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Public education, endogenous fertility
and economic growth

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Public education, endogenous fertility and economic growth*

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Abstract

Using a closed-economy overlapping generations model with endogenous fertility, child quality choice and human capital accumulation, this paper examines the effects of public investment in education on fertility rate and per capita output growth rate under a pay-as-you-go (PAYG) social security system. Parents face a trade-off between the quantity and quality of children. Differently from previous studies, this paper shows that there is an inverted U-shaped relation between public investment in education and fertility. Small sized public education policy stimulates fertility and impedes growth.

Keywords Public education • Private education • Fertility • Human capital

JEL-Classification J13 • I22 • H52

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1 Introduction

Recently many developed countries face the declining birth rate and increasing longevity. Decreasing young population is a matter of public concern in these countries because it decreases labor force in the future and reduces the contributions to the social security system. Policy makers aim at increasing the population growth rate and the ratio of economically active workers to total population by various public investments in children e.g., child allowances, social security system and public education. Some literature dealt with public financed child care policy in an overlapping generations model. For example, van Groezen et al (2003) analyzes the relation between child care subsidy, pay-as-you-go (PAYG) social security system and fertility. But these literature ignore a child quality issue. Becker and Lewis (1973) shows a quality-quantity trade-off of fertility choice. Parents face a trade-off between having many children and spending large resources on their quality e.g., health and education, of each children. If parents decide fertility and education jointly, government investment in children not only has an effect on their fertility choice also affects human capital accumulation.

Few studies examine the effects of public education policy on private investment in education and fertility and growth under PAYG social security system. de la Croix and Doepke (2004) extended Kaganovich and Zilcha (1992) to examine the choice of an education regime in an endogenous growth model with endogenous fertility choice. They show that at public education regime economic growth rate is higher and income inequality is lower than at private education regime. But their analysis is limited to two polar schooling systems i.e., the government or household only supplies the educational investment in their articles. Under public education regime, parents are individually not allowed to choose the level of education for their children. In reality, in many developed countries private education coexists with public education. Table 1 shows the percentage of private investment in education to total educational expenditure in OECD countries.

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Japan	10.2	65.6
Korea	19.2	76.7
USA	8.3	64
Italy	2.2	32.4
Germany	12.9	11.8
France	7.4	15.6
Sweden	0.1	8.7

Table 1: The percentage of private investment in education to total educational expenditure in OECD countries by level of education (primary, secondary and post-secondary non-tertiary education and tertiary education). The data are from OECD (2010a).

The issue addressed in this paper are as follows: how a change in the allocation between public investment in education and PAYG social security benefits affects fertility and growth ? This paper extends de la Croix and Doepke (2003) to investigate the effect of public education on fertility by introducing the parental fertility decision and the child quality issue into a three-period-lived overlapping generations model with PAYG social security system. Differently from previous works, I assume that private education coexists with public education. Government finances two public program; PAYG social security program and public education program. This paper investigates the effects of an increase in public funding for children's education on the parent's fertility and educational decision¹.

The main results are as follows. There is an inverted U-shaped relation between public investment in education and fertility. And an V-shaped relation between public education and growth appear. Small sized public education policy generates a substitution effect from quality of children to quantity of children in contradiction to Zhang (1997). On the contrary, in the case in which the size is sufficiently high, increasing public investment in education decreases fertility and stimulates per capita output growth for a given PAYG

¹But I ignore income inequality to maintain tractability of the model.

contribution rate by reducing PAYG social security benefits..

This paper is similar to some previous papers that study the relation between private and public investment in education. Kaganovich and Zilcha (1999) studies the effects of public education policy on private educational choice, welfare and growth. Golmm and Kaganovich (2003) explores the relationship between parental educational and public investment in human capital formation. But they ignore the response of fertility to public education policy. Omori (2010) built a overlapping generations model with two intergenerational transfer program and endogenous fertility choice. But with their modeling public education policy have a monotonic effect on fertility rate and it does not examine the effect of public education policy on per capita output growth.

