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Discussion Paper No.348

SPATIAL AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY MODEL AND ITS APPLICATION

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April 26, 2016

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SPATIAL AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY MODEL AND ITS APPLICATION

TAKAKI SATO* YASUMASA MATSUDA†

Abstract

This paper proposes spatial autoregressive conditional heteroscedasticity (S-ARCH) models to estimate spatial volatility in spatial data. S-ARCH model is a spatial extension of time series ARCH model. S-ARCH models specify conditional variances as the variances given the values of surrounding observations in spatial data, which is regarded as a spatial extension of time series ARCH models that specify conditional variances as the variances given the values of past observations. We consider parameter estimation for S-ARCH models by maximum likelihood method and propose test statistics for ARCH effects in spatial data. We demonstrate the empirical properties by simulation studies and real data analysis of land price data in Tokyo.

1 INTRODUCTION

In the field of finance, volatility is one of the important factor. Volatility is related to calculating Value at Risk or pricing of derivatives. Volatility has two features. First, we can't observe volatility directly. One approach about this problem is to estimate volatility from past data. Second feature of volatility is that there exists volatility clusters. This means that volatility is very high for certain time periods and very low for other periods. To estimate volatility of time series data, Engle (1982) proposed ARCH model. Bollerslev (1986) introduced GARCH model which is extension of ARCH model. These models are widely accepted and commonly used to estimate and forecast volatility. Dolde and Tirtiroglu (1997) deal with volatility of real estate. Real estate is a financial data, while at the same time has a property of spatial data.

An important property of spatial data sample is a spatial dependency. Spatial dependency means an observation of some spatial point has similar property of observations of near location. This property is called the first law of geography (Tobler (1970)). To estimate or predict better, many statistician propose some spatial statistics models which treat this spatial dependency. There are some spatial econometrics models. SAR (Spatial

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Autoregressive) model which is proposed in Ord (1975) is the most basic model in spatial econometrics. SAR model include spatial lag of dependent variables as spatial interaction. As a natural extension of SAR model, there are some models which has spatial lag of dependent variables or disturbance terms. Kelejian and Prucha (2010) proposed heteroscedasticity model which contains spatial lags in the dependent variable, exogenous variables, and the disturbance terms.

In this paper, we deal with spatial volatility model. This study has two motivate. First, estimating volatility serve to make real estate portfolio. Secondly, there is a chance that spatial model can extend to spatiotemporal model. We could estimate spatiotemporal ripple effect of volatility in spatial data and also time series data.

There has been little study done concerning spatial volatility model. Robinson (2009) apply idea of time-series stochastic volatility model to spatial model. On the other hand, Bera and Simlai (2005) propose ARCH type spatial volatility model but little attention has been given to statistical property of that model.

By extending ARCH model, we propose spatial econometrics model to estimate spatial volatility. That is spatial autoregressive conditional heteroscedasticity model (S-ARCH model). An important difference from previous study which try to apply ARCH model to spatial model is that volatility structure is described by log volatility form. This approach has advantage. We can change model into SAR model which has a special kind of error term. Therefore, condition that S-ARCH model can be estimated is same as SAR model.

The paper proceeds as follows. In section2, SAR model and S-ARCH model are presented. estimation methods of these models are presented in section3. Section 4 deals with empirical analysis of S-ARCH model. Both simulation study and real data analysis are reported. Section 5 provides the conclusions.

2 MODEL

In this section, we briefly review the SAR model and propose S-ARCH model. SAR model is basic spatial econometric model and can capture spatial correlation of dependent variable. In section 2.2, S-ARCH model is defined as natural extension of time series ARCH model.

2.1 SPATIAL AUTOREGRESSIVE MODEL

The SARmodels can be expressed as shown in (1).

$$\begin{aligned} \boldsymbol{y} &= \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \\ \boldsymbol{\epsilon} &\sim \boldsymbol{N}(\boldsymbol{0}, \sigma^2 \boldsymbol{I}_n), \end{aligned} \tag{1}$$

where n is the total number of spatial units, $\boldsymbol{y}$ is an $n \times 1$ vector of dependent variables, and $\boldsymbol{X}$ is an $n \times k$ matrix of independent variables. $\boldsymbol{W}$ is a specified constant spatial weights matrix. This matrix is based on physical distance or contiguity of spatial units. ρ is scalar parameter which express strength of spatial dependency, $\boldsymbol{\beta}$ is an $n \times 1$ vectors whose elements are parameters, and $\boldsymbol{\epsilon}$ is an error term. We assume $\boldsymbol{\epsilon}$ follows multivariate normal distribution with mean $\boldsymbol{0}$ and homoscedastic variance σ^2.

