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Shin-Ichi Nishiyama
Masao Ogaki

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TOHOKU ECONOMICS RESEARCH GROUP

GRADUATE SCHOOL OF ECONOMICS AND
MANAGEMENT TOHOKU UNIVERSITY
KAWAUCHI, AOBA-KU, SENDAI,
980-8576 JAPAN

The Cross-Euler Equation Approach in Estimating the Elasticity of Intertemporal Substitution for Food and Non-Food Consumption in Japan[1]

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Shin-Ichi Nishiyama
Graduate School of Economics and Management, Tohoku University

Masao Ogaki
Department of Economics, Keio University

Abstract

We use the standard two-good version of the life cycle/permanent income model in analyzing the intratemporal and intertemporal aspect of food and non-food expenditure in Japan. The empirical dilemma in identifying and estimating the parameters governing the intertemporal elasticity of substitution (IES) is addressed. In overcoming this empirical dilemma we employ the Cross-Euler equation approach proposed by Nishiyama (2005). The IES parameters are estimated by exploiting the cointegration restriction implied by the Cross-Euler equation and also from the standard Euler equation using GMM. Further, by comparing the IES estimates from the Cross-Euler equation to those from the standard Euler equation, we formally test the hypothesis whether food and non-food expenditure in Japan is affected by some factors that cause misspecification in the standard Euler equation approach, such as liquidity constraints or habit formation.

Key words: Cointegration, Elasticity of Intertemporal Substitution, Euler Equation
JEL Classification: C22, E21

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1. Introduction

Previous empirical studies often estimated the intertemporal elasticity of substitution (IES) of Japanese consumer in the context of single goods and found the IES to be considerably high,[2] implying that the consumers are willing to substitute current consumption for future consumption in response to small change in real interest rate. If these high estimates of IES are indeed true, then we should be observing a significant decrease in the consumption growth rate during an expansionary monetary policy and significant increase during a contractionary monetary policy in Japan. However, in reality, we do not observe this kind of phenomenon in Japan and, therefore, the counter-intuitively high estimates of IES from the previous literature remains to be a puzzle. In this paper, we estimate and test intertemporal and intratemporal implications of a two-good model with food and non-food goods with a representative consumer, using Japanese data.

The preference specification of the two-good model follows Atekeson and Ogaki (1996). In this model, the intertemporal elasticity of substitution (IES) for total consumption expenditure increases as the level of wealth increases. The intuition behind this is that the consumer is less willing to substitute the food consumption than the non-food consumption over time. Thus, the IES for the food good is smaller than that for the non-food good. The IES for the total consumption expenditure is a weighted average of the IES for the food and the IES for the non-food good, with the weights being the budget shares. Because the budget share for the food is larger when the consumer is poor, the IES is small when the consumer is poor. The IES increases as the wealth increases in this model. Also, the model has various implications on how macroeconomic variables behave as the Japanese economy grows out of the destruction during the World War II. For example, the model typically predicts that the saving rate in Japan was very low immediately after the World War II, and then started to increase as the economy grew (see, e.g., Ogaki, Ostry, and Reinhart (1996) for a description of the saving rate behavior of the model with wealth-increasing IES). Thus it is important to investigate the extent to which the model is consistent with the Japanese data.

As for the methodological strategy of this paper, we apply Nishiyama's (2005) Cross-Euler Equation approach to test intertemporal and intratemporal

[2] For instance, Hamori (1996) reports the IES of Japanese consumer to be well above 10.

implications of the model. This is a new approach to test Euler equation in two-good models. In many applications of two-good models, researchers have faced a methodological problem in estimating intratemporal first order condition and Euler equations. The Cross-Euler equation approach[3] can be a solution to this methodological problem.

The methodological problem in a two-good model with time separable preferences arises because the intratemporal first order condition holds without any forecasting error in the model. In order to use the first order condition to estimate and test the model, it is necessary to add measurement errors or preference shocks to the model. However, adding measurement errors or preference shocks causes problems for the Euler equation approach based on the Generalized Method of Moments (GMM) as pointed out by Garber and King (1983). Because of the nonlinearity of Euler equations, measurement errors or preference shocks make GMM estimators inconsistent. Nishiyama (2005) solved this problem by focusing on first order conditions involving a good at time t and the other good at time t+1. He calls such a first order condition a cross-Euler equation. A cross-Euler equation involves a forecasting error unlike the intratemporal first order condition. Hence statistical methods can be applied to the cross-Euler equation without adding measurement errors nor preference shocks to the model. The first step in his approach is to derive a long-run restriction from a cross-Euler equation. This long-run restriction implies a relationship between variables called cointegration. The cointegration relationship allows one to use a regression to estimate preference parameters.

Regarding the cointegration regression and the test of coingetration, Nishiyama (2005) used Park's (1992) Canonical Cointegration Regression (CCR) estimator, and Park's (1990) tests for the null hypothesis of cointegration in the CCR framework. In this paper, we use Stock and Watson's (1993) Dynamic Ordinary Least Squares (DOLS) estimator and Choi, Hu, and Ogaki's (2005) Hausman-type test for the null hypothesis of cointegration. If the parameterized endogeneity correction used in the DOLS is a good approximation, these methods

[3] Further, Nishiyama (2005) showed that the cointegration relationship implied by the cross-Euler equation is robust to many factors such as liquidity constraints and time non-separability of preferences. These factors are often pointed out as possible causes of empirical rejections of standard Euler equations. Thus, by comparing estimates from cross-Euler equations and those from standard Euler equations, it is possible to formally test the hypothesis that liquidity constraints or non-separability of preferences casue the misspecification of the standard Euler

have better small sample properties than the nonparametric CCR methods. On the other hand, CCR is expected to perform better when the prameterized endogeneity correction is misspecified. Because of a relatively small sample we used in this paper, we prefer to use the DOLS estimator. In the DOLS framework, only the Hausman-type cointegration test and Shin's (1994) test are available at this point (it should be noted that the popular augmented Dickey-Fuller test should be applied to the static OLS residual rather than the DOLS residual.) The Hausman-type test has better small sample properties than Shin's test as shown in Choi, et. Al.(2005).

The rest of the paper is organized as follows. In Section 2, we describe the two-goods version of life cycle/permanent income model (LCPIM) for food and non-food consumption. In Section 3, we explain the Cross-Euler equation approach and derive the cointegrating restriction among the forcing variables. In Section 4, we estimate the IES parameters exploiting the cointegration restriction implied by the Cross-Euler eqation. We also estimate the IES parameters from the standard Euler equations using GMM. In Section 5, we formally compare the IES estimates from the Cross-Euler to standard Euler equation. Section 6 summarizes this paper with a tentative conclusion.

2. Modeling Food and Non-Food Consumption Behavior

2.1. Setup of the Model

This paper adopts the standard two-goods version of the life cycle/permanent income model following Atkeson and Ogaki (1996). A representative agent is assumed to maximize his expected lifetime utility under his lifetime budget constraint. The dynamic optimization problem is formulated as follows,

$$\begin{gathered} \max E_t\left[\sum_{i=0}^{\infty}\beta^i U(F_t, NF_t)\right] \\ s.t.\ W_{t+1} = (1+r_t)\left(W_t - P_t^F F_t - P_t^{NF} NF_t\right) + Y_{t+1} \end{gathered} \qquad (1)$$

where F_t stands for food expenditure at period t, NF_t for non-durable non-food expenditure, W_t for assets held by the agent, Y_t for the stochastic labor income of the agent, r_t for the real interest rate from period t to t+1, P_t^F for the price of

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equations.

food, and P_t^{NF} for the price of non-food, respectively. β stands for the agent's discount rate.

We assume that period-by-period utility is time separable for this agent and also assume additive-separability between durable goods and non-durable goods. As for the period-by-period utility function, we employ a standard addi-log function following Houthakker (1960):

$$U(F_t, NF_t) = \frac{F_t^{1-\alpha}}{1-\alpha} + K\frac{NF_t^{1-\gamma}}{1-\gamma}. \qquad (2)$$

Under this specification, the FOCs will then be as follows:

$$\frac{P_t^F}{P_t^{NF}} = \frac{1}{K}\frac{F_t^{-\alpha}}{NF_t^{-\gamma}} \qquad (3)$$

$$E_t\left[\beta\left(\frac{F_{t+1}}{F_t}\right)^{-\alpha}(1+r_t)\frac{P_t^F}{P_{t+1}^F}\right] = 1 \qquad (4)$$

$$E_t\left[\beta\left(\frac{NF_{t+1}}{NF_t}\right)^{-\alpha}(1+r_t)\frac{P_t^{NF}}{P_{t+1}^{NF}}\right] = 1 \qquad (5)$$

It should be noted that under this addi-log specification, $1/\alpha$ and $1/\gamma$ can be interpreted as the intertemporal elasticity of substitution (IES) of food and non-food expenditure, respectively.

Some remarks should follow for these FOCs. As was pointed out by Ogaki and Park (1997), the specification of the intratemporal relationship eq. (3) turns out to be robust to several kinds of nuisance factors, such as liquidity constraint and/or habit formation in the utility function. However, the specification of Euler equations is very sensitive to the presence of liquidity constraint or habit formation. In other words, specification of the intratemporal relationship is robust, but the specification of Euler equations is not. Conversely, if by any method we can find evidence that the Euler equation is correctly specified, this will be strong evidence against the presence of liquidity constraint or habit formation.