The rest of paper is organized as follows: Section 2 presents the model used in this article. Section 3 examines fertility rate and per capita output growth rate in the balanced growth path. Section 4 devotes to show the effects of public education on fertility and growth analytically. The last section offers some concluding remarks.

2 Model

Time is discrete and goes from 0 to infinite. Consider a closed-economy that is populated by overlapping generations of people who live for three periods. Each individual lives for three periods; childhood, adulthood and old-age. He receives education in the first period, works, rears and educates children in the second period and retires in the third period. The length of each period is normalized to one. All decisions are made in the adulthood period of life. Individuals care about adulthood consumption c_t , old-age consumption d_{t+1} , their number of children n_t , and the human capital of children e_t . Total population P_t evolves over time according to $P_{t+1} = n_t P_t$.

2.1 Household

Preference of any individual in generation t are given by:

$$\log c_t + \beta \log d_{t+1} + \gamma \log n_t h_{t+1}, \quad (1)$$

where c_t , d_{t+1} are consumption profiles of generation t . The parameter $0 < \beta < 1$ is a psychological discount factor and $0 < \gamma < 1$ is a altruism factor. Raising one child takes a constant fraction $0 < \phi < 1$ of an adult's time. This fraction of time is exogenous and constant. An adult has to choose consumption profiles c_t and d_{t+1} , saving for old-age s_t , number of children n_t , and private educational investment per child e_t . The budget constraint for the adulthood period is²:

$$c_t = (1 - \tau) \{(1 - \phi n_t) w_t h_t - e_t n_t w_t h_t\} - s_t,$$

The budget constraint for the old-age period is:

$$d_{t+1} = R_{t+1} s_t + T_{t+1},$$

Using these budget constraints, I have a lifetime budget constraint as follows:

$$(1 - \tau) \{(1 - \phi n_t) w_t h_t - e_t n_t w_t h_t\} + T_{t+1}/R_{t+1} = c_t + d_{t+1}/R_{t+1}. \quad (2)$$

The human capital of generation $t + 1$ depends on the human capital of their parents h_t , the public education investment E_t and the private education investment e_t . The human capital of generation $t + 1$ is assumed to be produced as follows:

$$h_{t+1} = \mu(E_t + e_t)^\eta h_t. \quad (3)$$

²I assume that private educational spending for children is tax-deductible in the same way as de la Croix and Doepke (2003). This assumption is not essential to my main results.

Here $h \equiv H/P$ represents per capita human capital. μ is a efficiency parameter and $0 < \eta < 1$. The public educational investment in children is obtained for free and perfectly substitutes to the educational investment provided by their parents³. The private investment in children's education e has to be paid for by their parents.

Given the wage rate w_t , the price of physical capital R_{t+1} , the human capital level h_t , the child rearing time per child ϕ , the policies of government, individuals during adulthood period in generation t choose s_t, n_t, e_t to maximize their lifetime utility Eq. (1). To find the solution to the household maximization problem, I solve the following Kuhn-Tucker problem:

$$\begin{aligned} & \max_{c,d,e,n} \log c_t + \beta \log d_{t+1} + \gamma \log n_t h_{t+1} \\ & + \lambda_1 \{ (1 - \tau)(1 - \phi n_t w_t h_t - e_t n_t w_t h_t) + T_{t+1}/R_{t+1} - c_t - d_t/R_{t+1} \} \\ & + \lambda_2 e_t, \end{aligned}$$

where λ_1 and λ_2 are shadow prices. The Kuhn-Tucker conditions are:

$$\begin{aligned} \frac{1}{c_t} &= \lambda_1, \\ \frac{\beta}{d_{t+1}} &= \frac{\lambda_1}{R_{t+1}}, \\ \frac{\gamma}{n_t} &= \lambda_1(1 - \tau)(\phi + e_t)w_t h_t, \\ \frac{\gamma\eta}{E_t + e_t} &= \lambda_1(1 - \tau)n_t w_t h_t + \lambda_2, \\ c_t + \frac{d_{t+1}}{R_{t+1}} &= (1 - \tau)(1 - \phi n_t w_t h_t - e_t n_t w_t h_t), \\ \lambda_2 e_t &= 0. \end{aligned}$$