We can express equation (1) as $\boldsymbol{y} = (\boldsymbol{I}_n - \rho\boldsymbol{W})^{-1}(\boldsymbol{X\beta}+\boldsymbol{\epsilon})$. We need to check existence condition of $(\boldsymbol{I}_n - \rho\boldsymbol{W})^{-1}$. Basically, spatial weight matrix is symmetric matrix. Under assumption that spatial weight matrix is row-normalized symmetric matrix, $(\boldsymbol{I}_n - \rho\boldsymbol{W})^{-1}$ exsits if $\rho \in (-1, 1)$ (Lee (2004)). Therefore, ρ is restricted to the interval$(-1, 1)$.

2.2 SPATIAL AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY MODEL

We propose spatial autoregressive conditional heteroscedasticity model(S-ARCH model). S-ARCH model is natural extension of ARCH model. In ARCH model, conditional variance is described by past observations. On the other hand, conditional variance is described by surrounding area's observations in S-ARCH model.

The S-ARCHmodels can be expressed as shown in (2).

$$\begin{aligned}
\boldsymbol{y} &= \begin{pmatrix} \sigma_{11}\epsilon_1 \\ \vdots \\ \sigma_{nn}\epsilon_n \end{pmatrix}, \qquad (2)\\
\epsilon_i &\sim IID(0,1),\\
{y_i}^2 &= {\sigma_{ii}}^2\epsilon_i^2\\
log{y_i}^2 &= log{\sigma_{ii}}^2 + log\epsilon_i^2, \qquad (3)\\
log\sigma_{ii}^2 &= \alpha_0 + \alpha_1\sum_{j=1}^{n} w_{ij}logy_j^2, \qquad (4)
\end{aligned}$$

where n is the total number of spatial units, and $\boldsymbol{y}$ is an $n \times 1$ vector of dependent variables. ϵ is a independent and identically distributed random variables with mean zero and variance 1. α_0 and α_1 are scalar parameters, w_{ij} is an (i,j)element of spatial weight matrix, and y_j is a j-th element of $\boldsymbol{y}$.

We get next equation by substituting equation (4) to equation (3).

$$log{y_i}^2 = \alpha_0 + \alpha_1\sum_{j=1}^{n} w_{ij}logy_j^2 + log\epsilon_i^2, \qquad (5)$$

Equation (5) is same as SAR model. From discussion of section 2.1, α_1 is restricted to the interval$(-1, 1)$.

We check properties of S-ARCH model. Expectation, conditional expectation, variance conditional varianceof y_i is derived ,where i means some area of spatial units.

Let ψ_{-i} be the information set of neighborhood of area i. At first, we derive an expectation and a conditional expectation of y_i.

$$\begin{aligned}
E(y_i|\psi_{-i}) &= E(\sigma_{ii}\epsilon_i|\psi_{-i})\\
&= \sigma_{ii}E(\epsilon_i)\\
&= 0.
\end{aligned}$$

$$\begin{aligned}
E(y_i) &= E(E(y_i|\psi_{-i}))\\
&= 0.
\end{aligned}$$

Second, we derive a variance and a conditional variance of a_i.

$$\begin{aligned} Var(y_i|\psi_{-i}) &= E(\sigma_{ii}^2\epsilon_i^2|\psi_{-i}) \\ &= Exp(\alpha_0 + \alpha_1 \sum_{j=1}^{n} w_{ij} log y_j^2). \end{aligned}$$

$$\begin{aligned} Var(y_i) &= E[var(y_i|\psi_{-i})] + Var[E(y_i|\psi_{-i})] \\ &= E(Exp(\alpha_0 + \alpha_1 \sum_{j=1}^{n} w_{ij} log y_j^2)) + 0 \\ &= Exp(\alpha_0) E(Exp(\alpha_1 \sum_{j=1}^{n} w_{ij} log y_j^2)) \\ &= Exp(\alpha_0) E(Exp(log \prod y_j^{2\alpha_1 w_{ij}})) \\ &= Exp(\alpha_0) E(\prod y_j^{2\alpha_1 w_{ij}}). \end{aligned}$$