If the model is correct, then the intratemporal optimality condition (3) and Euler equations (4) and (5) will be correctly specified. Therefore, parameter estimates α and γ from eq. (3) and Euler equations (4) and (5) should be reasonably close. If the statistical test concludes that parameter estimates are significantly different from each other, then, by contrapositive logic, we can conclude that some of the assumptions we had made (i.e. addi-log type utility function, additive separability of durable and non-durable goods, non-existence of liquidity constraints, non-existence of habit formation, etc.) are implausible. Conversely, if a statistical test does not reject the null hypothesis that parameter estimates are equal, it will support or, at least, leave some possibility open for the joint assumption of addi-log utility specification without the presence of habit formation or liquidity constraints.

2.2. Complications in Estimating IES parameters

The empirical task is to first obtain the IES parameter estimates from the intratemporal relationship and from Euler equations. However, as Nishiyama (2005) pointed out, when the utility function is of the addi-log type, there is an empirical complication in estimating parameters.

A complication arises from the deterministic relationship of the intratemporal FOC (3). A natural way to estimate the IES parameters from the intratemporal relationship is to log-linearize eq. (3) as follows

$$\ln \frac{P_t^F}{P_t^{NF}} + const. + \alpha \ln F_t - \gamma \ln NF_t = 0 \qquad (6)$$

Provided that (log) relative price, (log) food expenditure, and (log) non-food expenditure all follow the difference stationary process, one may be tempted to exploit the cointegration restriction by adding the ad-hoc stationary error term on RHS of eq. (6). But then, in order to maintain coherence of the error structure within the model, the ad-hoc error term should also be incorporated in the Euler equations (4) and (5). Indeed, Nishiyama (2005) shows that, by introducing new kind of error term to the model, the conditional moment conditions implied by the Euler equations to be non-standard – i.e.,

$$E_t\left[\beta\left(\frac{F_{t+1}}{F_t}\right)^{-\alpha}(1+r_t)\frac{P_t^F}{P_{t+1}^F}\right] \neq 1 \text{ and}$$

$$E_t\left[\beta\left(\frac{NF_{t+1}}{NF_t}\right)^{-\alpha}(1+r_t)\frac{P_t^{NF}}{P_{t+1}^{NF}}\right] \neq 1.$$

As such, in the presence of ad-hoc error term, eq. (4) and (5) are no longer correctly specified and, therefore, the GMM estimation based on those misspecified conditional moment conditions yields inconsistent estimates of the IES parameters.

This is the point where one experiences an empirical difficulty in estimating the IES parameters. In order to overcome this difficulty, Nishiyama (2005) proposed the Cross-Euler equation approach in estimating the IES parameters. By exploiting the cointegrating restriction implied by the cross-Euler equation, it becomes possible to estimate the IES parameters from both standard Euler and cross-Euler equation without altering the error structure of the model.

3. The Cross-Euler Equation Approach

3.1. Economic Interpretation of the Cross-Euler Equation

In this section, we derive the Cross-Euler equations based on a time-separable utility function. Since the time-horizon of a representative agent's optimization problem is infinite, we can reformulate the problem using Bellman equation as follows,

$$V(W_t)=\max_{F_t,NF_t}\left\{U(F_t,NF_t)+\beta E_t V(W_{t+1})\right\}$$
$$s.t.\ W_{t+1}=(1+r_t)\left(W_t-P_t^F F_t-P_t^{NF}NF_t\right)+Y_{t+1}.$$

First, the FOCs with respect to F_t and NF_t will be

$$\frac{U_F(F_t,NF_t)}{P_t^F}=\beta(1+r_t)E_tV'(W_{t+1}) \quad \text{and} \tag{7}$$

$$\frac{U_M(F_t,NF_t)}{P_t^{NF}}=\beta(1+r_t)E_tV'(W_{t+1}), \tag{8}$$

respectively. Next, invoking the envelope theorem on the Bellman equation, we obtain the following relationship between the current and future shadow price of wealth;

$$V'(W_t)=\beta(1+r_t)E_tV'(W_{t+1}) \tag{9}$$

Substituting eq. (9) for eq. (7) and eq.(8), the FOCs for F_t and NF_t can be rearranged as

$$\frac{U_F(F_t,NF_t)}{P_t^F}=V'(W_t) \quad \text{and} \tag{10}$$

$$\frac{U_{NF}(F_t,NF_t)}{P_t^{NF}}=V'(W_t). \tag{11}$$

Conventionally, the standard Euler equation can be derived by updating eq. (10) (or eq. (11)) and substituting back to eq. (7) (or eq. (8)) which was the case in the previous section. Instead, we derive the Cross-Euler equation by updating eq. (11) and substituting it for eq. (7). After some manipulation, the Cross-Euler equation can be shown to be

$$E_t\left[\beta\frac{U_{NF}(F_{t+1},NF_{t+1})}{U_F(F_t,NF_{t+1})}(1+r_t)\frac{P_t^F}{P_{t+1}^{NF}}\right]=1. \qquad (12)$$

By the same token, another type of Cross-Euler equation can be derived by updating eq. (10) and substituting it for eq. (8)as follows,

$$E_t\left[\underbrace{\beta\frac{U_F(F_{t+1},NF_{t+1})}{U_{NF}(F_t,NF_t)}}_{\Theta}\underbrace{(1+r_t)\frac{P_t^{NF}}{P_{t+1}^F}}_{\Omega}\right]=1. \qquad (13)$$

Now, let us attempt to make an economic sense of the Cross-Euler equation taking the case of eq. (13). First, notice that the term Θ in eq. (13) stands for the cross-intertemporal marginal rate of substitution (CIMRS)[4] between goods NF_t and F_{t+1}. In other words, the term Θ represents the agent's perceived trade-off between current non-food goods and future food goods. Second, let us turn to the term Ω in eq. (13). The term Ω stands for the opportunity cost of obtaining F_{t+1} in terms of NF_t. The logic is as follows. By selling one unit of NF_t at period t, the agent can obtain P_t^{NF} amount of numeraire goods -- i.e., W_t in this context. By saving all of these numeraire goods at period t, the agent can obtain $(1+r_t)P_t^{NF}$ of numeraire goods at period t+1. By using all of these to buy F_{t+1}, the agent can buy $(1+r_t)P_t^{NF}/P_{t+1}^F$ units of F_{t+1}. Thus, the opportunity cost of F_{t+1} in terms of NF_t is $(1+r_t)P_t^{NF}/P_{t+1}^F$. Finally, if the agent is optimally trading NF_t to F_{t+1}, then the agent is equalizing the opportunity cost to CIMRS between F_{t+1} and NF_t, yielding the above Cross-Euler equation (16). This is the economic intuition behind the Cross-Euler equation.

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[4] The concept of cross-intertemporal marginal rate of substitution (CIMRS) is a key ingredient of the cross-Euler equation and is defined as follows. For more elaboration regarding the concept of CIMRS, see Nishiyama (2005).

Definition: Let $V(x_1^1,...,x_1^K,...,x_T^1,...,x_T^K)$ be a utility function defined upon K goods with T periods and let **x** be a $KT\times 1$ vector such that **x**=$(x_1^1,...,x_1^K,...,x_T^1,...,x_T^K)$. Then we call the following expression,

$$-\frac{\partial V(x)/\partial x_{t+1}^i}{\partial V(x)/\partial x_t^j},$$

as the cross-intertemporal marginal rate of substitution (CIMRS) between goods x_{t+1}^i and x_t^j, where $i\neq j$ and t = 1, ..., T-1.

In order to understand the structure of Cross-Euler equation further, it is useful to decompose the terms Θ and Ω. Decomposing the Cross-Euler equation (16), we obtain the following relationship

$$E_t\left[\underbrace{\beta\frac{U_{NF_{t+1}}}{U_{NF_t}}}_{\Theta_1}\underbrace{(1+r_t)\frac{P_t^{NF}}{P_{t+1}^{NF}}}_{\Omega_1}\underbrace{\frac{U_{F_{t+1}}}{U_{NF_{t+1}}}}_{\Theta_2}\underbrace{\frac{P_{t+1}^{NF}}{P_{t+1}^{F}}}_{\Omega_2}\right]=1. \tag{14}$$

It is possible to decompose CIMRS into IMRS component and MRS component. Exploiting this property, the term Θ can be decomposed to IMRS portion (denoted Θ_1 in the above equation) and MRS portion (denoted Θ_2 in the above equation). Turning to the term Ω, which stands for the opportunity cost of F_{t+1} in terms of NF_t, it is also possible to decompose it into two parts; the opportunity cost of NF_{t+1} in terms of NF_t (denoted Ω_1 in eq. (14)) and the opportunity cost of F_{t+1} in terms of NF_{t+1} (denoted Ω_2 in eq. (14)). Here, notice that IMRS Θ_1 and opportunity cost Ω_1 constitutes a standard Euler equation in the conventional context. Further, notice that MRS Θ_2 and opportunity cost Ω_2 constitutes an intratemporal FOC at period t+1. Thus, in this sense, the Cross-Euler can be interpreted as the composite optimal condition that embeds both intertemporal and intratemporal optimality conditions into one equation.

3.2. Cointegration Relationship Implied by the Cross-Euler Equation

Next, under certain assumptions, we show that the Cross-Euler equation implies the cointegrating restriction among the forcing variables. Further, we show that the Cross-Euler equation approach can overcome the empirical dilemma. In other words, the Cross-Euler equation approach allows us to estimate and compare the IES parameters to those from the standard Euler equations without altering the error structure of the model.

