is reasonable is determined only by the labor input coefficients. However, because not all the wage rates/prices are determined according to the IDL patterns, we have to adopt a different way from the linkage type. There are two methods.

We explain those in the case of the IDL patterns with  $l$  disconnections. In this case, countries are divided into  $l+1$  groups and the IDL has to be reasonable within each group and also among groups. Reasonableness of the IDL within each group is able to be checked easily because the relative wage rates are determined by linkage. For the IDL among groups to be reasonable, there is a condition. It is that relative wage rates between countries belonging to different groups have to be within a specific range. This wage rates constraint condition has to be met between all the combinations of two out of the  $l+1$  groups, or there are  ${}_{l+1}C_2$  constraint conditions relating wage rates. If wage rates satisfying all these conditions can exist under an IDL pattern, the IDL pattern is judged as reasonable (we show examples in subsubsection 3.1.4). Contrary, if these conditions are contradictory each other, the IDL pattern is judged as not reasonable. For convenience of explanation, we describe this method accompanied by identification of the range of wage rates as *the judging method 1*.

The other method called *the judging method 2* uses the identified IDL patterns of linkage type. If, while holding the condition that all the commodities are produced and all the countries produce at least one commodity, we remove one active point of a

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<sup>15</sup> See fn. 12. All the patterns from with one disconnection to with  $M-1$  disconnections are summed.

linkage type IDL pattern, one disconnection occurs and a limbo type IDL pattern with one disconnection is derived. Further, by adding the same operation to this newly obtained pattern, we can obtain an IDL pattern with two disconnections. By repeating the same operation up to  $M-1$  disconnections, we can identify all the limbo type IDL patterns (we show an example in subsubsection 3.1.4).

Next, after identifying the patterns, we formulate equations and solve them. When the number of disconnections is  $l$ ,  $l+1$  country groups are formed. While the wage rates/prices within each country group are determined by the IDL pattern itself, those between groups are not determined only by the pattern. To determine all the wage rates/prices, we have to add the wage rate of a country in each group which does not produce a numéraire commodity as the unknown<sup>16</sup>. The number of additional unknowns is  $l$ . On the other side, the number of active points diminishes by the number of disconnections in the limbo type (McKenzie, 1954a: see the judging method 2 in order to understand intuitively). Eventually, regardless of the number of disconnections, the total unknowns are still  $M+N-1$ , and we can solve all the sets of equations mathematically.

Finally, we have to select a set of solutions that fulfills the following two conditions: all the solutions are positive and the obtained wage rates are within the adequate range in the case of using the judging method 1 or the solution set passes *a competitive test* in the case of the judging method 2. This test is to check whether non-active points are competitive by comparing the production costs of non-active points with the commodity prices. As the entire wage rates/prices are already obtained, the test is simple. If at least one non-active point is competitive, the set is disqualified. Of course, the verification of the wage rates' range and the competitive test are equivalent. In any case, only one set satisfies these two conditions and this set is the equilibrium solution.

In the pattern that country  $A$  produces commodities 1 and 2, country  $B$  commodities 3 and 4, and country  $C$  commodity 4 only, the wage rates/prices and simultaneous equations are expressed as below. As some of wage rates are unknowns in the limbo type, we express them as  $x_i$  ( $i=A, B, C$ ). Here, the production volumes of five active points and country  $B$ 's wage rate ( $x_B$ ) are unknowns.

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<sup>16</sup> Although it is possible to add not wage rates but commodity prices as unknowns, we select wage rates in view of tractability.

Prices and wage rates:

$$p_1=1$$

$$p_2=a_{A2}/a_{A1}$$

$$p_3=a_{B3}*x_B$$

$$p_4=a_{B4}*x_B$$

$$w_A=1/a_{A1}$$

$$w_B=x_B$$

$$w_C=(a_{B4}/a_{C4})*x_B$$

Conditions of full employment:

$$a_{A1}*x_{A1}+a_{A2}*x_{A2}=L_A$$

$$a_{B3}*x_{B3}+a_{B4}*x_{B4}=L_B$$

$$a_{C4}*x_{C4}=L_C$$

Conditions of supply-demand balance (only three of the four are independent):

$$x_{A1}*p_1=w_A*L_A*b_{A1}+x_B*L_B*b_{B1}+(a_{B4}/a_{C4})*x_B*L_C*b_{C1}$$

$$x_{A2}*p_2=w_A*L_A*b_{A2}+x_B*L_B*b_{B2}+(a_{B4}/a_{C4})*x_B*L_C*b_{C2}$$

$$x_{B3}*p_3=w_A*L_A*b_{A3}+x_B*L_B*b_{B3}+(a_{B4}/a_{C4})*x_B*L_C*b_{C3}$$

$$x_{B4}*p_4+x_{C4}*p_4=w_A*L_A*b_{A4}+x_B*L_B*b_{B4}+(a_{B4}/a_{C4})*x_B*L_C*b_{C4}$$

If the solution set obtained fulfills the above-mentioned two conditions, this is the equilibrium solution. Otherwise, we have to calculate about other IDL patterns.

In the limbo type, a small change in demand may cause a change in the wage rates/prices. By solving the above equations for  $x_B$

$$x_B = L_A/a_{A1}*(b_{A3}+b_{A4})/\{a_{B4}/a_{C4}*L_C*(b_{C1}+b_{C2})+L_B*(b_{B1}+b_{B2})\}$$

This expression shows the following<sup>17</sup>: expenditure coefficients of all countries are involved in determination of the wage rates of countries *B* and *C* (it should be noted that although there is not e.g.  $b_{A1}$  or  $b_{A2}$  in the above expression, a change in these may brings about a change in  $b_{A3}$  or  $b_{A4}$ ); if country *A* increases (decreases) expenditure coefficients of commodities 3 and 4 produced in countries *B* and *C*, the wage rates of

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<sup>17</sup> Because this expression is effective as long as the IDL pattern does not change, the following explanation assumes that there is no change in the pattern.

countries  $B$  and  $C$  increase (decrease); conversely, if countries  $B$  and  $C$  increase (decrease) expenditure coefficients of commodities 1 and 2 produced in country  $A$ , wage rates of both countries decrease (increase). The reason is simple. In this IDL pattern, since quantity adjustment does not work between group of commodities 1 and 2 and group of commodities 3 and 4, national income and commodity prices, therefore wage rates have to change in order to absorb change in expenditure coefficients with full employment and trade equilibrium kept<sup>18</sup>.

### ***3.2 Analysis of the model by using the three-country four-commodity numerical example***

#### **3.2.1 Identification of the reasonable IDL patterns**

We set a three-country four-commodity numerical example and analyze characteristics of the Graham-type model by observing changes in the equilibrium values arising from changes in the given conditions. Labor input coefficients are given as Table 1. Here, country  $A$  symbolizes developed country, country  $B$  emerging country, and country  $C$  developing country. Units of the commodities are chosen in such a manner that all the labor input coefficients of country  $A$  are one and the commodities are numbered in order of diminishing country  $A$ 's comparative advantage between countries  $A$  and  $B$ <sup>19</sup>. Labor input coefficients of country  $C$  are given arbitrarily. The numéraire is commodity 1.