Third, we derive a covariance of y_i and y_j.

$$\begin{aligned} Cov(y_i, y_j) &= E(y_i y_j) \\ &= E(E(y_i y_j)|\psi_{-i}) \\ &= E(y_j(E(y_i|\psi_{-i}))) \\ &= 0. \end{aligned} \tag{6}$$

Therefore, we can write a covariance matrix of $\mathbf{y}$ as

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_{11} & 0 & \cdots & 0 \\ 0 & \sigma_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{nn} \end{pmatrix}.$$

3 ESTIMATION METHOD

In this section, estimation methods of two models which are introduced in section 2 is discussed. Many estimation methods of SAR model is suggested. However, each model is estimated by maximum likelihood estimation later in this paper. Therefor likelihood of them is introduced.

3.1 ESTIMATION METHOD OF THE SPATIAL AUTOREGRESSIVE MODEL

There are many estimation methods of SAR model. SAR model is estimated by maximum likelihood estimation(Ord (1975)), GS2SLS(Kelejian and Prucha (1998)), and bayesian estimation(LeSage (1997)). Moreover, Kelejian and Prucha (1998) and Lee (2004) prove the consistency of respectively the GS2SLS estimator and the ML estimator in the SAR model.

In this article, parameters of SAR model are estimated by maximum likelihood estimation about concentrated log-likelihood. The log-likelihood function for the SAR model takes the form in(7).

$$\begin{aligned} logL &= -\frac{n}{2}log(2\pi) - \frac{n}{2}log(\sigma^2) + log|\boldsymbol{I}_n - \rho\boldsymbol{W}| - \frac{\boldsymbol{\epsilon}'\boldsymbol{\epsilon}}{2\sigma^2} \\ \boldsymbol{\epsilon} &= \boldsymbol{y} - \rho\boldsymbol{W}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} \end{aligned} \tag{7}$$

where,$|\boldsymbol{I}_n - \rho\boldsymbol{W}|$is the determinant of this $n \times n$ matrix.

For reducing a multivariate optimization problem to a univariate problem, we use concentrated log-likelihood with respect to the parameters β, σ^2. Working with the concentrated log-likelihood yields exactly the same maximu likelihood estimates $\hat{\beta}$,$\hat{\sigma^2}$, and $\hat{\rho}$ as would arise from maximizing the full log-likelihood(LeSage and Pace (2009),p47). The maximum likelihood estimator about parameter $\boldsymbol{\beta}$ and σ^2 can be written as

$$\begin{aligned} \hat{\boldsymbol{\beta}} &= (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}\boldsymbol{y} - (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}\boldsymbol{W}\boldsymbol{y} \\ &= \boldsymbol{b_0} - \boldsymbol{b_L}, (\boldsymbol{b_0} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}\boldsymbol{y}, \boldsymbol{b_L} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}\boldsymbol{W}\boldsymbol{y}) \end{aligned} \tag{8}$$

$$\begin{aligned} \hat{\sigma^2} &= \frac{[(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{b_0}) - \rho((Wy) - \boldsymbol{X}\boldsymbol{b_L})]'[(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{b_0}) - \rho((Wy) - \boldsymbol{X}\boldsymbol{b_L})]}{n} \\ &= \frac{(e_0 - e_\rho L)'(e_0 - e_\rho L)}{n}. \end{aligned} \tag{9}$$

From above estimator, we get next concentrated log-likelihood with respect to the parameters β, σ^2.

$$logL_c = -\frac{n}{2}log(2\pi) - \frac{n}{2}log(\frac{(e_0 - e_\rho L)'(e_0 - e_\rho L)}{n}) + log|\boldsymbol{I}_n - \rho\boldsymbol{W}|. \tag{10}$$

The estimation process can proceed according to the following steps(Anselin (1988),p182):

1. carry out OLS of X on y: yields $\boldsymbol{b_0}$
2. carry out OLS of X on Wy: yields $\boldsymbol{b_L}$
3. compute residuals e_0 and e_L
4. given e_0 and e_L, find that maximizes $logL_c$
5. given ρ, compute $b = \boldsymbol{b_0} - \rho\boldsymbol{b_L}$ and $\sigma^2 = \frac{(e_0 - \rho e_L)'(e_0 - \rho e_L)}{n}$

3.2 ESTIMATION METHOD OF THE SPATIAL AUTOREGRESSIVE CONDITIONAL HETEROSCEDASTICITY MODEL

Parameters of S-ARCH model are estimated by maximum likelihood estimation about concentrated log-likelihood.

