TERG

Discussion Paper No. 273

The Cross-Euler Equation Approach to testing for the Liquidity Constraint: Evidence from Macro and Micro Data

Shin-Ichi Nishiyama

September 2011

TOHOKU ECONOMICS RESEARCH GROUP

GRADUATE SCHOOL OF ECONOMICS AND
MANAGEMENT TOHOKU UNIVERSITY
KAWAUCHI, AOBA-KU, SENDAI,
980-8576 JAPAN

The Cross-Euler Equation Approach to testing for the Liquidity Constraint: Evidence from Macro and Micro Data[1]

Shin-Ichi Nishiyama[2]
Graduate School of Economics and Management,
Tohoku University

This Version: September 2011
First Version: September 2002

[1] My deepest gratitude goes to Masao Ogaki for his advice and encouragement in pursuing this project. I would also like to thank Steve Cecchetti, C.Y. Choi, Paul Evans, Ryo Kato, Nelson Mark, Kiyoshi Matsubara, Hajime Miyazaki, Donggyu Sul, Kiyoshi Taniguchi, and Takayuki Tsuruga for their valuable comments. All remaining errors are my own.

[2] 27-1 Kawauchi, Aoba-ku, Sendai-shi, Miyagi-ken, 980-8576, JAPAN, Tel.: +81-22-795-3471, Email: `nishiyama@econ.tohoku.ac.jp`

Abstract

In this paper, we propose a new empirical method in testing for the existence of liquidity constraint utilizing the concept of Cross-Euler equation. The Cross-Euler equation represents the optimal consumption pattern of a good in the current period to another good at a future period. It can be interpreted as the composite optimal condition that embeds both intertemporal and intratemporal optimal consumption relationships into one equation. The Cross-Euler equation has an advantage over the standard Euler equation, in the sense that the cointegrating relationship is maintained even when the liquidity constraint is present in the agent's decision problem. Thus, by comparing the preference parameter estimates from the Cross-Euler equation to those from the standard Euler equation, it is possible to detect the existence of a liquidity constraint. We adopt standard two goods version of Life-Cycle model to study the consumption behavior of necessity goods and luxury goods. First, based on the aggregate data, the test rejects the null of no liquidity constraints for necessity goods, while accpeting the null for luxury goods. Since, by construction, large share of necessity goods are consumed by poor households, it is possible to interpret the results as evidence that poorer households are likely to be liquidity constrainted. Next, we construct synthetic panel data from the Consumer Expenditure Survey where households have been classified to cohorts by their ages and educational attainments. Again, taking the Cross-Euler equation approach in testing for the liquidity constraints, the test rejected the null of no liquidity constraint for low-education cohorts, while accepting the null for high-education cohorts. Taking an education level as a proxy for permanent income, the test results were consistent with the view that poorer agents are more likely to be liquidity constrained.

1 Introduction

Whether there are liquidity constraints in the economy or not is an important question. It is important because fiscal and monetary policy implications from Life-Cycle model will be considerably altered in the presence of liquidity constraints. As pointed out by Altonji and Siow (1987) and Shea (1995), consumption growth may respond asymmetrically to change in transitory income when liquidity constraints are binding. This, in turn, implies the asymmetric fiscal policy effect. Also, when liquidity constraints are binding, change in real interest rate will not cause any intertemporal substitution in consumption that expansionary monetary policy may not have any effect in inducing consumers to spend more. Ironically, since consumers' net wealth are often seriously damaged during the economic downturn - making liquidity constraints even more compelling -, monetary policy may be ineffective during the recession when the policy stimulus is desperately needed. In the presence of liquidity constraints, monetary policy will face a serious problem during an economic downturn. In this connection, the asymmetric monetary policy effect reported by Choi (1999) and Weise (2000) may well be stemming from the presence of liquidity constraints in the economy. Thus, whether there exist liquidity constraints or not is a serious question for policy makers.

Reflecting the importance of liquidity constraints, not surprisingly, considerable amount of research have been devoted in testing for the liquidity constraints. Based on aggregate data, Flavin (1981) and Campbell and Mankiw (1989,1999), among others, have conducted an excess sensitivity test and found that consumption growth rate to be significantly correlated with lagged or predicted income growth, which can be interpreted as evidence of liquidity constraint. Turning to the excess sensitivity test based on panel data, Hall and Mishkin (1982), Shapiro (1983) and Hayashi (1985) all found some evidence that lagged income change or real disposable income change to be significantly correlated with consumption growth. Mariger (1987), Altonji and Siow (1987) and Zeldes (1989) specifically take into account for the Kuhn-Tucker condition emerging from the liquidity constraint. Mariger (1987), based on cross-sectional surveys, reports that 19.4% of the households are liquidity constrained. Zeldes (1989), based on Panel Study of Income Dynamics (PSID), have found that less wealthy households to be more vulnerable to liquidity constraints. Runkle (1991), based on PSID, tests the over-identifying restrictions implied by Euler equation and reports little evidence of liquidity constraints. Attanasio and Webber (1993) and Meghir and Webber (1996) studied the validity of Life-Cycle model using the Consumer Expenditure Survey (CES) and report some evidence that younger households are liquidity constrained. DeJuan and Seater (1999), also based on CES, exploits asymmetric response of consumption growth to income change and reports little evidence of liquidity constraints. Recently, Gross and Souleles (2001), using unique panel data on credit card accounts, estimates the marginal propensity to consume out of liquidity and found their estimates to be higher for people whose credit limit is low, which they interpret as evidence of liquidity constraints.

In this paper, we propose a new empirical method in testing for the existence of liquidity constraint utilizing the concept of Cross-Euler equation. The Cross-Euler equation represents the optimal consumption pattern of a good in the current period to another good at a future period. It can be interpreted as the composite optimal condition that embeds both intertemporal and intratemporal optimal consumption relationships into one equation. Under addi-log type period-by-period utility function, we show that the Cross-Euler equation has an advantage over the standard Euler equation, in the sense that the cointegrating relationship is maintained even when the liquidity constraint is present in the agent's decision problem. Thus, by comparing the preference parameter estimates from the Cross-Euler equation to those from the standard Euler equation, it is possible to detect the existence of a liquidity constraint.

As long as the Life-cycle model is constructed in the context of multiple goods[1], the Cross-Euler equation approach is potentially feasible. In this paper, in order to explore the potential of the Cross-Euler equation approach in testing the liquidity constraints we adopt two goods version of Life-cycle model. Specifically, we study the consumption behaviors of necessity goods and luxury goods of the agents. Choice of necessity and luxury goods are, to some extent, arbitrary. However, this classification has its own motivation, especially in the context of aggregate data. In aggregate data, by construction, relatively larger share of luxury goods are consumed by "rich" agents, while relatively larger share of necessity goods are consumed by "poor" agents in the economy. Thus, by studying the behavior of standard Euler equations for both goods and also studying the behavior of the Cross-Euler equation linking both goods, there is a good possibility that we can infer which type of agents are more vulnerable to liquidity constraints even from the aggregate data. Naturally, since the poorer agents tend to be more vulnerable to the liquidity constraint, we expect that the Euler equation for the necessity goods to be misspecified, but the Euler equation for the luxury goods to be specified. Empirical evidence in this paper, though yielding mixed results, seemed to have supported this prediction.

However, the empirical results based on the aggregate time-series data implicitly assumes the existence of the representative agent. The existence of the representative agent requires that all the agents in the economy to share the identical and homothetic preferences, which is obviously a too strong assumption (See Kirman (1992) and Stoker (1993)). In an attempt to overcome this aggregation problem and to legitimately test for the liquidity constraints, we also conduct cohort analysis proposed by Deaton (1985) as a complement to the empirical study from aggregate data. Under the cohort analysis, the households are aggregated into cohorts who share the similar taste and, therefore, it has an advantage over the representative model in the sense that it is relatively free from the aggregation problem. Based on the Consumer Expenditure Survey from 1984 to 1998, we classify the households into cohorts by their age and educational attainment following Attanasio and Browning (1997). Classification by the age are necessary because the households are expected to have hetero-

[1] One good version of Life-cycle model implicitly assumes that Hicks' Composite Condition to hold for all consumption goods (See, for instance, Chapter 2 of Deaton (1996) for details). This assumption is obviously too strong and is unlikely to be met in reality. In this sense, multiple good version of Life-cycle model has natural advantage over one good version.

geneous consumption patterns over the life-cycle. Classification by education attainments was adopted as a mean to classify the households by their life-time income (i.e. permanent income). Then we estimate the preference parameters for each education cohorts based on their Cross-Euler and standard Euler equations. As a mean to formally compare the parameters estimates from both Euler equations, we conduct Cooley and Ogaki's LR type test (1996). Empirical evidence based on cohort analysis suggested that low-education cohort to be liquidity constrained, but not for the high-education cohort. Taking the education level as a proxy for lifetime income, the test results seems to support the view that poorer agents are more likely to be liquidity constrained.