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<sup>18</sup> We will give a supplementary explanation. Suppose that  $b_{A3}$  increases a little,  $b_{A1}$  decreases a little and others do not change. Then, if we define commodity  $j$ 's consumption volumes in country  $i$  as  $c_{ij}$ ,  $c_{A1}$  ( $=w_A L_A b_{A1}/p_1$ ) decrease,  $c_{A2}$  ( $=w_A L_A b_{A2}/p_2$ ) is unchanged, and  $c_{B1}$ ,  $c_{B2}$ ,  $c_{C1}$ , and  $c_{C2}$  moves in the same direction with  $w_B$  and  $w_C$ . In order to fulfill country  $A$ 's full employment,  $c_{B1}$ ,  $c_{B2}$ ,  $c_{C1}$ , and  $c_{C2}$ , therefore  $w_B$  and  $w_C$  must increase.

<sup>19</sup> Although, without this process, the number of the IDL patterns to be investigated reaches 432, the number diminishes to 112 owing to this process.

**Tab. 1** Labor input coefficients

	Comm.1	Comm.2	Comm.3	Comm.4
Country A	1	1	1	1
Country B	5	4	3	2
Country C	60	25	30	7

From this table, ten linkage type IDL patterns are determined as bellow. First parentheses show the IDL patterns and e.g. A123 means that country A produces commodities 1, 2, and 3, second parentheses show commodity prices in order from commodity 1 to 4, and third wage rates from country A to C.

#### Linkage type IDL patterns

4+1+1 type (a country produces 4 commodities and the other two 1 each)

- 1) (A1234; B4; C4) (1; 1; 1; 1) (1; 1/2; 1/7)
- 2) (A1; B1234; C4) (1; 4/5; 3/5; 2/5) (1; 1/5; 2/35)
- 3) (A1; B1; C1234) (1; 25/60; 1/2; 7/60) (1; 1/5; 1/60)

3+2+1 type (a country produces 3 commodities, another 2, and the other 1)

- 4) (A123; B34; C4) (1; 1; 1; 2/3) (1; 1/3; 2/21)
- 5) (A123; B3; C24) (1; 1; 1; 7/25) (1; 1/3; 1/25)
- 6) (A1; B123; C24) (1; 4/5; 3/5; 28/125) (1; 1/5; 4/125)
- 7) (A12; B234; C4) (1; 1; 3/4; 1/2) (1; 1/4; 1/14)
- 8) (A13; B3; C234) (1; 5/6; 1; 7/30) (1; 1/3; 1/30)
- 9) (A1; B13; C234) (1; 1/2; 3/5; 7/50) (1; 1/5; 1/50)

2+2+2 (all the countries produce 2 commodities each)

- 10) (A12; B23; C24) (1; 1; 3/4; 7/25) (1; 1/4; 1/25)

Next are the limbo type IDL patterns. The IDL patterns and wage rates (including their range) are shown below (commodity prices are omitted). In the patterns with two disconnections, country C needs to fulfill two wage rates constraints.

#### Limbo type IDL patterns with one disconnection

- 11) (A123; B4; C4) (1; 1/3-1/2;  $2w_B/7$ )
- 12) (A12; B34; C4) (1; 1/4-1/3;  $2w_B/7$ )
- 13) (A1; B234; C4) (1; 1/5-1/4;  $2w_B/7$ )
- 14) (A1; B23; C24) (1; 1/5-1/4;  $4w_B/25$ )

- 15) (A1; B3; C234) (1; 1/5-1/3;  $w_B/10$ )      16) (A12; B3; C24) (1; 1/4-1/3; 1/25)  
 17) (A13; B3; C24) (1; 1/3; 1/30-1/25)      18) (A1; B123; C4) (1; 1/5; 4/125-2/35)  
 19) (A1; B1; C234) (1; 1/5; 1/60-1/50)      20) (A123; B3; C4) (1; 1/3; 1/25-2/21)  
 21) (A1; B13; C24) (1; 1/5; 1/50-4/125)      22) (A12; B23; C4) (1; 1/4; 1/25-1/14)

Limbo type IDL patterns with two disconnections (PSPs)

- 23) (A12; B3; C4) (1; 1/4-1/3; 1/25-1/7,  $w_B/10-2w_B/7$ )  
 24) (A1; B23; C4) (1; 1/5-1/4; 1/60-1/7,  $4w_B/25-2w_B/7$ )  
 25) (A1; B3; C24) (1; 1/5-1/3; 1/25-1/7,  $4w_B/25-w_B/10$ )

From this list, we can confirm that there is one ( $_{1+1}C_2$ ) wage rates constraint in the limbo type patterns with one disconnection and three ( $_{2+1}C_2$ ) with two disconnections. We also give concrete examples of the judging method 2. We can derive e.g. the pattern 11) by removing A4 from the pattern 1). In the same way, 11) or 12) is derived by removing B3 or A3 from 4), 23) by removing B4 from 12), and so on. Furthermore, by paying attention to the wage rate of 11) which is derived from 1) and 4), we can see that it lies between those of 1) and 4). Such a relation of derivation and wage rates suggests that e.g. 1) and 4) adjoin each other and 11) forms the boundary between 1) and 4) on the world production frontier (hereafter, **WPF**). According to Shiozawa (2007), the WPF of multi-country multi-commodity has a shape of convex polytope which is covered by facets, the number of which is 10 in the case of three-country four-commodity, and each facet represents each IDL pattern of the linkage type, and joints of the facets represent the IDL patterns of the limbo type. In two-dimensional graphs of two-country (or multi-country) two-commodity, lines correspond to the linkage type patterns and vertexes the limbo type patterns, and in three-dimensional graphs of three-country four-commodity, surfaces correspond to the linkage type patterns and ridgelines the limbo type patterns<sup>20</sup>.

On closer examination of these patterns, we can see that there are many cases that some active points do not follow the grades of comparative advantage between two countries. For example, although country B's commodity 4 is the lowest comparative

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<sup>20</sup> See e.g. McKenzie (1954b, p. 151) about the two-dimensional graphs and Figure 2 in **3. 2. 5** about the three-dimensional graphs.

advantage between countries *B* and *C*, country *B* produces only commodity 4 together with country *C* in the patterns 1) and 11). In 3) and 19), country *B*, together with country *A*, produces only commodity 1 in which country *B* has the lowest comparative advantage between countries *A* and *B*. Similarly, in 5) , 8), 15), 16), 17), 20), 23), and 25), country *B* produces only commodity 3 in which country *B* does not have the highest comparative advantage between countries *A* and *B*, and also between countries *B* and *C*. Simple relationships between two countries disappear behind complicated relationships among three countries. This indicates that research into multi-country multi-commodity models have a intrinsic significance of its own in the same way that simplicity and clearness of two-country two-commodity and two-country multi-commodity models have a important meaning.

### 3.2.2 Wage rates and the IDL patterns

As country *A*'s wage rate is always 1, we pick up only wage rates of countries *B* and *C* and compile them into Table 2 by limiting to the linkage type. Wage rates (WR in the table) are arranged in the decreasing order, and also the number of active points (NAP) and the IDL patterns (Patterns) are shown.