³Public education policy act as the device that enhance the efficiency of human capital formation in Omori (2010) and Fanti and Gori (2011). In contrast, public investment in education is substituting for private input in Zhang (1997) and Azarnert (2010).

From these Kuhn-Tucker conditions, the saving level of generation t is given by:

$$s_t = \left(\frac{1}{1 + \beta + \gamma} \right) \{ \beta(1 - \tau)w_t h_t - (1 + \gamma)T_{t+1}/R_{t+1} \}. \quad (4)$$

If the return on private investment in education is higher than the return of quantity of children, parents invest a part of their disposal income in education $e_t > 0$. However, if the return on private investment in education is below the return of quantity of children, a corner solution $e_t = 0$ exists. An optimal non-corner solution for the number of children and the parental investment in children's education at time t is given by:

$$n_t = \hat{\gamma}(1 - \eta) \frac{(1 - \tau)w_t h_t + T_{t+1}/R_{t+1}}{(1 - \tau)(\phi - E_t)w_t h_t}, \quad (5)$$

$$e_t = \left(\frac{1}{1 - \eta} \right) \{ \eta(1 - \tau)\phi w_t h_t - E_t \}. \quad (6)$$

The relation between private and public investment in education is illustrated in Figure 1. Public education system generates a substitution effect from quality of children to quantity of children and there is a negative relation between private and public investment in education. The optimal corner solution for the number of children and the parental investment in education at time t is:

$$n_t = \hat{\gamma} \frac{(1 - \tau)w_t h_t + T_{t+1}/R_{t+1}}{(1 - \tau)\phi w_t h_t}, \quad (7)$$

$$e_t = 0, \quad (8)$$

where $\hat{\gamma} \equiv \frac{\gamma}{1 + \beta + \gamma}$. Notice that fertility at equilibrium is positively related to lifetime income i.e., the wage earned during the adulthood period of life and PAYG social security benefits during the old-age period of life, and negatively related to child-rearing costs i.e., the private investment in education and the fixed child rearing-cost.

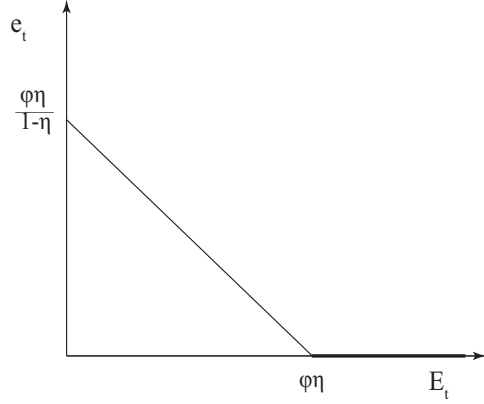


Figure 1: The relation between public and private investment in education

2.2 Firm

A single representative firm operates the technology:

$$Y_t = K_t^\alpha L_t^{1-\alpha},$$

where K_t is the aggregate physical capital and L_t is the aggregate labor supply at time t . Parameter $0 < \alpha < 1$ is expressed as the share of physical capital in production. Physical capital depletes completely after one period use in production. The firm chooses inputs by maximizing profits $Y_t - w_t L_t - R_t K_t$ at time t . From the first order conditions for profit maximization, factor prices are derived as follows:

$$w_t = (1 - \alpha) \left(\frac{K_t}{L_t} \right)^\alpha, \quad (9)$$

$$R_t = \alpha \left(\frac{K_t}{L_t} \right)^{\alpha-1}, \quad (10)$$

where w_t and R_t means the wage rate and the interest factor at time t respectively.