Another novelty of this paper is that we have adopted cointegration analysis in the setting of pseudo panel data. In part due to its relative youth, cohort technique developed by Deaton (1985) have traditionally found an application in $I(0)$ data setting, but not in $I(1)$ data setting. However, as long as number of observations within the cohort is sufficiently large, there is no problem in appealing to the asymptotic theory already established in the panel cointegration literatures. By exploiting the panel cointegration relationship in pseudo panel data, researchers can enjoy the reward of super-consistency (Engel and Granger (1987), Phillips and Moon (2000)) in estimating the preference parameters. More pragmatically, considering that time dimension of Consumer Expenditure Survey is quite limited, the notion of super-consistency seems to be extremely valuable. In this paper, in particular, we will estimate the preference parameters from (pseudo) panel cointegrating relationship using Mark and Sul's (2001) Panel Dynamic OLS estimation method.

The rest of paper is organized as follows. Section 2 describes the standard two goods version of the life-cycle model to study the consumption behavior of necessity goods and luxury goods. Section 3 describes the Cross-Euler equation approach in testing the liquidity constraints. In Section 4, we apply the Cross-Euler equation approach in testing the liquidity constraints using the aggregate data. In Section 5, we construct the cohort data set from Consumer Expenditure Survey and apply the Cross-Euler equation approach in testing for the liquidity constraints. Section 6 provides the concluding remark.

2 Model Description

This paper adopts the standard two-goods version of Life Cycle/ Permanent Income Model (LCPIM) as in Ogaki (1992). Representative agent is assumed to maximize his expected lifetime utility under his lifetime budget constraint. Stating mathematically,

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta^t U(N_t, L_t) \tag{1}$$

$$s.t. \quad A_t = (1+r_t)A_{t-1} + Y_t - P_t^N N_t - P_t^L L_t \quad \text{for } \forall t \geq 0 \tag{2}$$

where N_t stands for necessity goods at period t, L_t stands for luxury goods, A_t stands for the asset holding of the agent, Y_t stands for the labor income of the agent, r_t stands for the real interest rate from period $t-1$ to t, P_t^N stands for the price of a necessity good, and P_t^L stands for the price of an luxury goods. Finally, we parameterize agent's subjective discount rate as constant β.

We have assumed that period-by-period utility is time separable for this agent and have implicitly assumed the additive separability between durable goods and non-durable goods. Solving above optimization problem yields the following first order conditions (FOC).

$$\frac{P_t^N}{P_t^L} = \frac{U_{N_t}}{U_{L_t}} \quad \text{for } \forall t \geq 0 \tag{3}$$

$$E_0\left[\beta\frac{U_{N_{t+1}}}{U_{N_t}}(1+r_{t+1})\frac{P_t^N}{P_{t+1}^N} - 1\right] = 0 \quad \text{for } \forall t \geq 0 \tag{4}$$

$$E_0\left[\beta\frac{U_{L_{t+1}}}{U_{L_t}}(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L} - 1\right] = 0 \quad \text{for } \forall t \geq 0 \tag{5}$$

Eq. (3) represents the contemporaneous FOC for this representative agent. These FOC's follow if the agent is maximizing his utility given the contemporaneous price ratio of necessity and luxury goods. In other words, representative agent will equalize his contemporaneous marginal rate of substitution (MRS) to current price ratio of two goods. Eq. (4) represents the intertemporal FOC, the Euler equation, of necessity goods. This FOC will follow, if the agent is maximizing his expected utility over time given the discounted expected price ratio of necessity goods at period t to period $t+1$. The Euler equation for luxury goods (eq. (5)) holds by the parallel logic.

Next, in order to make the model econometrically estimable, we are going to parametrize the utility function. We specify the utility function as a standard addi-log function following Houthakker(1960).

$$U(N_t, L_t) = \frac{(N_t)^{1-\alpha}}{1-\alpha} + K\frac{(L_t)^{1-\gamma}}{1-\gamma} \tag{6}$$

This addi-log specification was used in Ogaki (1992). Houthakker's addi-log specification reveals the non-homothetic preference of the agent in general, but contains the homothetic preference as a special case when $\alpha = \gamma$. This non-homothetic preference is crucial in our model since we try to capture intertemporal aspects of necessity goods (which by definition requires the income elasticity to be smaller than 1) and luxury goods (which requires the income elasticity to be greater than 1). Homothetic preference, such as CES utility function, reveals the unit income elasticity that it is not an appropriate utility specification to adopt when modeling the behavior of necessity and luxury goods consumption at the same time. It should be noted that under this addi-log specification, $1/\alpha$ and $1/\gamma$ can be interpreted as the intertemporal elasticity of substitution (IES) of N_t and L_t respectively[2].

Under this specification, FOC will then be as follows.

$$\frac{P_t^N}{P_t^L} = \frac{1}{K}\frac{(N_t)^{-\alpha}}{(L_t)^{-\gamma}} \quad \text{for } \forall t \geq 0 \tag{7}$$

[2] This will not be the case if a utility function is time non-separable (e.g. allowing habit formation) or goods non-separable (e.g. CES type function). This was pointed out by Constantinides (1991). The general formula for deriving IES under a time non-separable utility function was shown by McLaughlin (1995).

$$E_0\left[\beta\left(\frac{N_{t+1}}{N_t}\right)^{-\alpha}(1+r_{t+1})\frac{P_t^N}{P_{t+1}^N}-1\right]=0 \quad \text{for } \forall t\geq 0 \tag{8}$$

$$E_0\left[\beta\left(\frac{L_{t+1}}{L_t}\right)^{-\gamma}(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L}-1\right]=0 \quad \text{for } \forall t\geq 0 \tag{9}$$

Given these specifications, we are now ready to actually estimate and test the implication of the model.

Some remarks should follow for these FOC's. As was pointed out by Amano and Wirjanto (1996) and Ogaki and Park (1998), the specification of the intratemporal relationship eq.(7) turns out to be robust to several kinds of nuisant conditions, such as liquidity constraint and habit formation in utility function. However, the specification of Euler equations are very sensitive to the presence of liquidity constraint or habit formation. In other words, specification of the intratemporal relationship is robust, but the specification of Euler equations are not. Conversely, if for any method we can find the evidence that Euler equation is specified, that will be a strong evidence against the presence of liquidity constraint or habit formation. This specification issue of Euler equations will be the central focus of the rest of our paper.

3 The Cross-Euler Equation Approach

If our model is correct, then the intratemporal relationship eq.(7) and Euler equations eq.(8) and eq.(9) will be specified. Therefore, the parameter estimates α and γ from eq.(7) and Euler equation (8) and (9) should be reasonably close. Thus, the main goal of this paper is to test whether these estimates from different equations are statistically close enough or not. If the statistical test concludes that parameter estimates are significantly different from each other, then, by the contrapositive logic, we should conclude that some of the assumptions we had made (i.e., addi-log type utility function, additive separability of durable and non-durable goods, non-existence of liquidity constraint, non-existence of habit formation etc.) are illegitimate. Unfortunately, statistical test will not be informative as to say exactly which assumption is wrong. Conversely, if the statistical test does not reject the null hypothesis that parameter estimates are equal, it will support or, at least, leaves some possibility open for the joint assumption of addi-log utility specification without the presence of habit formation or liquidity constraint.

Thus, the empirical task is to first obtain the parameter estimates from contemporaneous relationship (i.e. eq.(7)) and from Euler equations (i.e. eq.(8) and eq.(9)). Predecessors in this line of research have used cointegration analysis and/or GMM in estimating the parameters. However, when the utility function is a standard one, there will be an empirical complication in estimating parameters. The following section will address this complication and propose a remedy for this problem.