**Tab. 2** Wage rates, the number of active points, and the IDL patterns

Country <i>B</i>	WR	1/2	1/3	1/3	1/3	1/4	1/4	1/5	1/5	1/5	1/5
	NAP	1	1	1	2	2	3	1	2	3	4
	Patterns	1)	5)	8)	4)	10)	7)	3)	9)	6)	2)
Country <i>C</i>	WR	1/7	2/21	1/14	2/35	1/25	1/25	1/30	4/125	1/50	1/60
	NAP	1	1	1	1	2	2	3	2	3	4
	Patterns	1)	4)	7)	2)	5)	10)	8)	6)	9)	3)

There are very large wage rates differentials according to the IDL patterns. Generally, the fewer the number of active points is, the more advantageous the wage rate is. However, it is not absolutely so. The NAP of country *B* varies from 1 to 4 in four patterns with the same WR of 1/5, which is the most disadvantageous for country *B*. Also in the case of country *C*, there is one exception. Such a state never occurs in two-country multi-commodity. This is a peculiar phenomenon to multi-country multi-commodity.

The range of the wage rates differentials reaches out from the minimum to maximum of the productivity differentials of individual sectors between all the combinations of two out of the three countries, namely from  $1/2$  to  $1/5$  between countries *A* and *B*, from  $1/7$  to  $1/60$  between countries *A* and *C*, and from  $2/7$  to  $5/60$  between countries *B* and *C*, which can be easily calculated from Table 2. This is a matter of course, considering that wage rates differentials are productivity differentials of link commodities. This is merely another expression of the fact that all the commodities have a possibility to be link commodities at a point of time when only production techniques are given, or before volumes of usable labor and demand structures are given.

### 3.2.3 Determination of Wage rates, commodity prices, and production volumes

Here, by giving volumes of labor and expenditure coefficients, we determine the wage rates/ prices and production volumes. Assume that country *A* has 1000 labor, country *B* 1000, and country *C* 3000 and all the four expenditure coefficients in each country are 0.25. When we calculate production volumes of active points about all the patterns from 1) to 10), only 4) has all positive solutions, and others have one or two negative solutions. Under these conditions, the IDL pattern 4) is formed and production volumes and consumption volumes are as Table 3 (numerical values are rounded: hereafter the same shall apply), and differentials between both volumes are export-import volumes.

**Tab. 3** Production volumes and consumption volumes (1)

Pattern 4)	Production volumes				Consumption volumes			
	Comm.1	Comm.2	Comm.3	Comm.4	Comm.1	Comm.2	Comm.3	Comm.4
Country <i>A</i>	404.8	404.8	190.5		250.0	250.0	250.0	375.0
Country <i>B</i>			214.3	178.6	83.3	83.3	83.3	125.0
Country <i>C</i>				428.6	71.4	71.4	71.4	107.1

We change expenditure coefficients: for all countries, those of commodities 1 and 2 from 0.25 to 0.3 and those of commodities 3 and 4 from 0.25 to 0.2. Also at this time, the pattern having all positive solutions is only 4). Therefore, the wage rates/prices do not change at all. Production volumes and consumption volumes are as Table 4.

**Tab. 4** Production volumes and consumption volumes (2)

Pattern 4)	Production volumes				Consumption volumes			
	Comm.1	Comm.2	Comm.3	Comm.4	Comm.1	Comm.2	Comm.3	Comm.4
Country A	485.7	485.7	28.6		300.0	300.0	200.0	300.0
Country B			295.2	57.1	100.0	100.0	66.7	100.0
Country C				428.6	85.7	85.7	57.1	85.7

Changes in demand are adjusted by changes in production volumes and export-import volumes without price changes. What Graham indicated occurs.

### 3.2.4 Distribution of labor among countries and the IDL patterns

In this subsubsection, we investigate the effects of distribution of labor among countries on the IDL patterns. We Start from Table 4 and reduce country A's labor one by one from initial value 1000, remaining the others unchanged. Thereby, country A's production volumes diminish with diminishing labor. When the labor reaches 928, production volumes of commodity 3 become negative, and there are no linkage type IDL patterns having all positive solutions. As we can easily predict, only the limbo type IDL pattern 12) satisfies the above-mentioned two conditions (see 3.1.3).

Let us continue to reduce country A's labor. The pattern 12) is held for a certain time, and, during the time, the wage rates of countries B and C continue to decrease together with prices of commodities 3 and 4. When country A's labor reaches 696, country B's production costs of commodity 2 come to 0.9994 and fall below the commodity price (=1). Country B's industry of commodity 2 starts production activity and the IDL pattern 7) is formed. For reference, we show an example of the pattern 12) in Table 5.

**Tab. 5** Production volumes and consumption volumes (3)

Pattern 4)	Production volumes				Consumption volumes			
	Comm.1	Comm.2	Comm.3	Comm.4	Comm.1	Comm.2	Comm.3	Comm.4
Country A	400.0	400.0			240.0	240.0	185.7	278.6
Country B			309.5	35.7	86.2	86.2	66.7	100.0
Country C				428.6	73.8	73.8	57.1	85.7

This is the case that country A's labor is 800. The wage rates are (1; 0.29; 0.08), and

commodity prices (1; 1; 0.86; 0.57). In harmony with that the pattern 12) is derived from 4) and 7), the wage rates/prices of 12) are also between those of 4) and 7).

Thus, changes in distribution of labor among counties change the IDL patterns. All the transitions of the patterns which are induced by changes in country A's labor are as follows.

Country A's labor	the IDL patterns	wage rates
Over 3714	1): (A1234; B4; C4)	(1; 1/2; 1/7)
3714–2477	11): (A123; B4; C4)	(1; 1/3-1/2; $2w_B/7$ )
2476–929	4): (A123; B34; C4)	(1; 1/3; 2/21)
928–697	12): (A12; B34; C4)	(1; 1/4-1/3; $2w_B/7$ )
696–608	7): (A12; B234; C4)	(1; 1/4; 1/14)
607–230	22): (A12; B23; C4)	(1; 1/4; 1/25-1/14)
229–159	10): (A12; B23; C24)	(1; 1/4; 1/25)
158–127	14): (A1; B23; C24)	(1; 1/5-1/4; $4w_B/25$ )
Under 127	6): (A1; B123; C24)	(1; 1/5; 4/125)

From the observation above, we can indicate four points. First, in the situation that labor input coefficients and expenditure coefficients are fixed, changes in distribution of labor among countries change the IDL patterns and therefore the wage rates/prices. The cause is changes in shapes of the WPF. While changes in composition of demand volumes are very small because of the fixed expenditure coefficients<sup>21</sup>, shapes of the WPF change largely. As a result, demand points lie on the different facets (or the IDL patterns) of the WPF (see **Appendix 3**).

Second, the more country A's relative labor volumes to both countries B and C are, the higher the both countries' wage rates are. Although, there are some exceptions as stated above (see Table 2), ceteris paribus, the smaller labor volumes are, the more favorable wage rates are in general. Third, in the transition process of the IDL patterns, the linkage type and the limbo type appear alternately. Fourth, this transition process progresses along the patterns resembling mutually without a jump. This means that

<sup>21</sup> By expressing commodity j's demand volumes in country i as  $d_{ij}$ , we obtain  $d_{ij} = b_{ij} w_{ij} L_{ij} / p_j$ . Because  $w_{ij}$  and  $p_j$  change with changes in IDL patterns,  $d_{ij}$  changes a little.

changes in the wage rates/prices caused by changes in the IDL patterns are not radical but gradual unless changes in the given conditions are radical.