2.3 Government

I assume that the government behaves under a balanced budget restriction. Two policies are funded by income tax τ collected at constant rate. I define a parameter χ which represents the fraction of government revenue devoted to public investment in education. Public investments in education policy and PAYG social security policy in period t is given by:

$$T_t P_{t-1} = \tau(1 - \chi)L_t w_t, \quad (11)$$

$$E_t w_t h_t P_t = \tau \chi L_t w_t, \quad (12)$$

where T_t is the per capita PAYG social security benefit and E_t is the per child public financed education expenditure in period t . In the following discussions, the government predetermines the sequences of education tax rate τ and fraction χ .

2.4 Markets

The equilibrium conditions in the physical capital market and the labor market are written as:

$$K_{t+1} = s_t P_t, \quad (13)$$

$$L_t = P_t (1 - \phi n_t - e_t n_t) h_t. \quad (14)$$

Now I can define the competitive equilibrium in our model as follows:

Definition 1 *Competitive Equilibrium:*

Given the initial aggregate physical capital K_0 , the initial stock of aggregate human capital H_0 , the initial population size P_0 , an equilibrium consists of sequences of household's decision rules $\{c_t, d_{t+1}, s_t, n_t, e_t\}_{t=0}^{\infty}$, sequences of prices $\{w_t, R_t\}_{t=0}^{\infty}$, government expenditure $\{E_t, T_{t+1}\}_{t=0}^{\infty}$ and aggregate quantities $\{K_{t+1}, L_t\}_{t=0}^{\infty}$ such that

1. the households' decision rules $\{c_t, d_{t+1}, s_t, n_t, e_t\}_{t=0}^{\infty}$ maximizes lifetime utility function (1) subject to the lifetime budget constraint (2) and the human capital formation function (3);
2. the factor prices $\{w_t, R_t\}_{t=0}^{\infty}$ are such that markets clear, i.e., (9) and (10) hold;
3. the government expenditures $\{E_t, T_{t+1}\}_{t=0}^{\infty}$ hold (11) and (12);
4. the aggregate valuables $\{K_{t+1}, L_t\}_{t=0}^{\infty}$ hold (13) and (14).

3 Fertility and Growth

In this section, I derive the fertility rate and the per capita output growth rate in the balanced growth path.

3.1 Fertility

From Eq. (4) and (13), the equilibrium condition in the capital market at time t can be rewritten as:

$$K_{t+1} = \left(\frac{1}{1 + \beta + \gamma} \right) \{ \beta(1 - \tau)w_t h_t - (1 + \gamma)T_{t+1}/R_{t+1} \} P_t. \quad (15)$$

Using Eq. (9) and Eq. (11), the present value of per capita social security benefit at time t is given by:

$$T_{t+1}/R_{t+1} = \tau(1 - \chi) \left(\frac{1 - \alpha}{\alpha} \right) K_{t+1}/P_t. \quad (16)$$

Substituting Eq. (16) into (15) for the aggregated physical capital dynamics, I have:

$$K_{t+1} = S(\tau, \chi)(1 - \tau)w_t h_t P_t, \quad (17)$$

where $S(\tau, \chi)$ represents the saving rate at equilibrium:

$$S(\tau, \chi) \equiv \beta \left\{ (1 + \beta + \gamma) + (1 + \gamma) \left(\frac{1 - \alpha}{\alpha} \right) \tau(1 - \chi) \right\}^{-1}.$$

From Eq. (5), (6), (7), (8) and (14), the labor supply at time t is given by:

$$L_t = L(\tau, \chi) h_t P_t. \quad (18)$$

where $L(\tau, \chi)$ represents the working time of parents and is given by:

$$L(\tau, \chi) \equiv 1 - \hat{\gamma} \left\{ 1 + \tau(1 - \chi) \left(\frac{1 - \alpha}{\alpha} \right) S(\tau, \chi) \right\}.$$