We assume each ϵ follows independent and identically distributed standard normal sequence. From change of variables, a probability density function of $z = log\epsilon_i^2$ is

$$h(z) \sim \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}e^z + \frac{z}{2}}.$$

The log-likelihood function for the S-ARCH model takes the form in (11)

$$\begin{aligned} logL &= \sum(-\frac{1}{2}e^{f_i} + \frac{1}{2}f_i) + log|\boldsymbol{I}_n - \alpha_1 \boldsymbol{W}|, \\ x_i &= logy_i^2, \\ f_i &= x_i - \alpha_0 - \alpha_1 \sum_{j=1}^{n} w_{ij}x_j^2, \end{aligned} \tag{11}$$

where,$|\boldsymbol{I}_n - \alpha_1 \boldsymbol{W}|$is the determinant of this $n \times n$ matrix. The partial derivative of log-likelihood function with α_0 is

$$\frac{\partial logL}{\partial \alpha_0} = \sum(-\frac{1}{2} + \frac{1}{2}e^{f_i}). \tag{12}$$

The maximum likelihood estimator about parameter α_0 can be written as

$$\hat{\alpha_0} = log(\frac{\sum e^{x_i - \alpha_1 \sum_{j=1}^{n} w_{ij}x_j}}{n}). \tag{13}$$

From above estimator, we get next concentrated log-likelihood with respect to the parameters α_0.

$$\begin{aligned} logL_c &= (-\frac{1}{2}e^{f_i} + \frac{1}{2}f_i) + log|\boldsymbol{I}_n - \alpha_1 \boldsymbol{W}|, \\ f_i &= x_i - log(\frac{\sum e^{x_i - \alpha_1 \sum_{j=1}^{n} w_{ij}x_j}}{n}) - \alpha_1 \sum_{j=1}^{n} w_{ij}x_j^2. \end{aligned} \tag{14}$$

Estimation is accomplished by numerical optimize eauation (14) about α_1 and α_0 is gotten by substituting estimated α_1 into equation(13).

4 EMPIRICAL ANALYSIS

In this section, we report empirical analysis of S-ARCH model. First, simulation study of S-ARCH model is shown to investigate properties of the maximum likelihood estimator. Secondly, real data analysis of S-ARCH model is reported. We use Kanto area as spatial scenario in both case.

4.1 SIMULATION STUDY

To investigate properties of the maximum likelihood estimator of S-ARCH model, we do Monte Carlo simulation. We use Kanto area as spatial scenario. Spatial units used in this simulation are ward, city and town. Therefore there are 347 observations. Spatial weight matrix is based on the first order contiguity relations for the 347 regions and is row-normalized such that the elements of each row sum to one. For each case, there are 1000 repetitions.

<<Table 1>>

In the first case (S-ARCH 1), the sample data are generated with $\alpha_1 = 0$ and α_0 taken some value. We assume case that no spatial correlation exist in volatility. ME $\hat{\alpha_0}$ is mean of estimated α_0. ME $\hat{\alpha_1}$ is gotten in a similar fashion. MSE $\hat{\alpha_0}$ is mean squared error of estimated α_0. MSE $\hat{\alpha_1}$ is derived in the same way. Let λ be likelihood ratio under null hypothesis $H_0 : \alpha_1 = 0$. $-2log\lambda$ follows chi-square distribution with degrees of freedom 1. Last row ($LR > 3.84$) means percentages that likelihood ratio test statistics (i.e. $-2log\lambda$)under null hypothesis $H_0 : \alpha_1 = 0$ is over 3.84. 3.84 is value of 5 % point of chi-square distribution with degrees of freedom 1.

The sample data of the second case (S-ARCH2) are generated with $\alpha_0 = -0.5$ and α_0 taken some value. We check some cases that the strength of spatial correlation of volatility changed. power means percentages that likelihood ratio test statistics under null hypothesis $H_0 : \alpha_1 = 0$ is over 3.84.