### 3.2.5 Changes in demand and the IDL patterns

Now, we observe the movement of equilibrium solutions which is induced by changes in expenditure coefficients under the condition that the WPF is fixed. Labor input coefficients are the same as Table 1, country *A* has 1000 labor, country *B* 1000, and country *C* 3000. Each country's expenditure coefficients are equal for each commodity: those of commodities 2 and 3 are fixed at 0.25, and those of commodities 1 and 4 change 0.01 by 0.01 from 0.01 to 0.49. Then, the transition of the IDL patterns is as follows. Only expenditure coefficients of commodity 1 are shown and those of commodity 4 (0.5 – expenditure coefficients of commodity 1) are omitted.

Expenditure coefficients	the IDL patterns	wage rates
0.01	1): (A1234; B4; C4)	(1; 1/2; 1/7)
0.02–0.11	11): (A123; B4; C4)	(1; 1/3–1/2; $2w_B/7$ )
0.12–0.32	4): (A123; B34; C4)	(1; 1/3; 2/21)
0.33–0.41	20): (A123; B3; C4)	(1; 1/3; 1/25–2/21)
0.42–0.49	5): (A123; B3; C24)	(1; 1/3; 1/25)

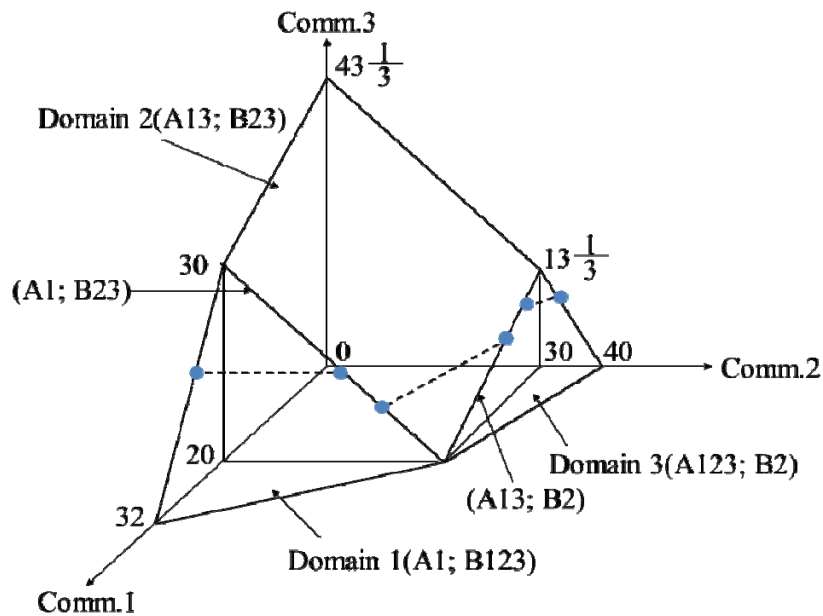
Changes in expenditure coefficients also change the IDL patterns. In the transition of the patterns, the linkage type and the limbo type appear alternately, and the transition progresses along the patterns resembling mutually. Let us pay attention to the movement of country *C*'s wage rates. The wage rates continue to decrease intermittently with the decline in the expenditure coefficients of commodity 4, in which country *C* has the highest comparative advantage. In general, the larger the expenditure coefficients of commodities having higher comparative advantage become, the more favorable the wage rates become.

Especially in the limbo type, there is another interesting point about the relation between demand volumes and the wage rates/prices. We explain this point by using a two-country three-commodity example. Labor input coefficients and volumes of labor are as Table 6, and each country's expenditure coefficients are equal for each

commodity: those of commodity 3 is fixed at 0.2, and those of commodity 1 increases 0.01 by 0.01 from 0.01 to 0.79, and those of commodity 2 decreases 0.01 by 0.01 from 0.79 to 0.01.

**Tab. 6** Labor input coefficients and volumes of labor

	Labor input coefficients			Volumes of labor
	Comm.1	Comm.2	Comm.3	
Country A	20	40	30	400
Country B	50	20	20	600



**Fig. 2** World production frontier and the locus of production point

The WPF and the locus of production points are drawn in Figure 2<sup>22</sup>. The three domains 1, 2, and 3 are the linkage type IDL patterns, and the two ridgelines are the limbo type IDL patterns, contents of which are shown in each parenthesis. Numerical values are the possible production volumes of each commodity at each point. The dotted line is the locus of production points, and starts from the point of right end, via

<sup>22</sup> This example and the figure are based on Shiozawa (2014).

domain 3, ridgeline, domain 2, ridgeline, and domain 1, and reaches the point of left end. It should be noted that the movement of the expenditure coefficients is converted into the movement of the physical quantities. The reason why, in spite of fixed expenditure coefficients, commodity 3's volumes change on each domain and ridgeline, is that the wage rates/prices change according to the IDL patterns<sup>23</sup>.

At first, we confirm the relative wage rates. When the wage rate of country *A* is 1, that of country *B* is 2 on the domain3,  $2 \rightarrow 1.5$  on the first ridgeline, 1.5 on the domain 2,  $1.5 \rightarrow 0.4$  on the next ridgeline, and 0.4 on the domain3. While, on each domain, production points move in proportion to expenditure coefficients without change in the wage rates/prices, on each ridgeline, those move with accompanied by change in the wage rates/prices. The transitions of production points from the ridgelines to the domains occur at the time when country *B*'s non-active points (of commodity 3 firstly, and next, of commodity 1) become competitive in the deterioration process of country *B*'s wage rate.

We explain in more detail about a state occurring on the ridgelines. On the first ridgeline, quantity adjustments without price changes are conducted between commodities 1 and 3. On the other hand, production volumes of commodity 2 of which expenditure coefficients decrease do not change, and its price changes. On the second ridgeline, a similar thing occurs. Thus, an important characteristic of Graham-type trade model that prices and quantities move independently is valid not only in the linkage type but also in the limbo type.

### 3.2.6 Changes in production techniques and the wage rates/prices

Here, we describe the effects of changes in production techniques on the wage rates/prices. Suppose that the IDL pattern 10) (*A*12; *B*23; *C*24) is formed. Commodity 2 is the common link commodity among three countries, wage rates are (1; 1/4; 1/25), and commodity prices (1; 1; 3/4; 7/25).

What happens when the labor productivity of commodity 4 in country *C* doubles (the labor input coefficient halves)? Nothing happens except that the price of commodity 4 halves. Country *C*'s real wage rate certainly increases due to the decline

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<sup>23</sup> See fn. 21.

in price of commodity 4, but this is common with the other countries in which labor productivity does not increase at all. This is not so good results for country *C*. The fruits of increases in labor productivity of commodities produced in only home country leak into foreign countries<sup>24</sup>.