3.1 An Empirical Dilemma

Let us turn back to the three FOC's (i.e. eq. (7), (8), and (9)) implied by the model. Since the conditional moment condition is established for eq. (8) and (9), there is no problem in applying GMM on these equations. If indeed these Euler equations are well specified, then the GMM will yield $O_P(T^{-1/2})$ consistent estimate of α and γ. However, unfortunately, the complication will arise from contemporaneous relationship eq.(7).

A natural way to estimate the parameters from intratemporal relationship is to log-linearize and rearrange the eq. (7) as follow.

$$\ln L_t + const. - \frac{1}{\gamma} \ln \frac{P_t^N}{P_t^L} - \frac{a}{\gamma} \ln N_t = 0$$

If the forcing variables $\ln L_t$, $\ln N_t$, and $\ln(P_t^N / P_t^L)$ follows the I(1) process, one is tempted to introduce some I(0) disturbance terms on RHS of the above equation in order to conduct the cointegration analysis.

$$\ln L_t + const. - \frac{1}{\gamma} \ln \frac{P_t^N}{P_t^L} - \frac{a}{\gamma} \ln N_t = \varepsilon_t \text{ where } \varepsilon_t \sim I(0) \text{ and } E(\varepsilon_t) = 0 \tag{10}$$

This error term can be an optimization error, measurement error, preference shock[3], etc. However, in so doing, one should also include this newly introduced error term into the existing Euler equations (8) and (9). In other words, in addition to the forecast error embedded in Euler equations, one is now introducing another kind of error term which is intrinsically different type of an error. It can be shown that when a new error term is introduced to Euler equations, no longer eq.(8) and eq.(9) are specified (See Appendix 1).

This is the point where one experience the dilemma. The objective is to estimate the parameters from both intratemporal relationship and Euler equations. If one introduces some arbitrary error term to the contemporaneous relationship in order to conduct the cointegration analysis, this newly introduced error term will affect the specification of Euler equation. Conducting GMM on standard Euler equations (8) and (9) will no longer yield a consistent estimates for α and γ. On the other hand, if one does not introduce any error term to contemporaneous relationship, Euler equation (8) and (9) will remain to be specified and GMM on these equations will yield consistent estimates assuming that the model is correct. But then, since the error term is not present for the contemporaneous relationship, one faces a illegitimacy in conducting the cointegration analysis.

3.2 The Cross-Euler Equation Approach

This section propose a remedy to the above empirical dilemma. The idea is to first define the concept called cross intertemporal marginal rate of substitution (CIMRS) and then to

[3] Clarida (1994) and Amano and Wirjanto (1996) adopts preference shock in their model, making cointegration analysis possible under mild conditions. However, it should be noted that if one adopts preference shock in their model, then as a trade-off, GMM estimation of Euler equation will not be implementable unless they have a data available for preference shock.

derive the corresponding first order condition which we will call the Cross-Euler equation. In this subsection, we will show how the concept of Cross-Euler equation can be a remedy for the above empirical dilemma.

Step 1: Defining CIMRS and deriving the Cross-Euler equations

Definition 1 (CIMRS) *Let $V(x_1^1, ..., x_1^K, ..., x_T^1, ..., x_T^K)$ be a utility function defined upon K goods with T periods. Then we call the following expression as*

$$-\frac{\partial V(\bullet)/\partial x_{t+1}^i}{\partial V(\bullet)/\partial x_t^j}$$

the cross intertemporal marginal rate of substitution (CIMRS) between goods x_{t+1}^i and x_t^j where $i \neq j$ and $t = 1, ..., T-1$.

The concept of CIMRS is just a simple extension of IMRS. It can be easily conceptualized as the IMRS defined upon different goods instead of same goods.[4] From the concept of CIMRS and from our model described, we can derive the "alternative" FOC. For convenience we will call the following FOC as the Cross-Euler equation.

$$E_0\left[\beta K\frac{(L_{t+1})^{-\gamma}}{(N_t)^{-a}}(1+r_{t+1})\frac{P_t^N}{P_{t+1}^L} - 1\right] = 0 \tag{11}$$

We can see the intuition of the above equation by thinking of the situation where the agent is trading N_t to L_{t+1}. Now the marginal rate of substitution between L_{t+1} and N_t (or CIMRS in our terminology) is defined as $-\beta U_{L_t,t+1}/U_{N_t,t}$ and takes the form of $-\beta K\,(L_{t+1})^{-\gamma}\,/\,(N_t)^{-a}$ under addi-log utility function. Next, let us consider the opportunity cost of obtaining L_{t+1} in terms of N_t. By selling one unit of N_t at period t, agent can obtain P_t^N of numeraire goods. By saving all of these numeraire goods at period t, agent can obtain $(1+r_{t+1})\cdot P_t^N$ of numeraire goods at period $t+1$. By using all of these to buy L_{t+1}, agent can buy $(1+r_{t+1})\cdot P_t^N/P_{t+1}^L$ units of L_{t+1}. Thus the opportunity cost of L_{t+1} in terms of N_t is $(1+r_{t+1})\cdot P_t^N/P_{t+1}^L$. If the agent is optimally trading N_t to L_{t+1}, then the agent is equalizing the opportunity cost to CIMRS between L_{t+1} and N_t, yielding the above Cross-Euler equation[5].

By similar fashion, we can derive the another version of Cross-Euler equation as follow.

$$E_0\left[\frac{\beta}{K}\frac{(N_{t+1})^{-a}}{(L_t)^{-\gamma}}(1+r_{t+1})\frac{P_t^L}{P_{t+1}^N} - 1\right] = 0 \tag{12}$$

In deriving the above Cross-Euler equation, we have equalized the CIMRS between N_{t+1} and L_t (i.e. $-(\beta/K)((N_{t+1})^{-a}\,/\,(L_t)^{-\gamma})$) to the opportunity cost of N_{t+1} in terms of L_t (i.e. $(1+r_{t+1})P_t^L/P_{t+1}^N$).

Step 2: Cointegration relationship implied by Cross-Euler equation

[4] Another way of saying is that IMRS of goods i is a special case of CIMRS between x_{t+1}^i and x_t^j where $i = j$.

[5] For a formal derivation, see Appendix 2.

Returning to the Cross-Euler eq. (11), it follows that

$$\beta K \frac{(L_{t+1})^{-\gamma}}{(N_t)^{-a}} (1 + r_{t+1}) \frac{P_t^N}{P_{t+1}^L} = 1 + e_{t+1} \tag{13}$$

where we defined e_{t+1} as

$$e_{t+1} \equiv \beta K \frac{(L_{t+1})^{-\gamma}}{(N_t)^{-a}} (1 + r_{t+1}) \frac{P_t^N}{P_{t+1}^L} - E_0 \left[\beta K \frac{(L_{t+1})^{-\gamma}}{(N_t)^{-a}} (1 + r_{t+1}) \frac{P_t^N}{P_{t+1}^L} \right]$$

Taking the log[6] on both side of eq.(13) will yield

$$const. + \ln \left[(1 + r_{t+1}) \frac{P_t^N}{P_{t+1}^L} \right] - \gamma \ln L_{t+1} + \alpha \ln N_t = \ln(1 + e_{t+1})$$

Assuming that the growth rate of both domestic and imported non-durable goods consumption (i.e. N_{t+1}/N_t and L_{t+1}/L_t), real interest rate (i.e. r_t) and the growth rate of the price level of both domestic and imported non-durable goods (i.e. P_{t+1}^N/P_t^N and P_{t+1}^L/P_t^L) are stationary[7], it can be shown that $\ln(1 + e_{t+1})$ will also be stationary (See Appendix 3).