Table 1 shows estimated value of each parameter is similar to true parameters. However, lower bias may exist because mean value of estimated parameter is lower than true parameters in each case. From last row of S-ARCH1 case, it is very likely that likelihood ratio statistic follows chi-square distribution. Therefore, when we check spatial ARCH effect in later section, we judge by whether likelihood ratio statistic is over 3.84.

4.2 REAL DATA ANALYSIS

In this section, we report five things. These are explanation of data, results of estimation of SAR model, Moran's I of residuals of SARmodel , results of estimation of S-ARCH model, and results of test of S-ARCH effect.

First, data for empirical analysis is explained. Change rate of land price data of Kanto area from 2003 to 2014 are used as dependent variable. This data consists of wards, cities and town unit data. There are 347 observations in the data. Basically, we use Publication of Land Prices as price data which can be gotten from National Land Numerical information download service. However, we used Prefectural Land Price Research as land price this time because there are towns which have no observation point in Publication of Land Prices.

<<Table 2>>

Second, we apply SAR model to the land price data and we get residuals of SAR model. Spatial scenario is same as section 4.1 and we use row-normalized first order contiguity spatial weight matrix. We assume that independent variable is only intercept term. Table 2 shows the result of SAR model. ρ express the strength of spatial dependence of observations. From the result, there are high spatial dependence in land price data. This means that if change rate of land price of certain area is high, then surrounding areas change rate is also high.

<<Table 3>>

Next, we check Moran's I of residuals. Moran's I is one of the index of spatial dependence. High Moran's I means that there are strong spatial correlations in observations. Moran's I is defined as

$$I = \frac{n}{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}} \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(y_i - \bar{y})(y_j - \bar{y})}{\sum_{i=1}^{n}(y_i - \bar{y})^2}. \tag{15}$$

Table 3 shows the Moran's I of residuals of SAR model. From table 3, it is considered that residuals of SAR model have little spatial dependence. However, squared residuals seems to have spatial dependence. Especially, the values from 2009 to 2012 is higher than the others. The cause of high Moran ' s I in those time is Lehman shock and The Great East Japan Earthquake. Lehman shock occurred in 2008 and Tohoku earthquake happened in 2011. These data are investigated in July 1 every year. The effect of Lehman shock appear in data of 2009 year.

<<Table 4>>

<<From Figure 1 to Figure 12>>

Then, we apply S-ARCH model to the residuals of SAR model. We use same spatial scenario and spatial weight matrix as SAR model.Table 4 shows the results of estimation of S-ARCH model. α_0 is parameter of intercept and α_1 express S-ARCH effect by which the effect of surrounding spatial units on given area's conditional variance is caught. Estimated α_1 is positive in every year and the estimated value from 2006 to 2013 are bigger than those of other years. It can be presumed that we can see clusters of high volatility and low volatility in the time that α_1 is larger.

Estimated volatility is displayed on the map in each figure. It provides a visual depiction of how values of volatility differ over space. As the result that estimated α_1 is low shows, each spatial unit take similar estimated volatility value in 2003 ,2004. On the other hand, we can see clusters of red and blue spatial units from 2005 to 2011. Japan economy was good from 2003 to 2007. As can be seen from these figures and table4, the average value of volatility changed significantly before and after 2007. The volatility after 2007 are small so the width of the change is also small.For this reason, it is hard to see that we find volatility clusters in 2013 and 2014 in spite of volatilities of neighboring areas are very similar. Lehman shock occurred in 2008. These data are investigated in July 1 every year. The effect of Lehman shock appear in data of 2009 year. From figure 7, we can find that an amount of change of land price is bigger in urban areas and west of Kanto area. The Great East Japan Earthquake occurred in 2011. Coastal areas suffered serious damage in that earthquake. we can find the width of change of land price in these areas is bigger from figure 9.

<<Table 5>>

Finally, we test the S-ARCH effect, which is to say that we test next hypothesis $H_0 : \alpha_1 = 0, H_1 : \alpha_1 \neq 0$. Table 4 shows the result of test of S-ARCH effect. Wald, LR and LM statistics are reported. From simulation study, we assume these statistics follow chi-square distribution with degrees of freedom 1. These three statistics is almost same in every year and over 3.84. Therefore, estimates of α_1 are statistically significant at the 5% level. It can be said that spatial ARCH effect exists in land price data in the same way as time series finance data.