Notice that the labor supply does not depend on the private educational decision of parents. Using Eq. (12) and (18), per child public education expenditure is given by:

$$E_t(\tau, \chi) = \frac{\tilde{E}(\tau, \chi)}{n_t},$$

where $\tilde{E} = \tau \chi L(\tau, \chi)$. Next I derive the fertility rate at the competitive equilibrium. First, suppose that parents in period t invest a part of their disposal income in education i.e., $e_t > 0$. I term this case as the mixed education case. From Eq. (5), the fertility rate at the equilibrium is given by:

$$n_t = \left\{ \hat{\gamma}(1 - \eta)(1 + T(\tau, \chi)) + \tilde{E}(\tau, \chi) \right\} \phi^{-1} \equiv n_{mix}^*, \quad (19)$$

where

$$T(\tau, \chi) \equiv \tau(1 - \chi) \left(\frac{1 - \alpha}{\alpha} \right) S(\tau, \chi).$$

Second, suppose that parents in period t invest nothing in their children's human capital accumulation i.e., $e_t = 0$. In this situation, parents find it

optimal to leave their children to the public education institution completely. They use all adulthood period resources to consumption, saving and child-rearing. I term this situation as the complete public education case. From Eq. (7), optimal number of children at the equilibrium is as follows:

$$n_t = \hat{\gamma}(1 + T(\tau, \chi))\phi^{-1} \equiv n_{pub}^*. \quad (20)$$

The number of children is constant over time in this model.

Lemma 1 *For all contribution rate $\tau \in (0, 1)$ and fraction $\chi \in [0, 1]$, a rise in the PAYG social security contribution rate $\tau(1 - \chi)$ increases fertility n^* .*

3.2 Investment in education

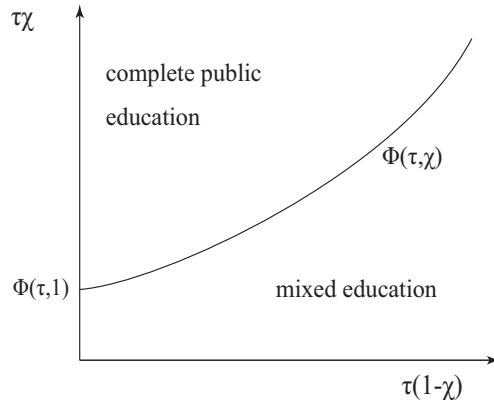


Figure 2: The function $\Phi(\tau, \chi)$

In order to investigate under what conditions parents privately invest in their children's education, I derive an inner solution condition for private education investment. From the Kuhn-Tucker conditions, the condition is

written as follows:

$$\eta\phi - \frac{\tilde{E}(\tau, \theta)}{n_{mix}^*} > 0.$$

Arranging above condition, the following lemma provides a necessary and sufficient condition for parental private investment in the balanced growth path.

Lemma 2 *Parents invest in their children's quality if and only if the size of public investment in education $\tau\chi$ satisfies:*

$$\tau\chi < \frac{\eta\hat{\gamma}(1 + T(\tau, \chi))}{1 - \hat{\gamma}(1 + T(\tau, \chi))} \equiv \Phi(\tau, \chi). \quad (21)$$

$\Phi(\tau, \chi)$ is a positive function of the PAYG social security contribution rate $\tau(1 - \chi)$. Figure 2 illustrates the threshold level $\Phi(\tau, \chi)$. $\Phi(\tau, 1)$ is the threshold value before applying the PAYG social security system. If the size of public education $\tau\chi$ is larger than the threshold level $\Phi(\tau, \chi)$, parents do not invest their children's quality privately. However, if the size of public education is lower than it, parents invest a part of their disposal income in their children's education. From Eq. (19) and (20) I obtain the fertility rate in the balanced growth path as follows:

$$n^* = \begin{cases} \{\hat{\gamma}(1 - \eta)(1 + T(\tau, \chi)) + E(\tau, \chi)\} \phi^{-1} \equiv n_{mix}^* & (\chi < \tilde{\chi}), \\ \hat{\gamma}(1 + T(\tau, \chi)) \phi^{-1} \equiv n_{pub}^* & (\tilde{\chi} \leq \chi). \end{cases} \quad (22)$$

where $\tilde{\chi}$ means the fraction which satisfies the equation $\tau\tilde{\chi} = \Phi(\tau, \tilde{\chi})$. Next I examine the dynamics in this model.

3.3 Dynamics

I define a capital/labor ratio as $k \equiv K/L$. Using Eq. (17), (18) and (22), I derive:

$$k_{t+1} = \left\{ \frac{(1-\tau)S(\tau, \chi)}{L(\tau, \chi)n^*(h_{t+1}/h_t)} \right\} (1-\alpha)k_t^\alpha. \quad (23)$$

From the assumption $0 < \alpha < 1$, this dynamics always converges to the capital/labor ratio \bar{k} monotonously.

$$\bar{k} = \left\{ \frac{(1-\tau)S(\tau, \chi)(1-\alpha)}{L(\tau, \chi)n^*(h_{t+1}/h_t)} \right\}^{\frac{1}{1-\alpha}}. \quad (24)$$

Using the Production technology and Eq. (18), per capita output growth rate at time t is written as follows:

$$\begin{aligned} 1 + g_t &= \frac{Y_{t+1}/N_{t+1}}{Y_t/N_t}, \\ &= \frac{h_{t+1}}{h_t} \left(\frac{k_{t+1}}{k_t} \right)^\alpha. \end{aligned}$$

In the balanced growth path, the capital/labor ratio is constant $k_t = k_{t+1} = \bar{k}$. Per capita output growth rate in the balanced growth path can be written as follows:

$$1 + g^* = \begin{cases} \mu\hat{\eta}(\phi - E_{mix}(\tau, \chi))^\eta \equiv 1 + g_{mix}^* & (\chi < \tilde{\chi}), \\ \mu E_{pub}(\tau, \chi)^\eta \equiv 1 + g_{pub}^* & (\tilde{\chi} \leq \chi), \end{cases} \quad (25)$$

where

$$\begin{aligned} E_{mix}(\tau, \theta) &\equiv \tilde{E}(\tau, \chi)/n_{mix}^*, \\ E_{pub}(\tau, \theta) &\equiv \tilde{E}(\tau, \chi)/n_{pub}^*, \\ \hat{\eta} &\equiv \frac{\eta}{1-\eta} \end{aligned}$$

4 Policy experiments

In this section, I examine the effect of reallocating public funds from PAYG social security benefits to public investment in education on fertility in the balanced growth path. I impose following parameter restrictions of the model:

Assumption 1

$$\hat{\gamma} < \alpha, \tau < 0.5, \eta < 0.5.$$

I can show the effect of public education on fertility in the balanced growth path as following proposition:

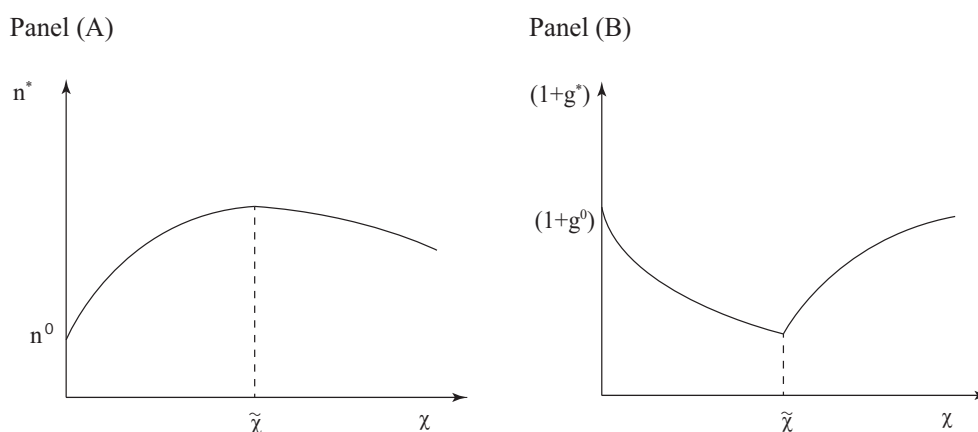


Figure 3: The effect of reallocating public funds from PAYG social security benefits to public investment in education on fertility rate and per capita output growth rate.

Proposition 1 *Assuming that the tax rate is constant, if the size of public investment in education is sufficiently small i.e., $\chi < \tilde{\chi}$, a reallocation public funds from PAYG social security benefits to public education increases fertility*

rate n_{mix}^* . On the contrary, if the size of public investment in education is sufficiently large i.e., $\tilde{\chi} \leq \chi$, a reallocation public funds from PAYG social security benefits to public education decreases fertility rate n_{pub}^* .

Proof 1 See Appendix.

Figure 3(A) summarizes the impact of public investment in education on fertility, where n^0 denotes the fertility rate if public education scheme does not exist. The intuition behind Proposition 1 is as follows. If $\chi < \tilde{\chi}$, reallocating public financial resources from PAYG social security policy to public education policy has three effects on fertility: (1) a positive direct effect that increases fertility by reducing per child education cost, (2) a negative direct effect that decreases fertility by reducing parent's lifetime budget, (3) a positive indirect effect that increases fertility by increasing per child public education expenditure. Two positive effects dominate a negative effect under Assumption 1. This results is similar to Azarnert (2010) which studies the effect of free public education on fertility, private educational investments, and human capital accumulation at different stages of economic development. In this paper, the availability of free public schooling crowds out public education investment, stimulates fertility and impedes growth at advanced stages of development. If $\tilde{\chi} \leq \chi$, a change in the allocation from PAYG social security policy to public education policy has a direct negative effect on fertility by reducing PAYG social security benefits.

Next, I examine how reallocating public financial resources from PAYG social security benefits to public investment in education affects per capita output growth rate in the balanced growth path. From Eq. (22), I derive the following proposition.

Proposition 2 Assuming that the wage tax rate is constant, if the size of public investment in education is small i.e., $\chi < \tilde{\chi}$, a reallocation public funds from PAYG social security benefits to public investment in education slows per capita output growth $(1 + g_{mix}^*)$. On the contrary, if the size of public investment is sufficiently large i.e., $\tilde{\chi} \leq \chi$, a reallocation public funds from

PAYG social security benefits to public investment in education stimulates per capita output growth $(1 + g_{pub}^)$.*

Proof 2 *See Appendix.*

Figure 3(B) summarizes the impact of public investment in education on per capita output growth rate, where $(1 + g^0)$ denotes the growth rate if public education scheme does not exist. The intuition behind Proposition 2 is as follows. From Eq. (25), per capita output growth rate depends on per child public investment in education E in the balanced growth path. If the size of public investment in education is small, a reallocating from PAYG social security benefits to public investment in education has three effects on per child public education expenditure: (1) a positive direct effect that increases public education expenditure by increasing the educational contribution rate, (2) a negative indirect effect that decreases public education expenditure by decreasing labor supply and the tax base, (3) a negative indirect effect that decreases per child public education expenditure by stimulating fertility rate. A first positive direct effect dominates other negative effects.

On the contrary, if the size of public investment in education is sufficiently large, reallocating public funds from PAYG social security benefits to public investment in education has three different effects on per capita public education expenditure: (1) a positive direct effect that increases public education expenditure by increasing the educational contribution rate, (2) a negative indirect effect that reduces household's labor supply and tax base, (3) a positive indirect effect that increases per capita out public education expenditure by reducing the number of child. Two positive effects always dominate a negative indirect effect.