In what follows, we assume the following stochastic processes for each variable.

Assumption 1: Log food and non-food goods expenditure follow the I(1) process.

Assumption 2: Log price index for food and non-food goods follow the I(1) process and they are not cointegrated.

Assumption 3: Real interest rate follows the I(0) process.

We parametrize the Cross-Euler equation (12) using Houthakker's (1960) addi-log utility function;

$$E_t\left[\beta K\frac{NF_{t+1}^{-\gamma}}{F_t^{-\alpha}}(1+r_t)\frac{P_t^F}{P_{t+1}^{NF}}\right]=1. \qquad (15)$$

In a similar fashion, we parametrize the another version of the Cross-Euler equation (13) as follows,

$$E_t\left[\frac{\beta}{K}\frac{F_{t+1}^{-\alpha}}{NF_t^{-\gamma}}(1+r_t)\frac{P_t^{NF}}{P_{t+1}^F}\right]=1. \qquad (16)$$

The forecast error from the Cross-Euler equation (15) can be defined as follows,

$$\begin{aligned} e_{t+1} &= \beta K\frac{NF_{t+1}^{-\gamma}}{F_t^{-\alpha}}(1+r_t)\frac{P_t^F}{P_{t+1}^{NF}} - E_t\left[\beta K\frac{NF_{t+1}^{-\gamma}}{F_t^{-\alpha}}(1+r_t)\frac{P_t^F}{P_{t+1}^{NF}}\right] \\ &= \underbrace{K\frac{\beta(1+r_t)}{F_t^{-\alpha}/P_t^F}}_{\text{non-stochastic at period t}}\underbrace{\left[\frac{NF_{t+1}^{-\gamma}}{P_{t+1}^{NF}} - E_t\left(\frac{NF_{t+1}^{-\gamma}}{P_{t+1}^{NF}}\right)\right]}_{\text{stochastic at period t}} \end{aligned}$$

It is useful to notice where the stochasticity of e_{t+1} is arising. Rearranging the definition of the forecast error as in the second line of the above equation, we see that e_{t+1} is composed of non-stochastic portion and stochastic portion as of period t. As can be seen, the stochasticity of e_{t+1} arises from the discrepancy between the realized marginal utility of NF_{t+1} and expected marginal utility of NF_{t+1}. This discrepancy is discounted to the present value and denominated by the marginal utility of F_t, which is non-stochastic as of period t. In this sense, the forecast error in the context of the Cross-Euler equation measures the magnitude of expectation error in future marginal utility from goods j in terms of current marginal utility from goods i.

From the definition of the forecast error, the Cross-Euler equation (15) can be rewritten as

$$\beta K\frac{NF_{t+1}^{-\gamma}}{F_t^{-\alpha}}(1+r_t)\frac{P_t^F}{P_{t+1}^{NF}}=1+e_{t+1}. \qquad (17)$$

Taking logarithm on both sides of eq. (17) will yield

$$const.+\ln\left[(1+r_t)\frac{P_t^F}{P_{t+1}^{NF}}\right]-\gamma\ln NF_{t+1}+\alpha\ln F_t=\ln(1+e_{t+1}).$$

Under the assumption that growth rate of food and non-food expenditure, the real interest rate, and the growth rate of the price level of both food and non-food are stationary, Nishiyama (2005) showed that $\ln(1+e_{t+1})$ to be stationary.

Exploiting the I(0) process of $\ln(1+e_{t+1})$, we obtain the following cointegrating restriction among the forcing variables,

$$\ln\left[(1+r_t)\frac{P_t^F}{P_{t+1}^{NF}}\right]+\alpha\ln F_t-\gamma\ln NF_{t+1}\sim I(0) \quad (18)$$

By the same token, the Cross-Euler equation (16) implies the following cointegrating restriction,

$$\ln\left[(1+r_t)\frac{P_t^{NF}}{P_{t+1}^{F}}\right]-\alpha\ln F_{t+1}+\gamma\ln NF_t\sim I(0) \quad (19)$$

Thus, we have derived the legitimate cointegrating restriction among the forcing variables based on the forecast error alone. Further, since we have not introduced any ad-hoc error structure to the model, such as an optimization error, measurement error, preference shock, the specification of the standard Euler equation (4) and (5) remain intact. In the sense that we now can compare the IES estimates from Cross-Euler equation to those from the standard Euler equations without altering the error structure of the model, our proposed approach successfully overcomes the empirical dilemma.

In addition, the log-linearized Cross-Euler equations can be shown to be robust against several nuisance factors (see Appendix). In particular, even in the existence of liquidity constraints or a certain type of habit formation, cointegration relationships (18) and (19) yield super-consistent estimates for α and γ, while Euler equations (4) and (5) are not guaranteed to yield consistent estimates. On the other hand, in the absence of liquidity constraints or habit formation, both log-linearized Cross-Euler equations and standard Euler equations yield super-consistent and consistent estimates of α and γ, respectively. This latter proposition, which basically states that the estimates of IES parameters from cointegration analysis and GMM to be close in the absence of liquidity constraints or habit formation, is particularly important since we can formally test this hypothesis using statistical methods such as Cooley and Ogaki's (1996) LR type test.

4. Estimating IES Parameters from the Cross-Euler Equations

4.1 Data Description

The data used in this paper is based on the Family Income and Expenditure Survey (FIES) from 1982 to 2004. In order to preserve enough number of observations, we focused on the expenditure behavior of both metropolitan and rural households excluding single-person households and agricultural households.[5] To smooth out the haphazard monthly movements, the expenditure data is transformed into quarterly data by summing monthly observations and the price data is transformed into to quarterly data by taking quarterly averages. The seasonal factor in both expenditure and price data have been removed using X-12 ARIMA method.[6] As a result of these adjustments, we have total of 90 observations (1982Q1 to 2004Q2) in estimating the IES parameters.

Although it is possible to define food expenditure in various ways, we construct food expenditure data by excluding 'alcoholic beverages' from 'all food' in FIES expenditure category. As Unayama (2003) reports, alcoholic beverages in Japan are well characterized as luxury goods rather than necessity goods. While other food related goods are characterized as necessity goods, the characteristic of alcohol beverages stands in sharp contrast to other food related goods. In order to retain the homogeneity among the food related goods so as to avoid the goods aggregation bias, we have deliberately excluded alcoholic beverages from the food category.

In constructing the non-food expenditure data, we have combined expenditure categories 'apparel' and 'other goods' in FIES. Again, Unayama (2003) found some empirical evidence that suggest 'apparel' and 'other goods' to be luxury goods. Based on this finding, the goods aggregation bias from combing 'apparel' and 'other goods' does not seem to be too problematic.

The price indexes for food and non-food have been constructed by taking the weighted average within the same category. The weight for each item corresponds to real expenditure share of each item within the category. Then those price indexes are used to deflate the nominal expenditure of food and non-food in order to convert them into real expenditure. Finally, the real expenditure for food and non-food are further adjusted to per-capita base. The Figure 1 and Figure 2 show the time-series plots for each variable.

4.2. Preliminary Analysis: Testing for Unit Root

As for a preliminary analysis before the cointegrating regression and

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[5] Family Expenditure and Income Survey started to include the single-person households and agricultural households in the survey starting from January 2000.

[6] The consumption tax, which was initially set at 3%, was introduced in April 1989 and subsequently raised to 5% in April 1997. The effects of consumption tax on the prices of goods have been removed by including the level shifts in X-12 ARIMA method.

testing, we test the null of unit root non-stationarity against the alternative of (trend) stationarity for the variables included in the cointegrating regression. In particular, we test the unit root non-stationarity of the following four variables: log food expenditure ($\ln F_t$), log non-food expenditure ($\ln NF_t$), opportunity cost of current food consumption against future non-food consumption ($\ln(1+r_t)*P_t^F / P_{t+1}^{NF}$), and opportunity cost of current non-food consumption against future food consumption ($\ln(1+r_t)*P_t^{NF} / P_{t+1}^F$). For the sake of visualization, the time series plot of the log food and non-food expenditure are provided in Figure 1, while the time series plot of the opportunity costs are provided in Figure 3. In Figure 3, the legend `Opp1` stands for the opportunity cost of current food consumption against future non-food consumption and `Opp2` stands for the opportunity cost of current non-food consumption against future food consumption.

The test results are reported in Table 0. For the tests setting the null as unit root non-stationarity, we have conducted Said and Dickey's (1984) augmented Dickey-Fuller (ADF) test and Phillips and Perron's (1988) PP test. The test results for ADF test and PP test are reported on the left hand side of Table 0. As can be seen from the table, the tests were not able to reject the null of unit root non-stationarity at the 5% significance level for all variables relevant to cointegrating regression. As for confirmatory analysis, we have tested the null of (trend) stationarity using KPSS method proposed by Kwiatkowski et al. (1992). The test results are reported on the right hand side of Table 0. The test rejects the stationarity of log food expenditure and `Opp1` -- conforming to the test results from ADF and PP tests --, but was not able to reject the stationarity of the log non-food expenditure and `Opp2`. We have also tested the null of trend stationarity using KPSS test. Here, the test rejects the trend stationarity for 3 out of 4 variables. Although, the hypothesis regarding the unit root non-stationarity regarding 4 variables are somewhat mixed, considering that ADF and PP tests were not able to reject the unit root hypothesis for all the cases and also based on the several rejections of (trend) stationarity from KPSS test, from this point forward, we regard four variables considered in the cointegration regression to be I(1) variables.