5 CONCLUSION

We have proposed the spatial autoregressive conditional heteroscedasticity (S-ARCH) model to estimate spatial volatility. Regarding log transformed S-ARCH models as spatial autoregressive models, we consider maximum likelihood estimators (MLE) for the parameters and test statistics for ARCH effects. Empirical studies by Kanto land price and simulated spatial data demonstrate that S-ARCH models work well to detect ARCH effects with reasonable size and power and to estimate spatial volatility. In the real data analysis, we found higher volatilities in the coastal area near Tohoku than those in other area, which may be considered as the quantitative evaluation of the effects of the big earthquake in Tohoku. In addition to the empirical properties, we are now considering to establish theoretical properties of the MLE, the consistency and asymptotic normality for them.

Finally, we will complete the paper by describing some future studies, In the empirical analysis for the land price, we took the first order contiguity relations of ward, city and town of Kanto area as spatial weight matrix. As (Beck, Gleditsch, and Beardsley (2006)) shows, spatial distances different from geographic distance can be interesting canditates to improve our volatility analysis by S-ARCH models. This paper only evaluated volatilities in spatial data by fixing time in the land price data, which is actually spatio-temporal data. Spatio-temporal extensions of S-ARCH model would provide more detailed analysis of the volatility structures in the land price.

APPENDIX

From page 10 to 15, two types of objects is listed. They are figures and tables. Figures shows maps of estimated spatial volatilities of each year. Tables shows five results of estimation and simulation.