Exploiting the I(0) process of $\ln(1 + e_{t+1})$, we can obtain the following cointegrating relationship.

$$\ln L_{t+1} + const. - \frac{1}{\gamma} \ln \left[(1 + r_{t+1}) \frac{P_t^N}{P_{t+1}^L} \right] - \frac{\alpha}{\gamma} \ln N_t \sim I(0) \tag{14}$$

By similar fashion, we can derive the following cointegration relationship from eq.(12).

$$\ln L_t + const. - \frac{1}{\gamma} \ln \left[\frac{1}{1 + r_{t+1}} \frac{P_{t+1}^N}{P_t^L} \right] - \frac{\alpha}{\gamma} \ln N_{t+1} \sim I(0) \tag{15}$$

Given these cointegrating relationships of log-linearized Cross-Euler equations, combined with GMM-estimable standard Euler equations (8) and (9), we now have a firm ground in comparing the estimates of α and γ. To summarize, under the weaker assumption which allows for the existence of liquidity constraint and/or certain type of habit formation, log-linearized Cross-Euler equation (14) and (15) will yield a super-consistent estimates for α and γ, while Euler equation (8) and (9) are not guaranteed to yield consistent estimates. Under the stronger assumption which does not allow for the presence of liquidity constraint or habit formation, both log-linearized Cross-Euler equations and standard Euler equations will yield super-consistent and consistent estimates of α and γ, respectively. This latter proposition, which basically states that the estimates of IES parameters from cointegration

[6] As was pointed out by Carrol (1997), ideally speaking, it is preferable to estimate the parameters without the log-linearization. However, since the possibly I(1) processes are present inside the conditional moment equations (11) and (12), again, it is likely that the fundamental assumption of GMM estimation method to be violated. This forced us to log-linearize the Cross-Euler equations.

[7] Empirical evidences seems to support this assumption. See, for instance, Clarida (1994), Amano et al (1996) and de la Croix et al (1998).

analysis and GMM to be close under the stronger assumption, is particularly important for us since we can formally test this proposition using statistical method such as Cooley and Ogaki's (1996) LR type test. The following table summarizes the main idea of this section.

	Log-linearized Cross-Euler Equation (Method: Cointegration)	Standard Euler Equation (Method: GMM)
Under weaker assumption	Super-consistent estimates for α and γ	Inconsistent
Under stronger assumption	Super-consistent estimates for α and γ	Consistent estimates for α and γ

Some remarks should follow for cointegration results. Since we are constructing I(0) error terms by leading the variables $\ln L_t, \ln N_t, \ln P^L, \ln P^N$, obviously there will be an endogeneity problem when estimating these cointegrating relationship. However, this problem could be handled by the estimation method such as Phillips and Hansen's (1991) FM-OLS or Park's (1992) CCR. Further, since the estimators in cointegrating regression will be super-consistent (i.e. $O_P(T^{-1})$ consistent), the endogeneity problem will not matter asymptotically. Thus, despite this endogeneity problem, we still can get consistent estimates for α and γ from eq.(14) or eq.(15).

4 Empirical Evidence from Macro Data

4.1 Data Description

The data we use in this paper are quarterly and seasonally adjusted U.S. non-durable goods consumption data covering the period from 1959 Q1 to 200 Q2 (166 observations). Under the classification of National Income and Product Accounts (NIPA), we use real per adult personal consumption expenditure (PCE) for food and tobacco[8] and real per adult PCE for non-durable goods excluding food and tobacco in estimating and testing of the model's implications. PCE for food and tobacco is set to be a proxy for necessity goods and PCE of non-durables excluding food and tobacco is set to be a proxy for luxury goods. Table 1 summarizes how we categorized the components of non-durable goods into necessity goods and luxury goods in this paper.

For a price measure of each goods (i.e. P^N and P^L), we adopted chain-type price index (base year 1996) reported in NIPA Table 7.5. "Chain-Type Price Indexes for PCE by Type of Product"[9]. In constructing real per adult consumption for food and tobacco (i.e. N_t), we deflated PCE for food and tobacco by the chain-type price index for PCE food and tobacco and further divided by U.S. population above 20 years old. It should be noted that by this

[8] Following Ogaki (1992), we have excluded alcohol beverages from food consumption expenditure.

[9] Chain-type price index of PCE for food and tobacco are published separately by BEA. In constructing the composite price index for food and tobacco, we simply computed the weighed average of two price index, where weight taken according to the food nominal expenditure and tobacco nominal expenditure. Details of this data manipulation can be found in Data Appendix of this paper.

Table 1: Categorizing Non-Durable Goods

Non – DurableGoods	
Necessity Goods (N_t)	Luxury Goods (L_t)
Food ex. alcoholic beverages	Clothing and shoes
Tobacco	Alcoholic beverages
	News and magazine
	Entertainment
	Other goods

Note: To be accurate energy goods stands for "Gasoline, fuel oil and other energy goods.". Definition of the type of consumption goods in this table follows NIPA's Table 2.6. "PCE by Type of Product".

manipulation, consumption of food and tobacco is now captured as the real *quantity* index rather than real expenditure. This manipulation is crucial because what we try to capture in the model is quantity of consumed goods rather than expenditure. The real per adult consumption for non-durable goods excluding food and tobacco (i.e. L_t) was constructed in a similar fashion. Finally, real interest rate (i.e. r) was constructed based on quarterly average of 3-month U.S. Treasury Bill Rate subtracting inflation rate, where the inflation rate was calculated from quarterly average of Consumer Price Index[10].

As for the preliminary step for the cointegration analysis, we tested the null of difference stationarity against the null of (trend) stationarity for the variables included in the cointegrating regressions. To be specific, we tested the difference stationarity of following four variables: log of necessity goods (i.e. $\ln N_t$), log of luxury goods (i.e. $\ln L_t$), log opportunity cost of current necessity goods in terms of future luxury goods (i.e. $\ln\left[(1+r_{t+1})P_t^N/P_{t+1}^L\right]$) and log opportunity cost of future necessity goods in terms of current luxury goods (i.e. $\ln\left[(1+r_{t+1})^{-1}P_{t+1}^N/P_t^L\right]$). The results of the unit root tests are reported on Table 2.

We used Said and Dickey's (1984) Augmented Dickey-Fuller (ADF) test, Phillips and Perron's (1988) PP test and Park and Choi's (1988) J test in testing for the null of difference stationarity. As can be seen from Table 2, the tests do not reject the null of difference stationarity against the null of stationarity at 10% significance level for all variables. Further, the tests do not reject the null of difference stationarity against the null of trend stationarity at 10% significance level for both log necessity and luxury goods. Thus, log of necessity and luxury goods may well be thought of as stochastic processes containing unit root with possible drift. However, for the log opportunity costs, the test results were rather mixed. It is not clear whether the log opportunity cost follows a difference stationary process or trend stationary process from this result. Keeping in mind the possibility of trend stationarity in log opportunity cost, we now proceed to the cointegration analysis of the log-linearized cross-Euler equations..

[10] Thus, real interest rate used in this paper is actually an ex-post real interest rate.

Table 2: Unit Root Test

Variable	ADF test		PP test		J-test	
	cst.	cst. & trd.	cst.	cst. & trd.	J(0,3)	J(1,5)
$\ln N_t$	-1.616	-1.822	-1.525	-1.707	1.240	1.804
$\ln L_t$	0.378	-1.306	0.765	-0.637	67.771	3.063
$\ln\left[(1+r_{t+1})P_t^N/P_{t+1}^L\right]$	0.197	-3.047	0.084	-5.284**	25.865	0.670
$\ln\left[(1+r_{t+1})^{-1}P_{t+1}^N/P_t^L\right]$	-0.722	-2.999	-0.661	-3.076	8.864	0.406*

Note: Lag order used for ADF test and PP test was four. The 10% critical values of ADF test and PP test with a constant is -2.576 and with constant and trend is -3.143. The 5% critical values are -2.879 and -3.438, respectively. The critical values are due to MacKinnon (1991). For J(0,3) test and J(1,5) test, 10% critical values are 0.577 and 0.452 and 5% critical values are 0.338 and 0.295, respectively.. The critical values are due to Park and Choi (1988). It should be noted under the J-test, the null of difference stationarity is rejected when the statistics are smaller than the critical value. * denotes the rejection of null hypothesis at 10% level. ** denotes the rejection of null hypothesis at 5% level.