On the contrary, increases in labor productivity of the link commodities raise the home countries' wage rates. Increases in wage rates, in turn, raise production costs of commodities of which increases in labor productivity are lower than those of the link commodities. Suppose that the labor productivity of commodity 2 in country *C* increases and consequently country *C*'s wage rate increases. Then, there can be two consequences. One is the case that the commodity 4 made in country *C* holds competitiveness in spite of price increases and commodity terms of trade of both countries *A* and *B* deteriorate. The other is the case that the commodity 4 made in country *C* loses competitiveness and the IDL pattern changes. Unlike changes in distributions of labor among countries and demand structures, changes in production techniques always change commodity prices. If changes in production technique occur in plural sectors of plural countries, and thereby the structures of comparative advantage change widely, the IDL patterns and the wage rates/prices also change widely.

### 3.2.7 Probability of the IDL patterns: Graham case versus Mill case

In the case of three-country four-commodity, there are 10 linkage type IDL patterns, 12 limbo type IDL patterns with one disconnection, and 3 limbo type IDL patterns with two disconnections (PSPs). Which patterns do have high frequency of appearance? As described before, it is clear that which patterns are formed is determined by the interaction among production techniques, distribution of labor, and demand structures. However, it is very difficult to calculate a probability of the appearance in a three-country four-commodity case. Therefore, we try it in a simple two-country three-commodity case (countries *A* and *B*, commodities 1, 2, and 3).

Labor input coefficients of country *A* are standardized as one and those of country *B* are expressed  $a_1$ ,  $a_2$ , and  $a_3$  ( $a_1 > a_2 > a_3 > 0$ ) in order from commodity 1 to 3. Volumes

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<sup>24</sup> Such a thing was indicated in Pasinetti (1981) and Lewis (1969).

of labor in country *A* are one, and those of country *B*  $L_B (> 0)$ . Expenditure coefficients of both countries are each  $1/3$  for all the commodities and the numéraire is commodity 1. Then, country *A*'s wage rate is always one, and there exist five reasonable IDL patterns: (A123; B3), (A12; B3), (A12; B23), (A1; B23), and (A1; B123). Of course, expenditure coefficients also have an effect on the probability of the IDL patterns. If an expenditure coefficient of a specific commodity is larger, the probability of the pattern which both countries produce the specific commodity is larger (see fn. 25). Here, by setting the coefficients uniform, we remove expenditure coefficients bias.

At first, let us examine the conditions that the pattern (A123; B3) is formed. In this pattern, commodity prices are (1; 1; 1) and the wage rates are (1;  $1/a_3$ ). By expressing production volumes of active points as  $x_{ij}$  ( $i=A, B$ ;  $j=1, 2, 3$ ), conditions of full employment and supply-demand balance are as follows.

$$\begin{aligned}x_{A1}+x_{A2}+x_{A3}&=1 \\a_3*x_{B3}&=L_B \\x_{A1}&=1/3+(1/3)*L_B/a_3 \\x_{A2}&=1/3+(1/3)*L_B/a_3 \\x_{A3}+x_{B3}&=1/3+(1/3)*L_B/a_3\end{aligned}$$

When all the solutions are positive, this pattern is formed. It is obvious that  $x_{A1}$ ,  $x_{A2}$ , and  $x_{B3}$  are positive, but  $x_{A3}$  is not so. By solving the above equations for  $x_{A3}$ , we obtain  $x_{A3}=1/3-(2/3)*L_B/a_3$ , accordingly,  $x_{A3} > 0 \Leftrightarrow L_B < a_3/2$ . Therefore, if  $L_B < a_3/2$ , the pattern (A123; B3) is formed.

Similarly, we examine the pattern (A12; B23). In this case, commodity prices are (1;  $1/a_3/a_2$ ), wage rates are (1;  $1/a_2$ ), and the conditions are as follows.

$$\begin{aligned}x_{A1}+x_{A2}&=1 \\a_2*x_{B2}+a_3*x_{B3}&=L_B \\x_{A1}&=1/3+(1/3)*L_B/a_2 \\x_{A2}+x_{B2}&=1/3+(1/3)*L_B/a_2 \\a_3/a_2*x_{B3}&=1/3+(1/3)*L_B/a_2\end{aligned}$$

It is obvious that  $x_{A1}$  and  $x_{B3}$  are positive, but  $x_{A2}$  and  $x_{B2}$  are not so. By solving the equations, we obtain  $x_{A2}=2/3-(1/3)*L_B/a_2$  and  $x_{B2}=(2/3)*L_B/a_2-1/3$ . From the both expressions, we obtain  $x_{A2}$  and  $x_{B2} > 0 \Leftrightarrow a_2/2 < L_B < 2a_2$ . Though we omit details, the conditions forming the pattern (A1; B123) is  $2a_1 < L_B$ .

Next is the limbo type pattern (A12; B3). The wage rate of country B ( $w_B$ ) is added to unknowns and the equations are as follows.

$$x_{A1}+x_{A2}=1$$

$$a_3*x_{B3}=L_B$$

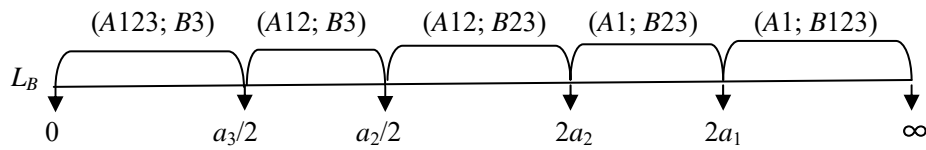
$$x_{A1}=1/3+(1/3)*w_B*L_B$$

$$x_{A2}=1/3+(1/3)*w_B*L_B$$

$$a_3*w_B*x_{B3}=1/3+(1/3)*w_B*L_B$$

In the limbo type, in addition to the condition that all the solutions are positive,  $w_B$  has to be within a specific range, namely,  $1/a_2 < w_B < 1/a_3$ . On the other hand, by solving the equations for  $w_B$ , we obtain  $w_B=1/(2L_B)$ . From the two expressions, we can understand that the conditions forming the pattern (A12; B3) is  $a_3/2 < L_B < a_2/2$ . Likewise, we can derive  $2a_2 < L_B < 2a_1$  for the pattern (A1; B23).

We can compile the above results into Figure 3.



**Fig. 3** The IDL patterns and differentials of labor productivity

This figure shows that we can calculate the probability of the IDL patterns by defining the range of  $L_B$  and values of  $a_1$ ,  $a_2$ , and  $a_3$  appropriately. If we assume that the range of  $L_B$  is from 0 to 10,  $a_1$  is 4,  $a_2$  3, and  $a_3$  2, the probability of the limbo type is 25%. If the range of  $L_B$  is from 1 to 8, within which both countries do not fail to gain from trading, the probability is roughly 36 %. Note that  $L_B$  is the relative labor volumes

of country  $B$  and  $a_j$  ( $j=1, 2, 3$ ) are the labor productivity differentials (LPD)<sup>25</sup>. Figure 3 also shows that the greater the LPD's differences between sectors are, the higher the probability of the limbo type is. With the increases in the number of countries and commodities, the probability of the limbo type becomes lower, because the LPD's differences between sectors are smaller.