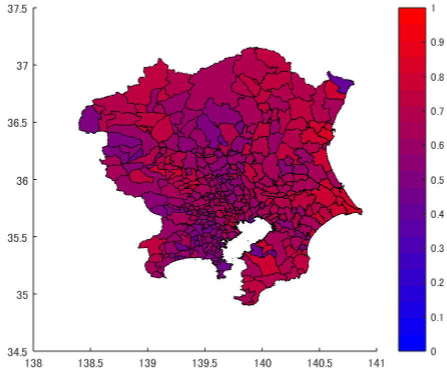

Figure 1: 2003 estimated volatilities map

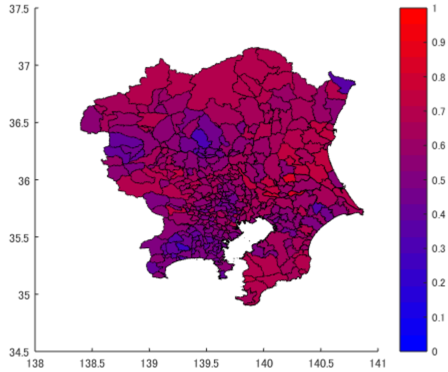

Figure 2: 2004 estimated volatilities map

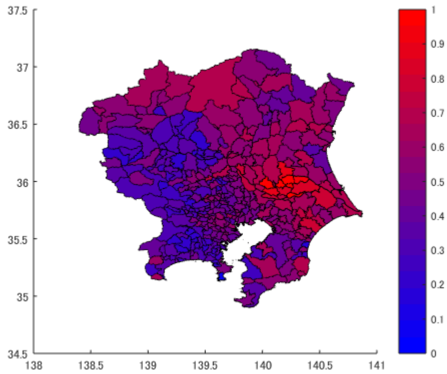

Figure 3: 2005 estimated volatilities map

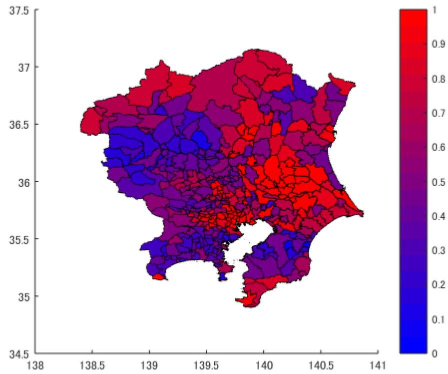

Figure 4: 2006 estimated volatilities map

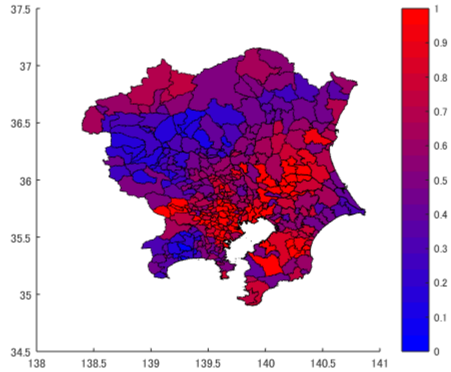

Figure 5: 2007 estimated volatilities map

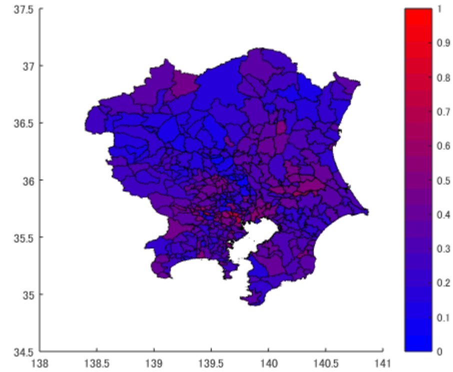

Figure 6: 2008 estimated volatilities map

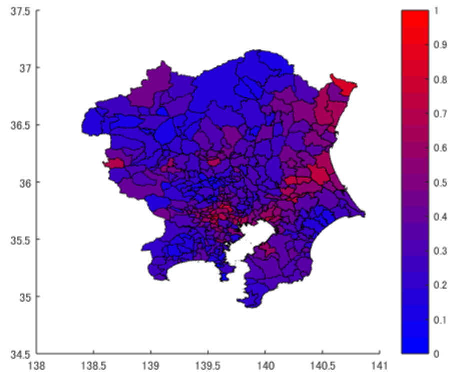

Figure 7: 2009 estimated volatilities map

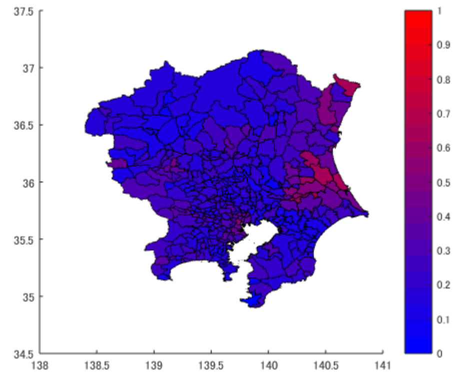

Figure 8: 2010 estimated volatilities map

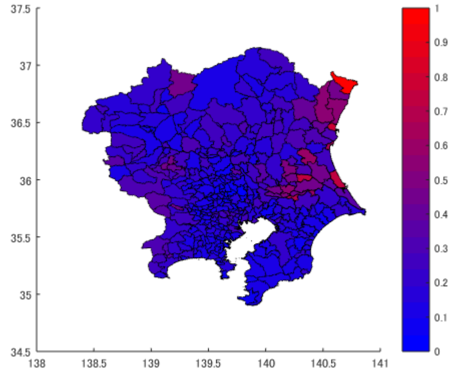

Figure 9: 2011 estimated volatilities map

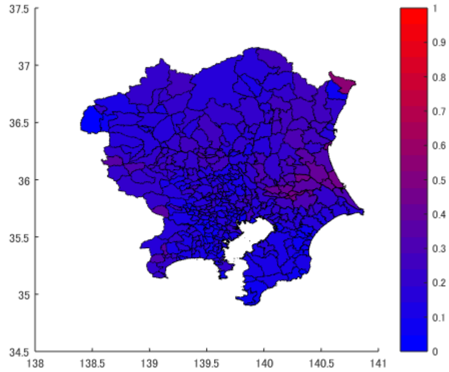

Figure 10: 2012 estimated volatilities map

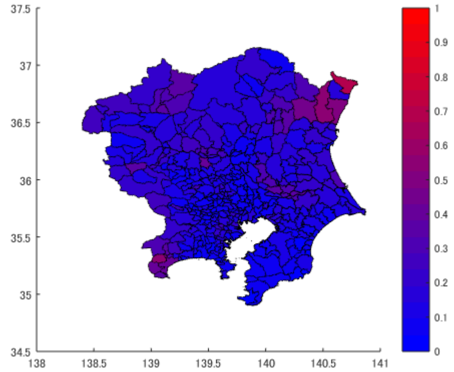

Figure 11: 2013 estimated volatilities map

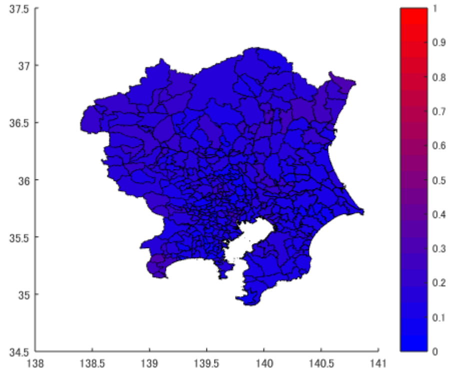

Figure 12: 2014 estimated volatilities map

Table 1: S-ARCH simulation

S-ARCH1	α_0			
$(\alpha_1 = 0)$	0.5	1.5	-0.5	-1.5
ME $\hat{\alpha_0}$	0.491	1.488	-0.513	-1.523
ME $\hat{\alpha_1}$	-0.005	-0.003	-0.004	-0.007
MSE $\hat{\alpha_0}$	0.008	0.006	0.018	0.037
MSE $\hat{\alpha_1}$	0.004	0.004	0.004	0.004
$LR > 3.84$	0.052	0.050	0.046	0.050
S-ARCH2	α_1			
$(\alpha_0 = -0.5)$	0.1	0.2	0.5	0.8
ME $\hat{\alpha_0}$	-0.524	-0.514	-0.530	-0.561
ME $\hat{\alpha_1}$	0.091	0.195	0.494	0.793
MSE $\hat{\alpha_0}$	0.022	0.022	0.032	0.069
MSE $\hat{\alpha_1}$	0.004	0.004	0.002	0.000
power	0.320	0.833	1	1

Table 2: The result of SAR model.

	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