4.3. Dynamic Regressions and the Hausman-type Cointegration Test

In this subsection, we use Stock and Watson's (1993) DOLS estimator in estimating the IES parameters from the log-linearized Cross-Euler equations[7]. We test the null hypothesis of cointegration.with Choi, Hu, and Ogaki's (2005) Hasusman-type Cointegration test in the DOLS framework.

The DOLS estimator is well known, but the Hausman-type cointegration test is not. Therefore, we mainly explain the idea of the Hausman-type cointegration test in this subsection. Consider the following dynamic regression where y_t is an I(1) variable, x_t is a 2-dimensional vector of I(1) variables, and k stands for the order of leads and lags.

$$y_t = \beta' x_t + \sum_{i=1}^{2} \sum_{j=1}^{k} (\gamma_{i,j} \Delta x_{i,t-j}) + e_t \qquad (20)$$

The leads and lags of the first difference of the regressors are added in this regression in order to correct for the endogeneity problem. Following Stock and Watson (1993), we assume that the endogeneity correction of adding leads and lags perfectly eliminates the endogeneity problem in that e_t is strictly exogenous with respect to the regressors in (20) in this paper[8].

Now, if the error term e_t is I(0), then this is a dynamic cointegrating regression. As shown by Stock and Watson (1993), the OLS estimator for this regression is super-consistent and asymptotically efficient under their regularity conditions.

On the other hand, if the error term e_t is I(1), then regression (20) is a spurious regression and, the Dynamic OLS is inconsistent for the coefficient β as shown in Choi et al. (2005). The Hausman-type cointegration test utilized these properties to discriminate between the situation in which e_t is I(0) and that in which e_t is I(1). In order to understand the idea of this test, it is useful to start with an analysis of the DOLS estimator by the Gauss-Markov Theorem, using Ogaki and Choi's (2001) framework. For this purpose, we consider a special case in which e_t is serially uncorrelated. In this case, Ogaki and Choi's conditional probability version of the Gauss-Markov Theorem applies[9], and the OLS applied to

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[7] Nishiyama (2005) adopted Park's (1992) Canonical Cointegration Regression (CCR) in estimating the IES parameters. If the parametric form of the endogenerity correction is a good approximation, then the Dynamic OLS Regression is more efficient than the CCR. .

[8] To ease our exposition, we assumed that the endogeneity correction is complete. We can relax this assumption as shown by Saikkonen (1991) under some regularity conditions.

[9] We implicitly assume that the error has finite second moments and that the design matrix is of full column rank given the realization of the regressors.

(20) is the Best Linear Unbiased Estimator (BLUE) given the realization of the regressors.

Now consider the case in which e_t is a random walk. Then all the assumptions of the condional probability version of the Gauss-Markov Theorem hold except for the spherical variance assumption. In this case, the OLS applied to (3.1) is unbiased (since we are assuming strict exogeneity), but is not efficient. In this case, we can apply the Generalized Least Squares (GLS) to (20) to obtain the BLUE. Applying GLS to (3.1) basically means that we apply OLS after taking the first difference of (20);

$$\Delta y_t = \beta' \Delta x_t + \sum_{i=1}^{2} \sum_{j=1}^{k} (\gamma_{i,j} \Delta^2 x_{i,t-j}) + \Delta e_t \qquad (21)$$

Choi et al. (2005) call this estimator the GLS corrected estimator.

In more general cases in which the error is serially uncorrelated, Choi et al. (2005) note that asymptotic theory shows that (a) the DOLS estimator is asymptically efficient if e_t is I(0), (b) the GLS corrected estimator is consistent, but is not as efficient as the DOLS estimator if e_t is I(0), (c) the DOLS estimator is inconsistent if e_t is I(1), and (d) the GLS corrected estimator is consistent if e_t is I(1).

These observations naturally lead to the idea of testing for cointegration by comparing the DOLS estimates and the GLS corrected estimates for β. Let the Hasuman-type cointegration be defined by

$$H_T = T\left(\hat{\beta}_{dgls} - \hat{\beta}_{dols}\right)\hat{V}_{\beta}^{-1}\left(\hat{\beta}_{dgls} - \hat{\beta}_{dols}\right),$$

where T stands for the sample size, β_{dols} stands for DOLS estimator in level regression, β_{dgls} stands for GLS corrected estimator in differenced regression, and V_β stands for a consistent estimator for the asymptotic variance of $\sqrt{T}\left(\hat{\beta}_{dgls} - \beta\right)$.

Under the null hypothesis that error term is I(0), both estimators β_{dols} and β_{dgls} are consistent and, therefore, they should be 'close' to each other. The test statistic, H_T, has an asymptotic chi-square distribution with 2 degrees of freedom. On the other hand, under the alternative hypothesis that the error term is I(1), the level regression will be spurious and, therefore, only the differenced regression will be consistent. Therefore, the estimates from these two estimators will be very different with a large probability. The test statistic, H_T, diverge in this case.

4.4. Estimation Results

Having explained the dynamic regression and Hausman-type Cointegration test, we are now in the position to estimate the IES parameters from the log-linearized Cross-Euler equation. The estimation results of the IES parameters are reported in Table 1.

First, we have estimated the IES parameters α and γ from eq. (18) using the Dynamic OLS. In order to check for the robustness of the estimate, we report the estimation results for several lag specifications. As can be seen from the left-side panel of the Table 1, level regression estimates of α and γ have theoretically 'correct' signs. The estimates for α ranges from 1.281 to 1.424, while γ ranges from 0.382 to 0.435. We have also estimated the IES parameters α and γ from the differenced regression. In a sharp contrast, the differenced regression estimates of α and γ turned out to be conspicuously smaller than those from the level regression. The estimates for α ranged from 0.148 to 0.515, while the estimates for γ ranged from 0.013 to 0.343. Although the difference between the estimates from level regression and differenced regression are evident, we formally conduct a statistical test using Hausman-type Cointegration test. As can be seen from the test results reported in Table 1, for all the lag specifications, the Hausman-type test rejects the null hypothesis of cointegration. Taking this test result for a face value, this implies that the log-linearized Cross-Euler eq. (18) is a spurious regression and, therefore, the estimates of α and γ from the level regression are likely to be inconsistent. As such, for the purpose of recovering the structural parameters from eq. (18), it seems to be reasonable to rely on the estimates from the differenced regression.

Second, by the same token, we have estimated the IES parameters α and γ from eq. (19) and the estimation results are reported on the right-side panel of Table 1. As can be seen from the table, both level and differenced regression yield theoretically 'correct' signs for α and γ. Turning to estimation results from the level regression, the estimates for α ranges from 0.167 to 0.519, while the estimates for γ ranges from 0.163 to 0.298. Next, turning to the estimation results from the differenced regression, the estimates for α ranges from 0.215 to 0.446, while the estimates for γ ranges from 0.1 to 0.337. Giving a cursory look at the estimation results, there seems to be no discernible difference between the estimates from the level regression and differenced regression. Again, to formally back this observation, we conduct the Hausman-type Cointegration test. Conforming to our guess, the Hausman-type test did not reject the null hypothesis of cointegration for all lag specifications, supporting the cointegration relationship of eq. (19). Provided this test result, for the sake of recovering the structural parameters from eq. (19), it seems to be reasonable to rely on the level regression, which is known to be more efficient than the differenced regression. Thus, thanks to the Hausman-type Cointegration test, we now have some guidance in which type of regression to rely on. In other words, for the Cross-Euler eq. (18), it seems to be reasonable to rely on the differenced regression, while for the Cross-Euler eq. (19), it seems to be better to rely on the level regression.

Now, the big puzzle remains. If indeed the permanent income /life-cycle model described in Section 2 is correct, we should be expecting *both* log-linearized Cross-Euler equation to be cointegrated. Likewise, if some assumptions pertaining to the cointegration restriction are violated, then it is natural for us to expect *both* log-linearized Cross-Euler equation to be spurious. However, the test results from the Hausman-type Cointegration test were perplexing. That is, for the log-linearized Cross-Euler eq. (18), the test rejected the null of cointegration, implying the regression to be spurious, while for the log-linearized Cross-Euler eq. (19), the test did not reject the null of cointegration. How should we interpreting this contradicting results?

It is indeed difficult to find a clear-cut answer to the above question. Although this deep puzzle remains, however, based on the observations that 1) differenced regression estimates for α and γ from both Cross-Euler equation are relatively similar, and 2) that the level regression from the Cross-Euler eq. (19) yields estimates that are close to the differenced regression estimates[10], in this paper, we simply regard the log-linearized Cross-Euler (18) to be spurious and disregard their estimates based on the level regression.

5. Estimating IES parameters from the Euler Equations

In this section, we conduct Hansen's (1982) GMM on eq. (4) and (5). Before reporting the GMM estimation results, we discuss the choice of instrumental variables (IV).

5.1. Choice of Instruments and Lag Order

[10] This “closeness” is statistically confirmed by the Hausman-type Cointegration test which is reported on the right-panel of Table 1.

As was pointed out by Hall (1993) and Ogaki (1993), it is well known that the estimate of GMM is very sensitive to the choice of instrumental variables. To test for the robustness of the estimates vis-a-vis the choice of instruments, we estimated the parameters under several types of instruments with varying time lags. The family of instrumental variables was chosen following the convention in applied GMM literature. The following table summarizes the choice of instrumental variables.