4.2 Estimation

4.2.1 Canonical Cointegration Regression

In this section we will explain the step in applying Park's (1992) Canonical Cointegration Regression (CCR) on log-linearized Cross-Euler eq. (14) and (15). The model implies the following cointegrating restriction for each equation:

$$\text{Cross-Euler eq. (11):} \quad \ln C_{t+1}^L - \frac{1}{\gamma}\ln\left[(1+r_{t+1})\frac{P_t^N}{P_{t+1}^L}\right] - \frac{\alpha}{\gamma}\ln C_t^N \sim I(0) \tag{16}$$

$$\text{Cross-Euler eq. (12):} \quad \ln C_t^L - \frac{1}{\gamma}\ln\left[\frac{1}{1+r_{t+1}}\frac{P_{t+1}^N}{P_t^L}\right] - \frac{\alpha}{\gamma}\ln C_{t+1}^N \sim I(0) \tag{17}$$

Following the result of unit root pretesting in the previous subsection, log necessity and luxury goods will be assumed to be a difference stationary process. However, for the log opportunity costs, the pretest results gave a mixed signal of difference stationary process and trend stationary process. Therefore, two cases must be considered in conducting the cointegration analysis: the case when deterministic trend is absent in the cointegrating system (i.e. log opportunity cost follows difference stationary process) and the case when deterministic trend is present in the cointegrating system (i.e. log opportunity cost follows the trend stationary process).

Case 2 *(Deterministic trend is not present inside the cointegrating system)*
Let y_t be a scalar of difference stationary process and let $\mathbf{x}_t$ be the $k \times 1$ vector of difference stationary process whose components are not stochastically cointegrated. If y_t and $\mathbf{x}_t$ satisfies the deterministic cointegration restriction, then the cointegrated system can be expressed as

$$y_t = \theta_c + \boldsymbol{\upsilon}_x'\mathbf{x}_t + \varepsilon_t \tag{18}$$

where θ_c is a scalar and ε_t is a stationary process with mean zero.

In our model, y_t can be thought of as the log luxury goods and $\mathbf{x}_t$ can be thought of vector containing log opportunity cost and log necessity goods. Under the case that deterministic trend is absent in the log opportunity cost, the model implies the deterministic cointegration among the variables $\ln C^L, \ln(1+r)\, P^N/P^L$ and $\ln C^N$ with cointegrating vector $(1, 1/\gamma, \alpha/\gamma)'$. This sets the ground for applying the CCR in the above regression form.

Some remarks are in order regarding to the CCR estimator. As is the case for any cointegrating regression, CCR will yield a super-consistent estimate of the parameters. In addition, by the non-parametric correction for the long-run variance of $(\Delta x_t, \varepsilon_t)'$, CCR is known to be asymptotically efficient and does not require the strictly exogeneity assumption in $(\Delta x_t, \varepsilon_t)'$. This latter property is crucial for our purpose since the regressor x_t is constructed by the leads lags of $\ln P^N, \ln P^L$, and $\ln C^N$ and ε_t also consists of leads and lags of similar variables. For instance, applying OLS estimator to above regression form following Engle and Granger's (1987) method, which assumes the strict exogeneity, will yield asymptotically biased, though consistent estimates of $1/\gamma$ and α/γ (see Phillips and Durlauf (1986) and Stock (1987)). Thus, applying the CCR in the above regression form will yield a super-consistent estimate of the intertemporal substitution parameters.

By applying Park's (1990) G(p,q) test on the residuals, we can obtain the Park's H(p,q) statistics. Under the null of cointegration, Park showed that H(p,q) statistics is asymptotically χ^2 distributed with $q-p$ degrees of freedom. Since we are interested in both deterministic and stochastic cointegration relationship, we conducted H(0,q) and H(1,q) tests in this paper. The results of Park's CCR estimates[11] are reported in Table 4. for cointegrating equation 14 and in Table 3 for cointegrating equation 15. In order to check the deterministic cointegration relationship, we have applied H(0, q) test for both equation. Also, to check for the stochastic cointegration relationship, H(1,q) test was also conducted.

Let us first turn to the estimation result of equation 14. Parameter estimate for α was 0.9924 and γ was 0.7663. Thus, IES for necessity goods (i.e. $1/\alpha$) was 1.0076 and IES for luxury goods (i.e. $1/\gamma$) was 1.3049. The test generally rejected the implication of the deterministic cointegration relationship, but was not able to reject the stochastic cointegration relationship.

Next, turning to the estimation result for equation 15, estimate for α was 0.9492 and for γ was 0.7504. Therefore the implied IES for necessity goods was 1.0535 and for luxury goods was 1.3324. We found that estimates of γ to be reasonably close between eq. 14 and 15. For the deterministic cointegration relationship, H(0,1) and H(0,3) test rejected the null hypothesis. For the stochastic cointegration relationship, only H(1,3) test rejected the null hypothesis of stochastic cointegration.

[11] In estimating the long-run covariance matrix of error term, we used Andrews and Monahan's (1992) VAR prewhitened HAC estimator. The choice of kernel was QS kernel as suggested by Andrews (1991). Following the Monte Carlo study of Han (1996), third stage CCR estimates are reported and H(p,q) statistics are based on fourth stage CCR.

Table 3: CCR Results

$\ln L_{t+1} = const. + \frac{1}{\gamma}\ln\left[(1+r_{t+1})P_t^N/P_{t+1}^L\right] + \frac{\alpha}{\gamma}\ln N_t + I(0)$					
Estimates			Implied Estimates		
const.	$1/\gamma$	α/γ		α	γ
-2.345	1.304	1.295		0.992	0.766
(1.220)	(0.042)	(0.147)			
Test Statistics					
H(0,1)	H(0,2)	H(0,3)	H(1,2)	H(1,3)	H(1,4)
5.479*	6.073*	7.355	0.593	1.875	2.168
[0.019]	[0.047]	[0.061]	[0.441]	[0.391]	[0.538]

Note: Numbers in parenthesis stand for the estimated standard error. Numbers in square brackets stand for p-value. * denotes the rejection of null of cointegration at 5% level. ** denotes the rejection of null of cointegration at 1% level.

Table 4: CCR Results

$\ln L_t = const. + \frac{1}{\gamma}\ln\left[(1+r_{t+1})^{-1}P_{t+1}^N/P_t^L\right] + \frac{\alpha}{\gamma}\ln N_{t+1} + I(0)$					
Estimates			Implied Estimates		
const.	$1/\gamma$	α/γ		α	γ
-2.096	1.332	1.264		0.949	0.750
(1.031)	(0.036)	(0.124)			
Test Statistics					
H(0,1)	H(0,2)	H(0,3)	H(1,2)	H(1,3)	H(1,4)
2.780	2.966	9.395*	0.185	6.614	7.470
[0.095]	[0.226]	[0.024]	[0.666]	[0.036]	[0.058]

Note: Numbers in parenthesis stand for the estimated standard error. Numbers in square brackets stand for p-value. * denotes the rejection of null of cointegration at 5% level. ** denotes the rejection of null of cointegration at 1% level.

4.2.2 GMM

In this section, we will conduct Hansen's (1982) GMM on eq. 8 and 9. Parameters α and γ will be estimated under single equation and system equation context. We will also discuss the choice of instrumental variables (IV) in this paper. Hansen's J test will also be reported.

As it was pointed out by Hall (1993) and Ogaki (1993), it is well known that the estimate of GMM is very sensitive to the choice of instrumental variables. To test for the robustness of the estimates against the choice of instruments, we estimated the parameters under several types of instruments with varying time lags. First family of the instrumental variables was chosen following the convention in applied GMM literature. As can be seen from the following table, six types of instrument sets were chosen.

IV Type	Euler Equation 8	Euler Equation 9
IV1	const., $\frac{C^N_{t+1}}{C^N_t}$	const., $\frac{C^L_{t+1}}{C^L_t}$
IV2	const., $\frac{P^N_{t+1}}{P^N_t}$	const., $\frac{P^L_{t+1}}{P^L_t}$
IV3	const., r_{t+1}	const., r_{t+1}
IV4	const., $\frac{C^N_{t+1}}{C^N_t}$, r_{t+1}	const., $\frac{C^L_{t+1}}{C^L_t}$, r_{t+1}
IV5	const., $\frac{C^N_{t+1}}{C^N_t}$, $\frac{P^N_{t+1}}{P^N_t}$	const., $\frac{C^L_{t+1}}{C^L_t}$, $\frac{P^L_{t+1}}{P^L_t}$
IV6	const., $\frac{C^N_{t+1}}{C^N_t}$, $\frac{P^N_{t+1}}{P^N_t}$, r_{t+1}	const., $\frac{C^L_{t+1}}{C^L_t}$, $\frac{P^L_{t+1}}{P^L_t}$, r_{t+1}

The next issue in conducting GMM estimation is to choose the lag order of the error term when estimating the variance-covariance matrix of GMM disturbance terms. According to the rational expectation hypothesis, it is known that the forecast error will be serially uncorrelated. Since our model is based on the representative agent with rational expectation, the economic theory suggests the lag order of zero. Nevertheless, taking into account for the time aggregation problem which was pointed out by Heaton (1995), we choose the lag order of one in estimating the variance-covariance matrix of GMM disturbance terms[12]. Also, to be consistent with the time aggregation issues, we have lagged the instrumental variables for two periods when conducting GMM estimations.