The above is the two-country three-commodity case. By using our three-country four-commodity example, we will try a simulation. Labor input coefficients are the same as Table 1 and all the expenditure coefficients are  $1/4$ . We generate 400 sets of random numbers which are composed of 3 positive integers fallen in the range of 100 – 30000 (within which all the 25 IDL patterns can be formed), regard the 3 integers as distributions of labor among countries, and identify the patterns formed under the distributions. The result is as follows: the linkage type is 317 sets (79%), the limbo type with one disconnection 82sets (21%), and the limbo type with two disconnections (PSPs) 1set.

When attention is focused on individual IDL patterns, the patterns which country  $A$  produces three or more commodities are many (315 sets) regardless of the type, and contrary, the patterns which country  $C$  produces three or more commodities are null. This stems from the assumption of labor input coefficients. Owing to the country  $C$ 's production techniques assumed to be very low level in comparison with both countries  $A$  and  $B$ , country  $C$  must have very large labor volumes to produce three or more commodities. Consequently, the patterns which country  $C$  produces only one commodity are exclusive, namely, account for 394 out of 400 sets. To itemize the patterns, the linkage type is 312 sets and the limbo type 82 sets. The patterns which country  $C$  produces only one commodity are not so many especially in the linkage type

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<sup>25</sup> If we define the labor volumes and labor input coefficients of both countries as  $L_i$  and  $a_{ij}$  ( $i=A, B; j=1, 2, 3$ ), the result of calculation is replaced as follows:  $L_B \rightarrow L_B/L_A$ ,  $a_1 \rightarrow a_{B1}/a_{A1}$ ,  $a_2 \rightarrow a_{B2}/a_{A2}$ ,  $a_3 \rightarrow a_{B3}/a_{A3}$ . Also, we show the result in the case of generalized expenditure coefficients. Suppose as follows: while labor volumes and labor input coefficients are the same as this subsection, expenditure coefficients expressed by  $b_j$  ( $j=1, 2, 3$ ) are different for each commodity, but are equal between both countries. Then, four boundary points of Figure 3 are replaced in order from left side as follows:  $a_3*b_3/(1-b_3)$ ,  $a_2*b_3/(1-b_3)$ ,  $a_2*(1-b_1)/b_1$ , and  $a_1*(1-b_1)/b_1$ . Therefore, if e.g.  $b_2$  is larger ( $b_1$  and  $b_3$  are smaller), the probability of the pattern ( $A12; B23$ ) increases.

(4/10). Rather, such patterns are more in the number and also larger in the ratio in the limbo type (8/15).

Based on the above examination, we can conclude that probability of the linkage type is considerably higher than that of the limbo type, however, that of the limbo type is not so low that we can ignore it, and that of the PSPs are extremely low. We call the aspect of the quantity adjustment of supply-demand balance *the Graham case* and the aspect of the price adjustment *the Mill case*. Then, the linkage type is exclusively the Graham case, the limbo type other than PSPs is coexistence of the Graham case and the Mill case, and PSPs are exclusively the Mill case. Therefore, in the adjustment of supply-demand balance in the world trade, given the condition of constant production costs, the Graham case overwhelms Mill case.

### 3.2.8 Summary of the results obtained through observation

We have observed the relation of equilibrium values and the given conditions. The results are summarized as follows.

1. The IDL patterns, which determined only by the condition of production techniques, are many and the relative wage rates differ according to the patterns. The width of the wage rates differentials extends from the minimum to the maximum of the productivity differentials of individual sectors.
2. In multi-country multi-commodity trade models, because simple relationships between two countries withdraw behind complicated relationships among many countries, there are many cases that some active points in the reasonable IDL patterns do not follow the grades of comparative advantage between two countries selected arbitrarily.
3. By giving labor volumes and demand structures in addition to productive techniques, a specific IDL pattern is determined. If it is the linkage type, the wage rates/prices are uniquely determined by the pattern itself.
4. In the linkage type, the wage rates/prices are highly stable in the face of changes in demand, and the quantity adjustment without changes in wage rates/prices (the Graham case) is conducted.

5. According to the three givens, disconnections of the linkage occur and the limbo-type IDL patterns are formed. The frequency is certainly low but not so low that we can ignore it.
6. Also in the limbo case, the linkages and, therefore, the Graham case exist. The price adjustment responding to changes in demand (the Mill case) is limited in places where there are the disconnections.
7. Generally, with some exceptions, the smaller labor volumes are and the larger expenditure coefficient of commodities having higher comparative advantage become, the more favorable the wage rates are.
8. Productivity increase in only commodities with comparative advantage does not raise wage rates. To raise them, productivity increase in link commodities is needed.

## 4 Concluding remarks

A keyword of the Graham-type trade model is link commodities. The possibility that the link commodities exist was indicated by some economist other than Graham and earlier than Graham (see **Appendix 2**). None of them, however, thought that the link commodities are important for trade theory. There were several reasons. First, they could not cast aside a stereotype that the price adjustments or the Mill case matter because Mill's prestige was great. Second, since there were no detailed world trade statistics, they did not think that the link commodities existed in real trading world. For example, Edgeworth (1894) wrote that the standard commodity (the link commodity in our terms) was a hypothetical commodity, and though existent in fact, would probably be insignificant in magnitude (p. 634). Although, with a development of world trade statistics after the World War II, the existence of commodities produced in common in more than one country has been recognized, they have not been treated as the link commodities, but have been discussed within a framework of intra-industry trade or differentiated products (Grubel, 1967; Krugman, 1980).

Third, it is not sufficiently understood that the number of the link commodities is only a small part of the number of trade items. As previously stated, the number of the

link commodities is, in theoretical trade models, at most “the number of countries minus one”. Present world trade statistics classify trade items on the base of the HS (Harmonized Commodity Description and Coding System) code. The number of the commodity groups is approximately 1200, based on 4-digit HS code (headings), and roughly 5000 on 6-digit HS code (subheadings). We may be able to group trade items that “absolute advantages are approximately equal and elasticities of substitution relatively high” (Chipman, 1965, p. 500) into one category. Similarly, we can group countries that income per capita and economic structure are approximately equal into one category. Considered comprehensively, contrary to Chipman, we can say that there are overwhelmingly more commodities than countries. Suppose a three-country thirty-commodity model. Then, the number of the link commodities is at most two or one fifteenth. Although the link commodities may be not so noticeable in real trading world, they carry out their functions adequately and sufficiently.

The link commodities, therefore the quantity adjustments or the Graham case matters. We need to free ourselves from undue emphasis on the PSPs.

In this paper, we provided the full employment version of the Graham-type model. In the real world, however, underemployment is a normal state. Needless to say, it is desirable that the assumptions of models reflect reality, and in fact, the Graham-type model which places high priority on quantity adjustments over price adjustments is essentially compatible with an underemployment in the following meaning: the movement of productive resources among domestic sectors, which requires a long time in the real, is indispensable for quantity adjustments under the condition of full employment, while changes in the operating rate and employment rate are sufficient for those under the condition of underemployment. We will provide the underemployment version of the Graham-type trade model in our next paper.