ρ	0.865	0.836	0.805	0.830	0.933	0.848	0.880	0.700	0.820	0.831	0.880	0.892
	(0.031)	(0.033)	(0.033)	(0.028)	(0.016)	(0.027)	(0.024)	(0.046)	(0.030)	(0.028)	(0.023)	(0.021)
β	-0.408	-0.423	-0.359	-0.096	0.037	-0.002	-0.270	-0.452	-0.250	-0.174	-0.083	-0.037
	(0.104)	(0.094)	(0.071)	(0.050)	(0.046)	(0.031)	(0.065)	(0.076)	(0.048)	(0.037)	(0.026)	(0.022)
σ^2	0.668	0.607	0.524	0.781	0.689	0.333	0.380	0.276	0.236	0.215	0.179	0.156
	(0.136)	(0.144)	(0.151)	(0.097)	(0.110)	(0.228)	(0.202)	(0.282)	(0.322)	(0.352)	(0.424)	(0.486)

Table 3: The result of Moran's I of residuals of SAR model.

	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
noraml	-0.075	-0.088	-0.078	-0.056	-0.063	-0.049	-0.043	-0.010	-0.036	-0.075	-0.082	-0.076
squared	0.076	0.055	0.142	0.062	0.090	0.048	0.152	0.322	0.295	0.246	0.101	0.072

Table 4: The result of S-ARCH model.

	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
α_0	-0.191	-0.222	-0.136	0.507	0.234	-0.295	-0.146	-0.423	-0.309	-0.647	-0.244	-0.974
	(0.016)	(0.021)	(0.015)	(0.015)	(0.014)	(0.030)	(0.021)	(0.019)	(0.026)	(0.020)	(0.036)	(0.041)
α_1	0.115	0.147	0.268	0.428	0.341	0.301	0.351	0.363	0.432	0.334	0.491	0.267
	(0.003)	(0.004)	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)	(0.001)	(0.003)	(0.003)

Table 5: The result of S-ARCH effect test.

	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Wald	4.735	5.621	38.494	98.027	67.700	30.570	56.431	83.730	83.756	77.226	88.585	22.489
LR	4.585	5.384	35.535	83.519	52.915	28.654	49.763	69.206	75.321	66.006	77.388	21.364
LM	4.738	5.612	38.644	93.177	67.147	30.039	55.529	78.791	81.962	77.158	86.408	22.274

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