IV Type	Euler Equation (4)	Euler Equation (5)
IV0	Const., F-lag, PF-lag	Const., NF-lag, PNF-lag
IV1	Const., F-lag, Int-lag	Const., NF-lag, Int-lag
IV2	Const., F-lag, NF-lag	Const., F-lag, NF-lag
IV3	Const., PF-lag, PNF-lag	Const., PNF-lag, PF-lag
IV4	Const., F-lag, PF-lag, Int-lag	Const., NF-lag, PNF-lag, Int-lag
IV5	Const., F-lag, NF-lag, PF-lag, PNF-lag	Const., F-lag, NF-lag, PF-lag, PNF-lag
IV6	Const., F-lag, NF-lag, PF-lag, PNF-lag, Int-lag	Const., F-lag, NF-lag, PF-lag, PNF-lag, Int-lag

Note: Following the convention in applied GMM literature, the instrumental variables have been constructed by lagging the forcing variables. Namely, constant (const.), lagged food consumption growth rate (F-lag), lagged non-food consumption growth rate (NF-lag), lagged price change in food (PF-lag) and non-food (PNF-lag), and lagged real interest rate (Int-lag).

Another issue in conducting GMM estimation is to choose the lag order of the error term when estimating the variance-covariance matrix of GMM disturbance terms. According to the rational expectation hypothesis, the forecast error will be serially uncorrelated. Since our model is based on the representative agent with rational expectation, economic theory suggests a lag order of zero. Nevertheless, taking into account the time aggregation problem which was pointed out by Grossman et al. (1987) and Heaton (1995) among others, we choose a lag order of one in estimating the variance-covariance matrix of GMM disturbance terms following Hansen and Heaton (1996). Also, to be consistent with time aggregation issues, we have lagged instrumental variables for at least two periods when conducting GMM estimations.

5.2. Estimation Results

GMM estimation was conducted using a family of conventional instruments. GMM estimation results of food Euler eq. (4) are summarized in Table 2. Similarly, GMM estimation results of non-food Euler eq. (5) are summarized in Table 3. Hansen's J-statistics for each GMM estimation are also reported.

Let us first turn to Table 2. As can be seen from the table, the estimates of α are close to zero for the most cases with occasional negative estimates. Also, for the estimates which were relatively apart from zero, there had been a tendency for those estimates to be accompanied by relatively high standard errors. Literally interpreting this estimation result, this implies that the representative agent is nearly risk-neutral with regard to the food consumption. Also, as for the intertemporal substitution of the food consumption, low estimates of α implies high IES. Even worth, the negative estimates of α implies the representative agent to increase current consumption by intertemporally substituting future consumption in response to a rise in real interest rate. In other words, the negative estimates of IES implies that the income effect from the rise of real interest rate dominate the substitution effect, which is quite unlikely to happen in practice. Finally, as for the specification check of the Euler equation, we now turn to Hansen's J-statistics. The test rejected the specification of the Euler equation for 12 out of 21 cases. Considering this frequent rejections by the Hansen's J-test, this can be considered as empirical evidence against the specification of the Euler equation for the food consumption.

Next, let us turn to Table 3. The estimates of γ are close to zero with some negative estimates. Interpreting this result, this implies that the representative agent to be risk-neutral with regard to the non-food expenditure. Also, as for the intertemporal substitution of the non-food consumption, low estimates of γ implies high IES. Again, literally interpreting, high IES for non-food expenditure means that the representative agent is willing to substitute current non-food consumption for future non-food consumption in response to a miniscule change in the real interest rate. Turning to J-test results, we found 14 rejections out of 21 cases which can be considered as evidence against the specification of the non-food consumption.

From our conventional wisdom, risk-neutral preference or extremely high value of IES (or even negative IES) is counter-intuitive and therefore it is hard to accept this GMM estimation result for face value. In addition, taking into account the strong evidence against the specification of the Euler equations for both food and non-food consumption from Hansen J-test, there seems to be little ground to believe that the GMM estimates for α and γ are consistent. However, at the same time, we should be aware of the possibility of size distortion since we have only used 90 sample periods in estimating the IES parameters. Due to the small sample size, it may well be the case that the J-test over-rejected the specification of the Euler equation. In order to verify the specification of the Euler equation further, we use likelihood ratio type test in the next section.

5.3.1. Further Specification Test

In the previous subsection, we have solely relied on Hansen J-test as for the specification check of the Euler equations. In this subsection, for the sake of additional specification check, we conduct the likelihood ratio type test proposed by Cooley and Ogaki (1996). The idea of the likelihood ratio type test is as follows.

If the model is correct under the assumption that there is no liquidity constraint or habit formation, log-linearized Cross-Euler equations will be correctly specified with cointegrating restriction. At the same time, standard Euler equations will also be correctly specified. Consequently, under the null hypothesis that the model is correctly specified, parameter estimates of α and γ from cointegration regression and GMM estimation should be statistically close. Under the test, the null hypothesis will be

$$H_0 : \alpha_{Euler} = \alpha_{Cross-Euler} \text{ and } \gamma_{GMM} = \gamma_{Cross-Euler} .$$

The rejection of the null implies that there exists at least one assumption that is violated. Unfortunately, the rejection of the null does not provide us much information about which assumption has been violated. On the other hand, the non-rejection of the null supports or, at least, leaves some possibility open for the plausibility of the joint hypothesis such that 1) the representative agent's utility function is of addi-log type, 2) the agent does not face liquidity constraints, and 3) the agent does not form habit.

5.3.2. Test Results

Here, we report the results of Cooley and Ogaki's (1996) LR-type test. In conducting the test, the same instruments from GMM estimation were used for both restricted and unrestricted GMM. We basically tested two types of null hypothesis. The first null hypothesis is $\alpha_{Euler}=\alpha_{Cross-Euler}$ and results are reported in

Table 4. The second null hypothesis is $\gamma_{Euler}=\gamma_{Cross\text{-}Euler}$ and results are reported in Table 5.

First, let us see the results under the null of $\alpha_{Euler}=\alpha_{Cross\text{-}Euler}$. Turning to Table 4, which reports the LR-type test results based on the DOLS estimate[11] from eq. (19), we found 9 rejections out of 21 cases. Considering this frequent rejection of the null hypothesis, the LR-type test does not seem to be supporting the specification of the Euler equation for food consumption. This test result is consistent with the test result from Hansen J-test.

Next, we turn to results under the null of $\gamma_{Euler}=\gamma_{Cross\text{-}Euler}$. Examining Table 5, which reports the test results based on the DOLS estimate,[12] again, from eq. (19), we found 8 rejections out of 21 cases. Again, conforming to the result from Hansen J-test, the LR-type test frequently rejected the null hypothesis of $\gamma_{Euler}=\gamma_{Cross\text{-}Euler}$, giving little support for the specification of the Euler equation for non-food consumption.

Thus, even for the alternative specification test utilizing LR-type test, we found an evidence against the specification for both food and non-food Euler equations. Based on the test results from Hansen J-test and LR-type test, there seems to be no strong ground in believing that the Euler equations are correctly specified. Since the Euler equations are unlikely to be correctly specified, consequently, the IES estimates from GMM estimation are unlikely to be consistent as well.

6. Concluding Remarks

In this paper, we adopted the Cross-Euler equation approach in estimating the IES following Nishiyama (2005). In particular, based on the two-goods (food and non-food consumption) version of the LCPIM, we exploited the cointegrating restriction implied by the Cross-Euler equation in estimating the IES of the

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[11] The estimate for $\alpha_{Cross\text{-}Euler}$ is based on the DOLS estimation of the log-linearized Cross-Euler eq. (19) with the lag order of four. The reason why we have relied solely on the Cross-Euler eq. (19) is because the regression of the log-linearized Cross-Euler eq. (18) is deemed spurious. Regarding the lag order of the DOLS estimation, we have noticed an influence possibly from the endogeneity problem. As can be seen from the Table 1 right-hand side panel, the DOLS estimate for α increases as the lag order increases from zero to four. Since the endogeneity bias can be mitigated by including higher order of lags, we have decided to pick lag order of four in this particular exercise.

[12] For the same reason, the estimate for $\gamma_{Cross\text{-}Euler}$ is based on the DOLS estimation of the log-linearized Cross-Euler eq. (19) with the lag order of four.

representative consumer.

We used Stock and Watson's (1993) dynamic OLS in estimating the cointegrating regression. From the level regression estimates, the IES for food consumption was around 0.76 (i.e., estimates of α was around 1.3) from one Cross-Euler specification and was around 3.33 (i.e., estimates of α was around 0.3) from another Cross-Euler specification. As for the IES for non-food consumption, the estimates were around 2.5 for one Cross-Euler specification and were around 5 for another specification. Thus, we encountered significantly different sets of IES estimates depending upon the specification of the Cross-Euler equations. In order to discern which set of estimates are more reliable, we conducted Hausman-type Cointegration test proposed by Choi et al. (2005). According to the test results, it turned out that one of the Cross-Euler equation specifications to be spurious and the other to be cointegrated. Simply disregarding the estimation results from a spurious regression, the sensible estimates of IES seems to be around 3.33 for food consumption and around 5 for non-food consumption. Although these IES estimates are not as large as the previous estimates of the IES, however, they still remain to be oddly high.