5 Conclusion

This paper examines the effects of an increase in public funding for education on the parental fertility and educational decision and per capita output

growth rate in the balanced growth path. The model used in this study is the three-period-lived overlapping generation model, incorporating both the trade-off between the quality and quantity of children. Government finances PAYG social security program and public education program. Similar to Zhang (1997) and Azarnert (2010), this paper assumes public input in education is substituting for private input at producing children's human capital.

First, this paper showed that under what conditions parents invest in their children's quality privately. Differently from previous studies, there are two different cases. When the size of public education expenditure is sufficiently low and per child public investment in education is not large, parents educate their children privately. On the other hand, when per child public educational investment is sufficiently large, parents leave their children's education to public education institution completely.

Second, I examine the effect of reallocating public funds from PAYG social security benefits to public investment in education on fertility and per capita output growth in the balanced growth rate. When parents invest their children's education privately, public investment in education increases fertility and impede growth. However when parents do not invest their children's education at all, a higher public investment in education decreases fertility rate and stimulate growth.

Appendix

Proof of Proposition 1

1. Assuming that $\chi < \tilde{\chi}$, the proof of the first part of proposition 1 uses the following derivative:

$$\begin{aligned} \frac{\partial n_{mix}^*}{\partial \chi} &= \phi^{-1} \left[\{\gamma(1 - \eta) - \tau\chi\gamma\} \frac{\partial T(\tau, \chi)}{\partial \chi} + \tau \{(1 + \beta) - \gamma T(\tau, \chi)\} \right], \\ &= \phi^{-1} \\ &\times \left[1 - \hat{\gamma} + \hat{\gamma}\hat{\alpha} \left\{ \{\tau(1 - \chi) + (1 - \eta) - \tau\chi\} S(\tau, \chi) + \{(1 - \eta) - \tau\chi\} (1 - \chi) \frac{\partial S(\tau, \chi)}{\partial \chi} \right\} \right], \end{aligned}$$

where $\hat{\alpha} \equiv \frac{1-\alpha}{\alpha}$. Under assumption 1 the first part of proposition 1 is obvious.

2. Now the proof of second part of Proposition 1 straightforwardly derives by the following derivative:

$$\frac{\partial n_{pub}^*}{\partial \chi} = \hat{\gamma}\phi^{-1} \frac{\partial T(\tau, \chi)}{\partial \chi}.$$

Since $\frac{\partial T(\tau, \chi)}{\partial \chi} < 0$, the second part of proposition 1 is obvious. \square

Proof of Proposition 2

1. I derive the following derivative from Eq. (25):

$$\frac{\partial E_{mix}}{\partial \chi} = \hat{\gamma}(1 - \eta)\phi^{-1} \left\{ \frac{\partial \tilde{E}(\tau, \chi)}{\partial \chi} (1 + T(\tau, \chi)) - \frac{\partial T(\tau, \chi)}{\partial \chi} \tilde{E}(\tau, \chi) \right\} n_{min}^{*-2}.$$

Since $\frac{\partial T(\tau, \chi)}{\partial \chi} < 0$ and $0 < \frac{\partial E(\tau, \chi)}{\partial \chi}$, the first part of proposition 2 is obvious.

2. Next I consider the latter part of proposition 2. I calculate the following derivative from Eq. (25):

$$\frac{\partial E_{pub}}{\partial \chi} = \hat{\gamma}\phi^{-1} \left\{ \frac{\partial \tilde{E}(\tau, \chi)}{\partial \chi} (1 + T(\tau, \chi)) - \frac{\partial T(\tau, \chi)}{\partial \chi} \tilde{E}(\tau, \chi) \right\} n_{pub}^{*-2}.$$

In the same way as the first part of proposition 2, the second part is obvious.
□

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