### *Appendix 1: On Jones' perfect specialization numerical example*

Jones (1961) has provided a Ricardian three-country three commodity numerical example and showed which pattern is efficient among PSPs. The answer is the pattern which minimizes the product of labor input coefficients of active points. This is valid in a general case of M-country M-commodity. These models are the complete opposite of the Graham-type model in the sense that there are no link commodities. Table A1-1 is the Jones' numerical example: country names, commodity names, and arrangements are changed.

**Table A1-1** Jones' numerical example

	Labor input coefficients		
	Comm.1	Comm.2	Comm.3
Country	2	<b>3</b>	10
Country	4	5	<b>10</b>
Country	<b>3</b>	7	10

The labor input coefficients of the effective PSP's active points is printed in boldface. The product of these is 90, and smaller than the other combinations. This is surely one of the reasonable IDL patterns. In addition to this, the six linkage type patterns and the six limbo type patterns with one disconnection are reasonable. As already mentioned, the total 13 reasonable IDL patterns are determined only by production techniques. However, it is the distributions of labor and the demand structures to identify a specific pattern out of the 13 patterns. As mentioned later soon, when these two conditions are given arbitrary, the linkage type has the highest probability, the limbo type with one disconnection is next to it, and the PSP is uncommon. However, since the probability of the PSP is not null, we can provide examples of the PSP. Here, we provide an example under the conditions that full employment and trade equilibrium are fulfilled.

What is the condition that the PSP is formed? It is that the active points are competitive. Concretely,

$$3w_C < 2w_A \text{ and } 3w_C < 4w_B \quad (\text{commodity 1 in country C})$$

$$3w_A < 5w_B \text{ and } 3w_A < 7w_C \quad (\text{commodity 2 in country A})$$

$$10w_B < 10w_A \text{ and } 10w_B < 10w_C \quad (\text{commodity 3 in country B})$$

By simplifying,

$$3w_A/5 < w_B < w_C < 2w_A/3$$

If we can give the distributions of labor and the demand structures so as to meet this condition, the PSP is formed. Although combinations of the both to meet the condition are countless, it is considerably difficult to identify them practically. There is an idea to simplify. Assume that all the expenditure coefficients are 1/3. Then, conditions of supply-demand balance are expressed as follows.

$$x_{C1} * p_1 = w_A * L_A * (1/3) + w_B * L_B * (1/3) + w_C * L_C * (1/3)$$

$$x_{A2} * p_2 = w_A * L_A * (1/3) + w_B * L_B * (1/3) + w_C * L_C * (1/3)$$

$$x_{B3} * p_3 = w_A * L_A * (1/3) + w_B * L_B * (1/3) + w_C * L_C * (1/3)$$

By replacing production volumes of these expressions with “labor volumes divided by labor input coefficients” and commodity prices with “labor input coefficients multiplied by wage rates” and by arranging adequately, the following is obtained.

$$2L_C * w_C = w_A * L_A + w_B * L_B$$

$$2L_A * w_A = w_B * L_B + w_C * L_C$$

$$2L_B * w_B = w_A * L_A + w_C * L_C$$

By subtracting second from first expression and deforming it, we obtain  $w_A/w_C = L_C/L_A$ . We can understand that the inverse of the relative wage rates are equal to the relative labor volumes. Therefore, the above inequalities of wage rates are replaced with the follow inequalities of labor volumes.

$$3L_A/2 < L_C < L_B < 5L_A/3$$

When country A's labor is e.g. 600, those of countries B and C must be in the range of 900 – 1000 and “country C's labor < country B's labor”. Under the other distribution of labor, the PSP with full employment and trade equilibrium is not realized. If these two conditions are not important, only the fulfillment of the above inequalities of wage rates is the condition realizing the PSP, which may be accompanied by unemployment and (or) trade imbalance. We give an example of the PSP satisfying two conditions. Labor input coefficients are the same as Table A1-1, labor volumes are 600, 990, and 960 in order from country A to C, expenditure coefficients are all 1/3, and the numéraire is commodity 1. Production volumes, consumption volumes, and wage rates are as Table A1-2, and commodity prices are 1, 8/5, and 320/99 in order from commodity 1 to 3.

**Tab. A1-2** An example of the perfect specialization

	Production volumes			Consumption volumes			Wage rates
	Comm.1	Comm.2	Comm.3	Comm.1	Comm.2	Comm.3	
Country A		200		320/3	200/3	33	8/15
Country B			99	320/3	200/3	33	32/99
Country C	320			320/3	200/3	33	1/3

Thus, we can bring all the IDL patterns into existence by manipulating values of labor volumes and expenditure coefficients. However, the frequency of the IDL patterns is not equal. We will try a simulation. Labor input coefficients are the same as Table A1-1 and all the expenditure coefficients are  $1/3$ . We generate 200 sets of random numbers which are composed of 3 positive integers fallen in the range of 100 – 10000 (within which all the 13 IDL patterns can be formed), regard the 3 integers as distributions of labor among countries, and identify the patterns formed under the distributions. The result is as follows: the linkage type is 167 sets, the limbo type with one disconnection 32sets, and the PSPs only 1 set. The frequency of the PSPs is very low.

Under other labor input coefficients, the frequency certainly differs, but the probability of the PSPs decreases with the increase of the number of countries, because the number of the wage rates' constraints increases. It would be far more productive to research the IDL patterns including the link commodities than the PSPs.

## ***Appendix 2: On von Mangoldt's theory of international values***

Hans von Mangoldt (1824-68), a German Economist, presented two-country three commodity and two-country five-commodity numerical examples, which are almost identical to the Graham-type model, in his book *Grundriß der Volkswirtschaftslehre* published in 1863. These examples are dealt with in Edgeworth (1894), thereby becoming widely known. In 1975 when a long time had passed after that, Appendix II (pp. 185-224) of the book, in which the numerical examples were included, was translated into English and appeared as von Mangoldt (1975) in *Journal of International Economics* (according to editor, the first English translation). Although von Mangoldt's numerical examples are a little complicated, F. Y. Edgeworth gave a clear explanation about the two-country three-commodity case. Here, we explain the two-country five commodity case with simplifying.

Tow countries I and II produce five commodities A, B, C, D, and E. Both countries have a given volume of productive resources (although terms of productive force, productive power, and productive factor are used, we can also regard them as labor volumes), by which all the five commodities can be produced at constant costs. Real production costs of each commodity and pre-trade consumption volumes of both countries are given as Table A2-1. Although the both countries' volumes of productive resources do not showed clearly, we can calculate them from Table A2-1: 1900 in country I and 1200 in country II.

**Tab. A2-1** Real production costs and pre-trade consumption volumes

	Real production costs					Consumption volumes				
	A	B	C	D	E	A	B	C	D	E
Country I	4	7	6	8	5	100	60	60	40	80
Country II	5	9	3	7	4	50	40	60	30	50

With a start of trade, both countries' demands increase. Mangoldt make a peculiar assumption that demands increase only in commodities of which relative prices decline, and demand volumes are given in such a way that full employment is fulfilled. A result is as Table A2-2.