For the sake of comparison, we also estimated the IES for food and non-food consumption based on the standard Euler equation using GMM estimation method, which has been a popular method in the preceding studies. Conforming to the preceding studies, the IES estimates for both food and non-food consumption turned out to be conspicuously high (i.e., CRRA coefficients to be close to zero). Further, Hansen J-test and Cooley and Ogaki's LR-type test frequently rejected the specification of the Euler equations.

Taking into account the possible miss-specification of the standard Euler equation based on the test results from J-test and LR-type test, it is likely that GMM estimation for IES parameters to be inconsistent. Given this test result, instead of relying on those GMM estimation results, one may be naturally tempted to count on the IES estimation results from the cointegration regression. Considering the robustness of the log-linearized Cross-Euler equation against the existence of liquidity constraint or habit formation, there seems to be a good reason to count on the IES estimates from the cointegration regression. However, at the same time, one should bear in mind the fact that one of the specification of the Cross-Euler equation turned out to be spurious in this paper. If indeed the assumptions pertaining to LCPIM model adopted in this paper were all plausible, then we should be expecting both specifications of the Cross-Euler equation to be cointegrated. The mixed evidence from Hausman-type Cointegration test is perplexing and may be implying that some of the assumptions we made in this paper to be violated. For this reason, the IES estimates from the cointegration regression reported in this paper should be interpreted with caution.

Appendix: Robustness of the Cointegration Relationship implied by the Cross-Euler Equation

Unlike the standard Euler equation which specifies the intertemporal optimality condition in terms of the *growth rate* of the forcing variables, the log-linearized Cross-Euler equation specifies the intertemporal optimality condition in terms of *level*. By the virtue of long-run equilibrium relationship in terms of level, the cointegration regression on (18) and (19) can be shown to be robust against certain types of nuisant factors -- i.e., the cointegration regression yields a super-consistent estimates for the IES parameters. In this appendix, based on Nishiyama (2005), we show the robustness of cointegration relationship against two types of nuisant factors discussed in the consumption literature: liquidity constraints and habit formation.

A.1. Robustness against Liquidity Constraints

Among the various types of liquidity constraints, we adopt the simplest form -- i.e., a borrowing constraint that requires the net worth of the agent to be always non-negative. Specifically, following Zeldes (1989), we impose the constraint such that $W_t - P_t^F F_t - P_t^{NF} NF_t \geq 0$ to the agent's dynamic optimization problem. Then, the Bellman equation can be reformulated as follows

$$\begin{aligned} &V(W_t) = \max\left\{U(F_t, NF_t) + \mu_t\left(W_t - P_t^F F_t - P_t^{NF} NF_t\right) + \beta E_t V(W_{t+1})\right\} \\ &s.t.\ W_{t+1} = (1+r_t)\left(W_t - P_t^F F_t - P_t^{NF} NF_t\right) + Y_{t+1} \end{aligned} \tag{A. 1}$$

where Lagrangian multiplier μ_t takes a positive value when the liquidity constraint is binding and becomes zero when the constraint is not binding.

As a preliminary step in showing the robustness of a cointegration relationship, we first show that the Lagrangian multiplier is stationary. The Euler equations for food and non-food goods can be shown to be

$$\frac{U_F(F_t, NF_t)}{P_t^F} = \mu_t + \beta(1+r_t)E_t\frac{U_F(F_{t+1}, NF_{t+1})}{P_{t+1}^F} \quad \text{and}$$

$$\frac{U_{NF}(F_t, NF_t)}{P_t^{NF}} = \mu_t + \beta(1+r_t)E_t\frac{U_{NF}(F_{t+1}, NF_{t+1})}{P_{t+1}^{NF}},$$

respectively. Now, following the treatment by Zeldes (1989), we normalize the Lagrangian multiplier into two types:

$$\mu_t^F \equiv \frac{\mu_t}{\beta(1+r_t)E_t\left[U_F(F_{t+1}, NF_{t+1})/P_{t+1}^F\right]} \quad \text{and}$$

$$\mu_t^{NF} \equiv \frac{\mu_t}{\beta(1+r_t)E_t\left[U_{NF}(F_{t+1}, NF_{t+1})/P_{t+1}^{NF}\right]}.$$

As can be seen, μ_t^F stands for the Lagrange multiplier denominated by the present-valued marginal utility derived from F_{t+1}, and μ_t^{NF} stands for another normalization in terms of NF_{t+1}. Here, it should be noted that μ_t^F and μ_t^{NF} are both non-negative and inside the information set available at period t, since the denominators used for the normalization are positive and non-stochastic as of period t.

Using the new normalization of the Lagrangian multiplier and with addi-log utility function, the Euler equation for food and non-food goods can be conveniently rearranged as

$$\beta(1+r_t)\left(\frac{F_{t+1}}{F_t}\right)^{-\alpha}\frac{P_t^F}{P_{t+1}^F}(1+\mu_t^F) = 1+\varepsilon_{t+1}^F \quad \text{and} \tag{A. 2}$$

$$\beta(1+r_t)\left(\frac{NF_{t+1}}{NF_t}\right)^{-\gamma}\frac{P_t^{NF}}{P_{t+1}^{NF}}(1+\mu_t^{NF}) = 1+\varepsilon_{t+1}^{NF}. \tag{A. 3}$$

where ε_{t+1}^F and ε_{t+1}^{NF} represent the forecast errors arising from the discrepancy between the expected and realized marginal utility. By assuming that the forecast errors are stationary over time, the Lagrangian multipliers μ_t^F and μ_t^{NF} can be shown to be stationary as well.

Given the stationarity of the Lagrangian multipliers, we next demonstrate the robustness of the cointegration relationship. The specification of the Cross-Euler equation in the presence of liquidity constraint can be shown to be

$$\frac{U_F\left(F_{t+1}, NF_{t+1}\right)}{P_t^F} = \mu_t + \beta(1+r_t)E_t\frac{U_{NF}\left(F_{t+1}, NF_{t+1}\right)}{P_{t+1}^{NF}} \quad \text{and} \tag{A. 4}$$

$$\frac{U_{NF}\left(F_{t+1}, NF_{t+1}\right)}{P_t^{NF}} = \mu_t + \beta(1+r_t)E_t\frac{U_F\left(F_{t+1}, NF_{t+1}\right)}{P_{t+1}^F}. \tag{A. 5}$$

For convenience, let us take the case of eq. (A. 4) to show the robustness of the cointegration relationship against the liquidity constraint. Dividing both side of eq. (A. 4) by $\beta(1+r_t)E_t\left[U_{NF}(F_{t+1},NF_{t+1})/P_{t+1}^{NF}\right]$ and rearranging the equation yields the following relationship

$$\beta(1+r_t)\frac{P_t^F}{P_{t+1}^{NF}}\frac{U_{NF}(F_{t+1},NF_{t+1})}{U_F(F_t,NF_t)}(1+\mu_t^{NF})=1+\tilde{e}_{t+1}$$

where μ_t^{NF} stands for the new normalization of the Lagrangian multiplier defined above, and $\tilde{e}_{t+1}$ stands for the forecast error. Taking the logarithm on both sides and by the assumption of addi-log utility function, it follows that

$$\ln(1+r_t)\frac{P_t^F}{P_{t+1}^{NF}}-\gamma\ln NF_{t+1}+\alpha\ln F_t=\ln(1+\tilde{e}_{t+1})-\ln(1+\mu_t^{NF}) \qquad \text{(A. 6)}$$

Now, since the forecast error follows the I(0) stochastic process and the normalized Lagrangian multiplier follows the I(0) process as well, the RHS of eq. (A. 6) will be I(0). As a consequence of this stationarity restriction, the LHS of eq. (A. 6) will also be I(0), which is exactly the cointegration relationship shown in eq.(18). Thus, under the addi-log utility function, the cointegration relationship (18) turns out to be robust against liquidity constraints and, therefore, allows us to estimate the IES parameters super-consistently even in the presence of liquidity constraints. A similar argument holds for the cointegration relationship (19).

The intuition behind the robustness of the cointegration relationship is as follows. The Lagrangian multiplier term, which is an unobservable variable, enters the regression as an error term in addition to the forecast error. However, since the Lagrangian multiplier term is stationary over time, its presence does not affect the cointegration relationship among I(1) variables -- i.e., $\ln NF_{t+1}$, $\ln F_t$, and the opportunity cost $\ln(1+r_t)P_t^F/P_{t+1}^{NF}$. Putting it another way, the perturbation resulting from the Lagrangian multiplier term is of I(0) stochastic order which distorts the *growth rate* of optimal consumption (which is the case for the standard Euler equation), but remains relatively innocuous vis-a-vis the *level* of the optimal consumption (which is the case for the log-linearized Cross-Euler equation).

Of course, in the small sample, there is an endogeneity problem in estimating the cointegrating relationship, since the Lagrangian multiplier μ_t^{NF} and future consumption NF_{t+1} are obviously correlated. However, this endogeneity problem could be handled by an estimation method such as Phillips and Hansen's (1991) FM-OLS or Park's (1992) CCR. Further, since the estimators in cointegrating regression will be super-consistent, i.e. $O_P(T^{-1})$ consistent, the endogeneity problem will not matter asymptotically. Thus, despite the endogeneity problem stemming from the presence of liquidity constraints, we can still obtain consistent estimates for α and γ from the cointegration relationship (18) and (19).