4.2.3 Result

GMM estimation was conducted using family of conventional instruments. The GMM estimation results for Euler equation 8 is summarized under Table 5. Similarly, the GMM estimation result for Euler equation 9 is summarized under Table 6. Hansen's J-statistics for each regression are also reported.

Let us first interpret the estimation result of Euler equation for necessity goods consumption. We first observe the large variance in the estimates of α. The estimates for α ranges from -11.917 to 15.136. This wide dispersion can also be confirmed from the estimated standard error for the estimator $\hat{\alpha}$. We can think of two possibilities that have contributed to these odd estimation results. First possibility is the weak instruments problem, i.e. if the instruments and the forcing variables in the regression are weakly correlated, the variance of the estimator will be large. It might be the case that in our GMM estimation, the conventional instruments were weakly correlated to the forcing variables.

Second possibility comes in when Euler equation is misspecified. The easiest way to check for the misspecification is to look at Hansen's J statistics. However, to our surprise, Hansen's J test does not reject the null hypothesis that Euler equation 8 is specified for all cases. Does this mean that Euler equation (8) is correctly specified? Statistically speaking, we cannot deny this possibility. But then the odd estimates of α in Table 5. does not conform with the result of Hansen's J test. Or it might be the case that the low power of

[12] Since the lag order was explicity chosen, we will use HAC estimator with truncated kernel when estimating the variance-covariance matrix of the GMM disturbance terms.

Table 5: GMM Results for Necessity Goods

$$E_t \left[\beta \left(\frac{N_{t+1}}{N_t} \right)^{-\alpha} (1 + r_{t+1}) \frac{P_t^N}{P_{t+1}^N} - 1 \right] = 0$$

IV Type	β	α	J-statistics	D.F.
IV0	0.991	9.745	- -	Just Identified
	(0.007)	(16.023)		
IV1	0.991	15.136	- -	Just Identified
	(0.009)	(7.346)		
IV2	0.971	-11.917	- -	Just Identified
	(0.035)	(22.504)		
IV3	0.991	15.485	0.066	1
	(0.009)	(7.348)	[0.797]	
IV4	0.984	-0.791	1.911	1
	(0.005)	(3.957)	[0.588]	
IV5	0.988	12.740	1.570	2
	(0.008)	(5.934)	[0.456]	
IV6	0.988	11.779	2.031	4
	(0.007)	(4.606)	[0.730]	

Note: All instruments are lagged for two periods. Numbers in parenthesis represent the estimated standard errors. Numbers in brackets represent the p-values. * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

Hansen's J test resulted in the under-rejection of the null. As such, we propose to use the likelihood ratio type test proposed by Cooley and Ogaki (1996), which will be the topic of the next section.

To the sharp contrast to the estimation result of Euler equation for necessity goods, the estimation results of Euler equation for luxury goods have an intuitive result. As can be seen from Table 6, we can observe the relative tightness in the estimates of γ. The estimates of γ range from 0.368 to 3.778, with an exception of 16.780 under IV1. This observation is consistent with the conspicuously small estimated standard error of $\hat{\gamma}$ compared to that of $\hat{\alpha}$. However, turning to Hansen's J test, the test rejected the specification of the Euler equation 9 for 3 out of 4 cases, which is counter-intuitive given the stable estimates of $\hat{\gamma}$. Or it may well be the case that the rejection came from the size distortion of Hansen's J test. As such, we will rely on Cooley and Ogaki's LR type test in testing the specification of the Euler equation 9.

4.3 Test of Liquidity Constraint

In this section, we will discuss why Cooley and Ogaki's (1996) test best suits for our purpose and also report the result of the test. Before we discuss Cooley and Ogaki's LR type test, it

Table 6: GMM Results for Luxury Goods

	$E_t\left[\beta\left(\frac{L_{t+1}}{L_t}\right)^{-\gamma}(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L}-1\right]=0$			
IV Type	β	γ	J-statistics	D.F.
IV0	0.985	0.368	- -	Just Identified
	(0.007)	(1.114)		
IV1	1.062	16.780	- -	Just Identified
	(0.065)	(15.111)		
IV2	1.002	3.778	- -	Just Identified
	(0.009)	(1.864)		
IV3	1.003	2.800	13.357	1
	(0.006)	(1.037)	[0.000]	
IV4	0.991	1.355	3.088	1
	(0.006)	(0.960)	[0.588]	
IV5	0.991	1.003	17.616	2
	(0.006)	(0.969)	[0.000]	
IV6	0.991	0.913	18.430	4
	(0.006)	(0.966)	[0.001]	

Note: All instruments are lagged for two periods. Numbers in parenthesis represent the estimated standard errors. Numbers in brackets represent the p-values. * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

may be useful to review the standard LR type test in the GMM literatures. For simplicity, we impose some linear restriction on the GMM estimator. In the most general linear form, the null hypothesis can be expressed as follow.

$$H_o:\ \mathbf{R}\hat{\boldsymbol{\theta}}_{GMM}=\mathbf{q}$$

where $\mathbf{q}$ is $q\times 1$ vector of constant and $\mathbf{R}$ is some $q\times k$ matrix. Then the LR type statistics, denoted as QLR, is defined as follow and can be shown that it will be asymptotically χ^2 distributed with q degrees of freedom.

$$QLR = T\cdot J_{restricted} - T\cdot J_{unrestricted}\ \xrightarrow{d}\ \chi^2(q)$$

where T stands for the number of observations and J stands for the minimized objective function under GMM. Now, it should be noted that under the standard LR type test, $\mathbf{q}$ was simply a vector of *constants*.

The punch line of Cooley and Ogaki's LR type test is that they replaced $\mathbf{q}$ with the *estimator* of cointegrating vector $\hat{\mathbf{q}}_{coint}$. By exploiting the super-consistency of $\hat{\mathbf{q}}_{coint}$, they show that QLR will again be asymptotically χ^2 distributed with q degrees of freedom[13].

[13] If, instead, the estimator $\hat{\mathbf{q}}$ were only consistent (i.e. $O(T^{-1/2})$ consistent), then one have to calculate

Restating mathematically,

$$H_o : \mathbf{R}\hat{\boldsymbol{\theta}}_{GMM} = \hat{\mathbf{q}}_{coint} \text{ and } QLR \xrightarrow{d} \chi^2(q).$$

Since our model involves the cointegration analysis and GMM in estimating the parameters α and γ, Cooley and Ogaki's LR type test seems to be the best candidate for our specification test.

We basically tested two types of null hypothesis. First null hypothesis is $H_0^1 : \hat{\alpha}_{GMM} = \hat{\alpha}_{coint}$ and results are reported under Table 7. Second null hypothesis is $H_0^2 : \hat{\gamma}_{GMM} = \hat{\gamma}_{coint}$ and results are reported in Table 8. Note again, if indeed eq.7, eq. 8 and eq. 9 are all well specified, then the test is likely to accept all of the above null hypotheses. We will interpret the results under three different nulls one by one.

Table 7: LR-type Test Results: Necessity Goods

$H_0 : \hat{\alpha}_{CCR} = \hat{\alpha}_{GMM}$		
IV Type	QLR statistics	P-value
IV0	1.660	[0.197]
IV1	15.396**	[0.000]
IV2	4.214*	[0.040]
IV3	16.101**	[0.000]
IV4	2.267	[0.132]
IV5	14.590**	[0.000]
IV6	14.502**	[0.000]

Note: * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

First let us turn to the results under the null of $H_0^1 : \hat{\alpha}_{GMM} = \hat{\alpha}_{coint}$. As can be seen from Table 7, QLR statistics exceeds the critical value for most of the cases, which implies the rejection of the null hypothesis. Indeed, the test rejects 5 out of 7 cases. This evidence suggests that eq. 7 and/or eq. 8 are misspecified.