**Tab. A2-2** Production volumes and demand volumes under trade (1)

	Production volumes					Demand volumes under trade				
	A	B	C	D	E	A	B	C	D	E
Country I	171+3/7	118+38/49	0	47+6/7	0	100	60	105	40	87.5
Country II	0	0	165	22+1/7	137.5	71+3/7	58+38/49	60	30	50

Commodity D is the link commodity (terms of “standard commodity” are used) and is exported from country I to country II. Commodity prices are  $1/2$ ,  $7/8$ ,  $3/7$ , 1 (numéraire),  $4/7$  in order from commodity A to E. Full employment and trade equilibrium are fulfilled.

Furthermore, Mangoldt presents additional three cases by changing suppositions. In all cases, country I’s pre-trade consumption volumes of commodity E increase (from 80 to  $115+3/7$ , 128, and 144 respectively), therefore, country I’s volumes of productive resources increase (from 1900 to  $2077+1/7$ , 2140, and 2220 respectively) and there is no change in other pre-trade situations than this. Then, equilibrium solutions (IDL patterns, production volumes, and commodity prices) are obtained as follows. Also in these cases, full employment and trade equilibrium are fulfilled.

Additional case 1: limbo type (IABD; IICE), prices ( $1/2$ ,  $7/8$ ,  $3/7$ , 1,  $4/7$ )

	Production volumes					Demand volumes under trade				
	A	B	C	D	E	A	B	C	D	E
Country I	$171+3/7$	$118+38/49$	0	70	0	100	60	105	40	126.25
Country II	0	0	165	0	176.25	$71+3/7$	$58+38/49$	60	30	50

Additional case 2: limbo type (IABD; IICE), prices ( $1/2$ ,  $7/8$ ,  $75/164$ , 1,  $100/164$ )

	Production volumes					Demand volumes under trade				
	A	B	C	D	E	A	B	C	D	E
Country I	$176+9/41$	$122+206/287$	0	72.012	0	100	60	98.4	40	131.2
Country II	0	0	158.4	0	181.2	$76+9/41$	$62+206/287$	60	30.012	50

Additional case 3: linkage type (IABDE; IICE), prices ( $1/2$ ,  $7/8$ ,  $15/32$ , 1,  $5/8$ )

	Production volumes					Demand volumes under trade				
	A	B	C	D	E	A	B	C	D	E
Country I	$178+1/8$	$124+2/7$	0	$72+13/16$	11	100	60	96	40	144
Country II	0	0	156	0	183	$78+1/8$	$64+2/7$	60	$32+13/16$	50

Mangoldt's theory of international values differs from that of Graham in two points, except for the number of countries. Firstly, Mangoldt gives demand conditions in terms of physical quantities instead of expenditure coefficients. Secondary, while Graham ignores the limbo type, Mangoldt treats equally the linkage type and the limbo type.

It is really astonishing that, as early as 1863, these numerical examples were presented. Mangoldt's this splendid idea, however, was not developed into an innovation of trade theory not only by himself but also by Edgeworth and Viner who evaluated Mangoldt highly. The primary reason would be that they considered the existence of the link commodities, the standard commodities in their terms, a mere device for explaining international values or a special case which is rare in the real world. For Mangoldt, it was no matter whether the link commodities exist or not. He gave the above numerical examples as exemplification that international values is determined by the equation of international demand and supply, as symbolized in the title of the English translation paper, *On the Equation of International Demand*.

J. S. Chipman surveyed trade theory in the mid 1960s, and evaluated Mill highly, Mangoldt favorably, and Graham severely (Chipman, 1965). Difference of attitude toward Mill, therefore toward mainstream trade theory<sup>26</sup>, we think, brought about difference of evaluation by Chipman. How did Graham himself evaluate Mangoldt? Although Graham had praised Mangoldt in Graham (1932), he changed the evaluation later, and eventually said "Mangoldt missed the right road only by a hair but a wrong turning eventually led him to far astray" (Graham, 1948, p. 69)<sup>27</sup>.

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<sup>26</sup> Gomes (1990) has designated A. A. Cournot and von Mangoldt as two forerunners of neoclassical trade theory, and has referred to Graham as a critic of neoclassical trade theory.

<sup>27</sup> Some economists other than Mangoldt: e.g. Sidgwick (1887, pp. 205-209), Edgeworth (1894, pp. 619-621, 630-634), Nicholson (1897, pp. 301-310), and Bastable (1903, p. 43), also referred to the possibility of existence of the link commodities relatively early. However, none of them realized the commodities' innovative significance for trade theory. See also Shiozawa (2017b).

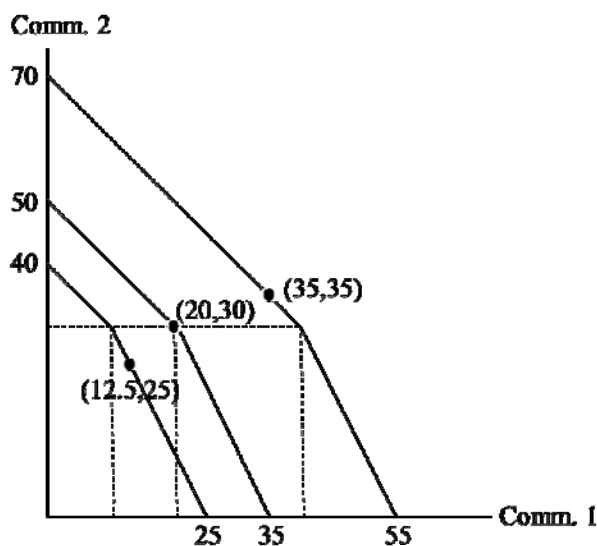
**Appendix 3: Distribution of labor among countries and the IDL patterns:  
Explanation by using two-country two-commodity**

In the subsection 3.2.4, we described the reason that changes in distribution of labor among countries change the IDL patterns. However, the explanation may be hard to understand. We will illustrate it by using two-country two-commodity case. Labor input coefficients and expenditure coefficients are set as Table A3-1 and fixed. While country *B*'s labor are fixed at 60, country *A*'s labor change from 40 to 20 and further to 10.

**Tab. A3-1** Labor input coefficients and expenditure coefficients

	Labor input coefficients		Expenditure coefficients	
	Comm.1	Comm.2	Comm.1	Comm.2
Country <i>A</i>	1	1	0.5	0.5
Country <i>B</i>	4	2	0.5	0.5

Then, world production frontiers and demand points are illustrated as Figure A3-1.



**Fig. A3-1** World production frontiers and the demand points

World production frontiers are cases that country *A*'s labor is 40, 20, and 10 in order from outer side, and points on each frontier are demand points. Of six triangles shown

by using auxiliary lines, the lower three are country *B*'s production frontiers with the same size, and the upper three are country *A*'s production frontiers with different sizes. The reason why compositions of demand volumes, which are expressed by inclinations of the straight lines (not drawn) connecting demand points and the origin, change a little despite of fixed expenditure coefficients is because the wage rates/prices change according to the IDL patterns (see fn. 21). As shown in Figure A3-1, owing to the large change of the production frontier's shape, demand points lie on different facets or a vertex. This means that the IDL patterns change from ( $A_{12}$ ;  $B_2$ ) to ( $A_1$ ;  $B_2$ ), further to ( $A_1$ ;  $B_{12}$ ) in order from outer side.

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