A.2. Robustness against Habit Formation

Next, we show the robustness of the cointegration relationship (18) and (19) in the presence of habit formation in the agent's preference. Among the various types of habit formation, we follow the specification used by Amano and Wirjanto (1996). Assuming that $\ln F_t$, $\ln NF_t$, and $\ln(P_t^{NF}/P_t^F)$ follows the I(1) process and that the agent has habit forming preference as follows

$$E_t\left[\sum_{i=0}^{\infty} U\left(F_{t+i}^*, NF_{t+i}^*\right)\right]$$

where $U(F_{t+i}^*, NF_{t+i}^*)$ takes the addi-log type utility function and habit formation variable defined as $F_t^* \equiv \sum_{j=0}^{\infty} \delta_j F_{t-j}$ and $NF_t^* \equiv \sum_{j=0}^{\infty} \delta_j NF_{t-j}$, Amano and Wirjanto (1996) shows the stochastic relationship among the intratemporal relative price, current food and non-food goods as follows,

$$\frac{1}{K}\frac{P_t^{NF}}{P_t^F}\frac{F_t^{-\alpha}}{NF_t^{-\gamma}} \sim I(0)$$

Taking the logarithm on both sides of the above stochastic relationship, the intratemporal relative price, food and non-food goods reveal the following cointegration relationship,

$$const. + (\ln P_t^{NF} - \ln P_t^F) - \alpha \ln F_t + \gamma \ln NF_t \sim I(0)$$

Now, by adding $\gamma \ln NF_{t+1}$ and $\ln P_{t+1}^{NF}$ on both sides and subtracting $\gamma \ln NF_t$, $\ln P_t^{NF}$, and $\ln(1+r_t)$ from both sides, we obtain the following relationship

$$const. - \ln(1+r_t) - (\ln P_t^F - \ln P_{t+1}^{NF}) - \alpha \ln F_t + \gamma \ln NF_{t+1}$$
$$\sim \gamma(\ln NF_{t+1} - \ln NF_t) + (\ln P_{t+1}^{NF} - \ln P_t^{NF}) - \ln(1+r_t) + I(0)$$

By assumption, the growth rate of NF_t , P_t^{NF} and the real interest rate is stationary and, therefore, the RHS of the above relationship is I(0). By the stationarity restriction on the RHS, consequently, the LHS will follow the I(0) process which is exactly the cointegration relationship we derived in eq. (18). Thus, cointegration relationship (18) turns out to be robust even in the presence of habit formation in the agent's preference. A similar argument holds for cointegration relationship (19).

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Figure 1: Log Food and Non-Food Expenditure

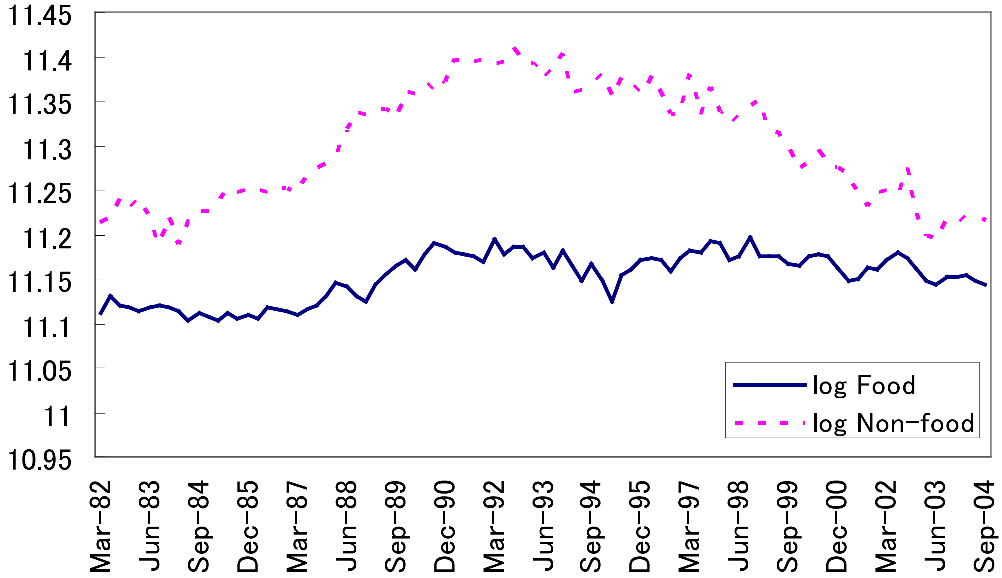

Figure 2: Real interest rate, Food and Non-Food Price Index

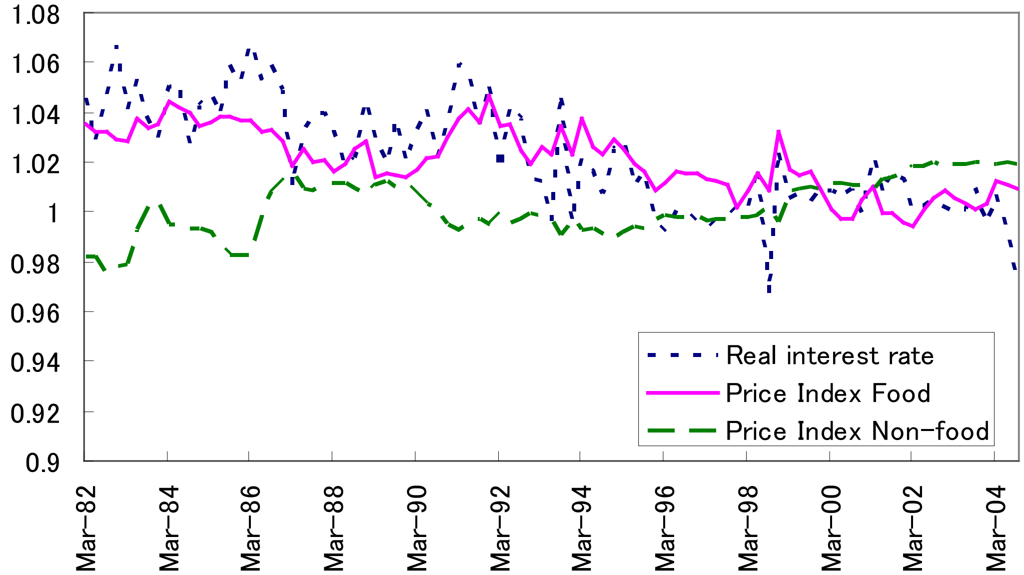

Figure 3: Opportunity Costs

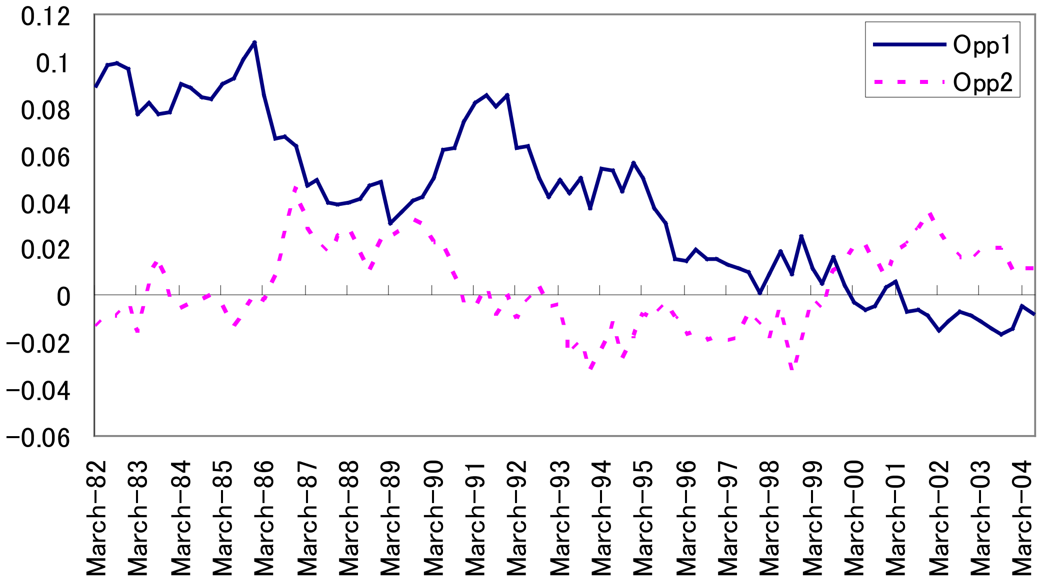

Table 0

	Null of Unit Root				Null of Stationary	
	ADF test		PP test		KPSS test	
	cst.	cst.&trend	cst.	cst.&trend	cst.	cst.&trend
$\ln F_t$	-2.257	-2.310	-2.257	-2.032	0.702*	0.242**
$\ln NF_t$	-1.383	-0.783	-1.201	-0.872	0.315	0.301**
$\ln\left[(1+r_t)\frac{P_t^F}{P_{t+1}^{NF}}\right]$	-1.164	-2.819	-1.149	-3.038	1.086**	0.077
$\ln\left[(1+r_t)\frac{P_t^{NF}}{P_{t+1}^{F}}\right]$	-2.703	-2.682	-2.612	-2.589	0.160	0.167*

Note:
Lag order used for ADF test was chosen based on Schwartz Information Criteria.
* denotes the rejection of null hypothesis at 5% level.
** denotes the rejection of null hypothesis at 1% level.