Next we will turn to the results under the null of $H_0^2 : \gamma_{GMM} = \gamma_{coint}$. To the sharp contrast to the former test, as one can see from Table 8, the test is not able to reject the null hypothesis, except for the case when we use the instrumental variable set, IV1. Except for this case, the QLR statistics are below the critical values. According to this result, the test seems to support the hypothesis that both eq. 7 and eq. 9 are specified.

the covariance of $\hat{\boldsymbol{\theta}}_{GMM}$ and $\hat{\mathbf{q}}$ in order to conduct the statistical inference. For details, see Ogaki (1993).

Table 8: LR-type Test Results: Luxury Goods

$H_0 : \hat{\gamma}_{CCR} = \hat{\gamma}_{GMM}$		
IV Type	QLR statistics	P-value
IV0	0.130	[0.717]
IV1	15.302**	[0.000]
IV2	2.666	[0.102]
IV3	2.460	[0.116]
IV4	0.193	[0.660]
IV5	0.239	[0.624]
IV6	0.592	[0.441]

Note: * denotes the rejection of null at 5% level. ** denotes the rejection of null at 1% level.

5 Empirical Evidence from Consumer Expenditure Survey

5.1 Motivation for Cohort Analysis

In the previous section, we have assumed an existence of representative agent and used aggregate data in estimating the preference parameters. However, when estimating the preference parameters consistently from the aggregate data, extremely stringent requirements have to be met. For instance, in order for the representative agent to exist, all the agents in the economy need to share the identical preference and also their utility function have to be homothetic (See Kirman (1992) and Stoker (1993) for more details). In reality, it is very unlikely that these conditions are met. Turning to micro data, several rich data set, such as Panel Study for Income Dynamics (PSID), Consumer Expenditure Survey (CES) and Family Expenditure Survey (FES), are available. However, these micro data set, despite its richness, contain some serious defects when testing for life-cycle model. For instance, PSID only tracks the consumption expenditure of individuals, which is not too useful especially when we are interested in the consumption pattern of non-durable goods and services as a whole. On the other hand, CES and FES, albeit its richness in the categories of consumption goods, are cross-sectional survey data that it is does not allow researchers to track same individuals over time. In reconciling this dilemma, Deaton (1985) proposed a method in tracking 'cohorts' from time series of cross-sectional survey. Cohort is a subset of households sharing the identical qualities that are unchanged over time. These qualities, for instance, can be year of birth, race, gender and so on. By classifying the cross-sectional observations into these cohorts and tracking them over time, Deaton (1985) showed that it is possible to construct synthetic panel (or pseudo panel) data that allows researchers to estimate the cohort-specific parameters consistently[14]. Moreover, since the households are aggregated into cohorts who share the similar taste, it is relatively free from the aggregation problem compared to representative agent model. Inspired by the virtues, not surprisingly, the cohort technique has been widely used in the empirical life-cycle literatures. The predecessor in the line of empirical cohort analysis have constructed the cohorts in several ways. Browning, Deaton, and Irish (1985), Moffit (1993), Attanasio and Webber (1993, 1995), have classified the households according to their ages. Attanasio et al. (1999) have classified the households according to their ages and education attainments.

In this paper, we adopt this cohort technique in estimating the preference parameters and testing for the liquidity constraints. In constructing the cohort data, careful attention must be exerted. As pointed out by Verbeek (1996), legitimate cohorts are constructed only if taste and demographic observations of households in each cohort are drawn from the identical probability distribution. Thus, the observational difference among each observation within the same cohort must be idiosyncratic and should be "averaged out" after taking average. To comply strictly with this requirement, households should better be classified into cohorts distinguished by the finest information available from the data, so that non-idiosyncratic effects are isolated. However, due to the limitation of sample observations from the CES - approximately 5,000 household observations each quarter -, there exist a trade-off between the construction of finer cohorts and sample observations per cohort. Some judgment must be exerted to strike the "optimal" balance between two requirements

In constructing the cohorts, we basically follow Attanasio et al. (1999) - that is to classify the households by ages and educational attainments. Specifically, I classified agents into 10 cohorts by their age and education level. This classification can be justified as follows: 1) Age classification is necessary because the agents reveal the heterogeneous consumption patterns (i.e. life-cycle in consumption) depending on their age. 2) Classification by education level is adopted as a mean to classify the agents by their life-time income (i.e. permanent income). By the construction of LCPIM, an agent will face a liquidity constraint when his net wealth decreases to some certain level. Naturally, an agent with low permanent income (i.e. poor agent) is more likely to face this bound over his course of life than an agent with high permanent income (i.e. rich agent). Thus, in order to be perfectly consistent with the theory implication, it is ideal to classify agents by their permanent income. However, since the permanent income of each agent is not directly observable from the data, I have compromised to proxy the permanent income by the education level. The cohort data for each type of agent have been constructed from the series of Consumer Expenditure Survey from 1984 to 1998.

Finally, whenever making a statistical inference in this section, we assume that number of cross-sectional cohorts is fixed in pseudo panel (i.e. H is fixed), assume that household observations within the cohort is large (i.e. $N_c \to \infty$) and assume that time series observations are large (i.e. $T \to \infty$). Under these assumption, consistency of the pooled estimates in pseudo

[14] For the survey regarding the cohort technique, see Verbeek (1996). For recent development, see Verbeek and Nijman (1993), Moffit (1993), and Collado (1997).

panel are known to yield consistent estimates (See Verbeek (1996) for more discussion).

5.2 Data Description

5.2.1 Construction of Cohorts

As a first step, the households are classified into 5 categories by their ages at 1984. Age intervals for each categories are 25-30, 31-36, 37-42, 43-48, and 49-54, respectively. Here, we are taking intervals of 6 years for each categories, which is slightly different from the convention previously used by Browning et al. (1985) and others. This practice has been adopted in consideration to the limitation on sample observations, but we believe that the heterogeneity biases relative to conventional intervals are minor.[15]. As a second and final step, we further classify the households into two categories according to their educational attainments. Households who dropped out from a high school or whose highest educational attainment is high school has been classified to the category called "High School." A Household who at least holds a college degree or higher has been classified to the category called "College." It should be noted that we have intentionally omitted households who have dropped out from a college. This practice has been adopted in order to preserve the sharp contrast between the "High School" and "College" categories. Thus, as a consequence of this two-step classification, we have constructed 10 cohorts. The following tables summarize the average observations per quarter for each cohort.

Table 9: Average Cohort Size per Quarter: 1984Q1 to 1999Q1

High School*				
HS1	HS2	HS3	HS4	HS5
322.43	273.21	255.77	234.61	231.34
College**				
Col1	Col2	Col3	Col4	Col5
242.02	253.41	196.43	120.41	95.82

Note: *) Highest educational attainment of a household head is less than or equivalent to high school diploma. Within this educational cohort, households were further classified by their ages. HS1 was born between '54 to '59, HS2 between '48 to '53, HS3 between '42 to '47, HS4 between '36 to '41 and HS5 between '30 to '35. **) Highest educational attainment of a household head is more than or equivalent to college degree. Col1 was born between '54 to '59, Col2 between '48 to '53, Col3 between '42 to '47, Col4 between '36-to '41 and Col5 between '30 to '35.

The top panel of the Table 9 reports the average cohort size for High School cohorts. HS1 stands for the High School cohort who was 25-30 years old, HS2 stands for the cohorts who was 31-36 years old, HS3 for the cohorts who was 37-42 years old, HS4 for the cohorts who was 43-48 years old, and HS5 for the cohorts who was 49-54 years old at 1984. The

[15] We have also tried 5 years intervals. However, the average observations for some cohorts, especially for older cohorts, were smaller than 100. In order to ensure that each cohort size is over or around 100 observations, we decided to classify the households by 6 years of interval.

bottom panel of Table 9 reports the average cohort size for College cohorts. Col1 stands for the College cohorts who was 25-30 years old, Col2 for the cohorts who was 31-36 years old, and so on. As can be seen, the cohort size for most of the cohorts were more than or approximately 200, except for Col4 and Col5. This apparently small cohort size for Col4 and Col5 is probably due to the low college enrollment for older generations[16].