Table 1

	Cross-Euler Equation Ver.1					Cross-Euler Equation Ver.2				
	DOLS (level)		GLS-corrected (difference)		Hausman-Test	DOLS (level)		GLS-corrected (difference)		Hausman-Test
	α	γ	α	γ		α	γ	α	γ	
Lag 0	1.314	0.382	0.333	0.013	189.82**	0.167	0.163	0.329	0.1	3.13
	(0.184)	(0.03)	(0.005)	(0.001)		(0.068)	(0.011)	(0.008)	(0.003)	
Lag1	1.381	0.414	0.313	0.248	26.54**	0.23	0.191	0.297	0.288	0.22
	(0.252)	(0.038)	(0.042)	(0.006)		(0.069)	(0.010)	(0.019)	(0.009)	
Lag2	1.424	0.435	0.515	0.343	11.37**	0.351	0.239	0.386	0.289	0.02
	(0.349)	(0.049)	(0.072)	(0.016)		(0.064)	(0.009)	(0.054)	(0.021)	
Lag3	1.384	0.427	0.343	0.275	13.44**	0.48	0.289	0.215	0.29	0.69
	(0.483)	(0.062)	(0.075)	(0.011)		(0.083)	(0.011)	(0.101)	(0.027)	
Lag4	1.281	0.413	0.148	0.168	9.46**	0.519	0.298	0.446	0.337	0.04
	(0.556)	(0.060)	(0.135)	(0.018)		(0.128)	(0.015)	(0.112)	(0.025)	

Note:
Number in brackets represents the estimated standard error.
* denotes the rejection of null hypothesis at 5% level.
** denotes the rejection of null hypothesis at 1% level.

Table 2

GMM Result:
Food Euler eq.(2.4)

$$E_t\left[\beta\left(\frac{F_{t+1}}{F_t}\right)^{-\alpha}(1+r_{t+1})\frac{P_t^F}{P_{t+1}^F}-1\right]=0$$

IV Type	Lag	β	α	J statistics	D.F.
IV0	(-2)	0.978	-0.538	1.501	1
		(0.003)	(0.175)	[0.220]	
	(-3)	0.979	0.665	2.262	1
		(0.003)	(0.210)	[0.132]	
	(-4)	0.979	1.847	0.831	1
		(0.003)	(0.543)	[0.361]	
IV1	(-2)	0.979	-0.044	20.219**	1
		(0.003)	(0.191)	[0.000]	
	(-3)	0.981	0.681	12.763**	1
		(0.003)	(0.226)	[0.000]	
	(-4)	0.979	2.342	5.319*	1
		(0.003)	(0.493)	[0.021]	
IV2	(-2)	0.978	-0.310	0.246	1
		(0.003)	(0.288)	[0.619]	
	(-3)	0.979	0.536	1.068	1
		(0.003)	(0.257)	[0.301]	
	(-4)	0.978	0.084	2.702	1
		(0.003)	(0.189)	[0.100]	
IV3	(-2)	0.978	-0.972	5.638*	1
		(0.004)	(0.742)	[0.017]	
	(-3)	0.970	-2.769	4.131*	1
		(0.008)	(1.114)	[0.042]	
	(-4)	0.972	3.857	0.680	1
		(0.007)	(1.078)	[0.409]	
IV4	(-2)	0.979	-0.141	20.809**	2
		(0.003)	(0.146)	[0.000]	
	(-3)	0.981	0.667	12.711**	2
		(0.003)	(0.210)	[0.001]	
	(-4)	0.979	2.304	5.603	2
		(0.003)	(0.489)	[0.060]	
IV5	(-2)	0.980	-0.375	6.170	3
		(0.003)	(0.272)	[0.103]	
	(-3)	0.982	0.413	11.986**	3
		(0.003)	(0.222)	[0.007]	
	(-4)	0.978	0.124	11.899**	3
		(0.003)	(0.172)	[0.007]	
IV6	(-2)	0.981	0.314	18.068**	4
		(0.003)	(0.188)	[0.001]	
	(-3)	0.983	0.473	15.820**	4
		(0.003)	(0.218)	[0.003]	
	(-4)	0.982	0.127	18.447**	4
		(0.002)	(0.168)	[0.001]	

Note:
Number in brackets represents the estimated standard error.
Number in square brackets represents the p-value.
* denotes the rejection of null hypothesis at 5% level.
** denotes the rejection of null hypothesis at 1% level.

Table 3

GMM Result:
Non-food Euler eq. (2.5)

$$E_t\left[\beta\left(\frac{NF_{t+1}}{NF_t}\right)^{-\gamma}(1+r_{t+1})\frac{P_t^{NF}}{P_{t+1}^{NF}}-1\right]=0$$

IV Type	Lag	β	γ	J statistics	D.F.
IV0	(-2)	0.952	2.994	4.482*	1
		(0.010)	(0.718)	[0.034]	
	(-3)	0.981	0.433	4.523*	1
		(0.002)	(0.208)	[0.033]	
	(-4)	0.979	-0.171	6.588*	1
		(0.002)	(0.199)	[0.010]	
IV1	(-2)	0.973	1.069	6.128*	1
		(0.003)	(0.225)	[0.013]	
	(-3)	0.978	0.762	5.994*	1
		(0.002)	(0.148)	[0.014]	
	(-4)	0.979	0.413	23.447**	1
		(0.002)	(0.144)	[0.000]	
IV2	(-2)	0.971	-1.918	0.247	1
		(0.011)	(1.284)	[0.619]	
	(-3)	0.979	0.386	0.261	1
		(0.002)	(0.273)	[0.609]	
	(-4)	0.979	-0.562	0.255	1
		(0.003)	(0.383)	[0.613]	
IV3	(-2)	0.972	1.533	0.000	1
		(0.003)	(0.363)	[0.981]	
	(-3)	0.982	0.132	6.664**	1
		(0.002)	(0.435)	[0.009]	
	(-4)	0.977	0.362	8.092**	1
		(0.002)	(0.080)	[0.004]	
IV4	(-2)	0.972	1.075	9.225**	2
		(0.003)	(0.220)	[0.009]	
	(-3)	0.979	0.696	7.679*	2
		(0.002)	(0.140)	[0.021]	
	(-4)	0.979	0.399	23.835**	2
		(0.002)	(0.143)	[0.000]	
IV5	(-2)	0.972	1.331	2.532	3
		(0.003)	(0.310)	[0.469]	
	(-3)	0.982	0.371	8.363*	3
		(0.002)	(0.227)	[0.039]	
	(-4)	0.979	0.345	11.295*	3
		(0.002)	(0.062)	[0.010]	
IV6	(-2)	0.974	1.039	4.000	4
		(0.003)	(0.200)	[0.405]	
	(-3)	0.980	0.619	11.777*	4
		(0.002)	(0.155)	[0.019]	
	(-4)	0.980	0.343	16.715	4
		(0.002)	(0.062)	[0.149]	

Note:
Number in brackets represents the estimated standard error.
Number in square brackets represents the p-value.
* denotes the rejection of null hypothesis at 5% level.
** denotes the rejection of null hypothesis at 1% level.

Table 4

LR-type Test Result:

$$H_0 : \alpha_{Euler} = \alpha_{Cross-Euler}$$

IV Type	Lag	Restricted	Unrestricted	QLR
IV0	(-2)	15.034	1.501	13.533**
	(-3)	13.244	2.262	10.982**
	(-4)	17.380	0.831	16.549**
IV1	(-2)	15.034	20.219	-5.185
	(-3)	13.244	12.763	0.481
	(-4)	17.380	5.319	12.061**
IV2	(-2)	5.118	0.246	4.872*
	(-3)	0.549	1.068	-0.519
	(-4)	5.618	2.702	2.916
IV3	(-2)	7.843	5.638	2.205
	(-3)	9.090	4.131	4.959*
	(-4)	5.950	0.680	5.27*
IV4	(-2)	15.044	20.809	-5.765
	(-3)	13.654	12.711	0.943
	(-4)	18.437	5.603	12.834**
IV5	(-2)	12.209	6.170	6.039*
	(-3)	9.244	11.986	-2.742
	(-4)	12.365	11.899	0.466
IV6	(-2)	15.667	18.068	-2.401
	(-3)	13.883	15.820	-1.937
	(-4)	18.456	18.447	0.009

Note:

* denotes the rejection of null hypothesis at 5% level.

** denotes the rejection of null hypothesis at 1% level.

Table 5

LR-type Test Result:

$$H_0 : \gamma_{Euler} = \gamma_{Cross-Euler}$$

IV Type	Lag	Restricted	Unrestricted	QLR
IV0	(-2)	2.513	4.482	-1.969
	(-3)	4.056	4.523	-0.467
	(-4)	5.734	6.588	-0.854
IV1	(-2)	15.446	6.128	9.318**
	(-3)	15.087	5.994	9.093**
	(-4)	15.489	23.447	-7.958
IV2	(-2)	1.172	0.247	0.925
	(-3)	0.278	0.261	0.017
	(-4)	3.773	0.255	3.518
IV3	(-2)	14.076	0.000	14.076**
	(-3)	6.115	6.664	-0.549
	(-4)	7.278	8.092	-0.814
IV4	(-2)	16.817	9.225	7.592**
	(-3)	15.239	7.679	7.56**
	(-4)	16.398	23.835	-7.437
IV5	(-2)	15.649	2.532	13.117**
	(-3)	7.548	8.363	-0.815
	(-4)	10.953	11.295	-0.342
IV6	(-2)	18.321	4.000	14.321**
	(-3)	16.454	11.777	4.677*
	(-4)	17.585	16.715	0.870

Note:

* denotes the rejection of null hypothesis at 5% level.

** denotes the rejection of null hypothesis at 1% level.