For each cohort, information regarding consumption and demographics have been extracted and were used in constructing the average cohort data[17]. Following Section 3, luxury goods were composed of alcohol beverages, apparel, transportation excluding vehicle, entertainment, reading and other goods. Necessity goods were composed of food and tobacco. In calculating the real expenditure, we have used the common price index used in Section 3. In other words, all cohorts are assumed to face the common opportunity cost intratemporally and intertemporally[18]. In addition to expenditure information, we have also calculated an average number of adults and children per cohort. The descriptive statistics regarding the consumption growth is reported in Table 10.

5.2.2 Age Profile and Preliminary Testing

In order to visualize the consumption pattern of the cohorts, so called "Age Profile" regarding luxury goods and necessity goods expenditure have been laid out in Figure 1.

Age profile plots the life-cycle consumption pattern of same education of cohorts with different age groups on the same diagram. The left-top panel plots the log real luxury goods expenditure and the righ-top panel plots the log real necessity goods expenditure of high school cohorts. As can be seen from these figures, consumption pattern for both goods reveals a hump over the life-cycle. For the luxury goods, consumption expenditure peaks out in the mid 40's and reveals a sharp decline towards the end of the life-cycle. In a similar fashion, necessity goods expenditure peaks out during late 30's to early 40's, although the magnitude of a hump seems to be milder than that of luxury goods. The left-bottom panel shows the age profile of the log real luxury goods expenditure and the right-bottom panel shows the age profile of the log real necessity goods expenditure of college cohorts. Again, hump shape in the consumption pattern can be clearly observed. For the college cohorts, luxury goods and necessity goods expenditure seem to peak out in the late 40's. Interestingly,

[16] Col4 is the genration who was born between 1936-1941 and Col5 is the generation who was born between 1930-1935. Considering the effect of Great Depression and WWII, probably only the wealthy families were able to send their children to colleges. Small cohort size for Col4 and Col5 will apparently introduce a large standard error when constructing a cohort's average data. As explained in Deaton (1985) and Verbeek (1996), by rendering number of cross-sectional cohorts to infinity (i.e. $H \to \infty$) with fixed cohort size (i.e. N_c is fixed), it is possible to obtain the consistent estimator in the presence of standard error in average cohort data. But then, due to the limitations in total household observations, one face a same dillemma in claiming a large sample in terms of H. Thus, obviously there is a trade-off in choosing $H \to \infty$ or $N_c \to \infty$. In this paper, we assume $Nc \to \infty$.

[17] All the average cohort data have been seasoanlly adjusted using seasonal dummies.

[18] Though this assumption offers us a simplicity in calculating the real expenditure for each cohort, this assumption may be too strong. Constructing Stone Price Index for each cohort is often favored solution. See Attanasio and Webber (1995) for instance.

Table 10: Descriptive Statistics of Real Expenditure Growth: 1984Q1 to 1999Q1

	High School				
	HS1	HS2	HS3	HS4	HS5
	Luxury Goods Expenditure Growth (% per Quarter)				
Mean	0.367	0.106	-0.289	-0.739	-0.755
Std. Dev.	6.298	6.391	6.103	7.393	6.826
	Necessity Goods Expenditure Growth (% per Quarter)				
Mean	0.532	-0.004	-0.311	-0.370	-0.397
Std. Dev.	4.030	3.565	4.373	4.703	3.728
	College				
	Col1	Col2	Col3	Col4	Col5
	Luxury Goods Expenditure Growth (% per Quarter)				
Mean	0.519	0.119	-0.232	-0.553	-0.802
Std. Dev.	5.898	5.609	7.178	8.819	11.53
	Necessity Goods Expenditure Growth (% per Quarter)				
Mean	0.925	0.461	-0.155	-0.434	-0.569
Std. Dev.	4.655	5.007	6.152	6.366	7.678

Note: Real expenditures are seasonally adjusted using seasonal dummies. Expenditure growth rates not adjusted for adult equivalent scale.

necessity goods expenditure reveals a sharp increase in the beginning of the life-cycle and stays persistent toward the end of life-cycle, while increase and decrease in luxury goods seems to be fairly symmetric.

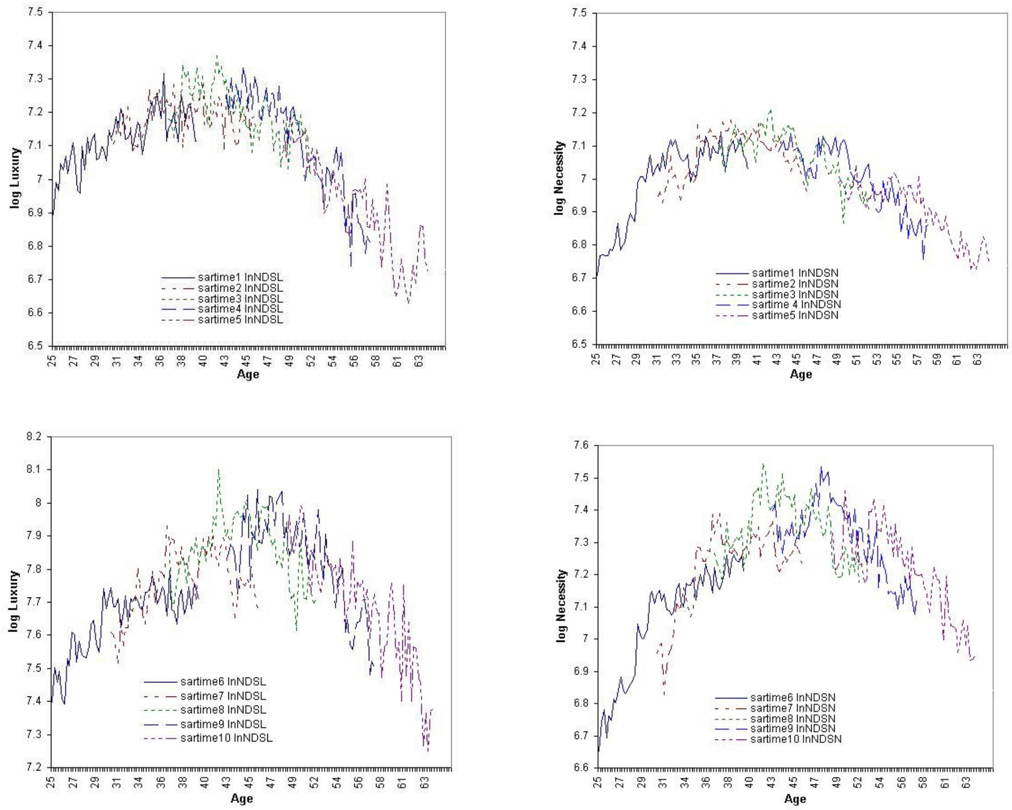
The main characteristic to be noted here is the clear violation of consumption smoothing, which is one of the main implications of life-cycle model. Is this hump shape in the consumption pattern solely induced by liquidity constraint? Not likely. Thus, before we can legitimately test for the existence of liquidity constraints, the first task is to account for this obvious hump shape over the life-cycle. As pointed out by Attanasio and Webber (1995 and 1997) and Attanasio et al. (1999), one of the main cause for this hump-shaped consumption pattern is due to demographic factors. As presented in their papers, the number of adults and children in the household shows similar humped-shape over the life-cycle. Mostly likely, as they argue, the hump-shape in consumption pattern can be attributed to these demographic factors. As for a quick and ad-hoc adjustment, we have adjusted consumption expenditure by the number of adult equivalent members[19] in the household. The age profiles

[19] The number of adult equivalent members in the household has been calculated according to the following formula.

$$\#\ of\ adult\ equivalent = 1 + 0.75 * \#\ of\ adults + 0.4 * \#\ of\ children$$

Child is defined as a household member with age below 18 years old and adult is defined as a household member with age above 18 years old. The forumula is a composition of the scale used by Attanasio (1995) and Blundell, Browning and Meghir (1994).

Figure 1: Age Profile of Luxury and Necessity Goods (Demographic Unadjusted)

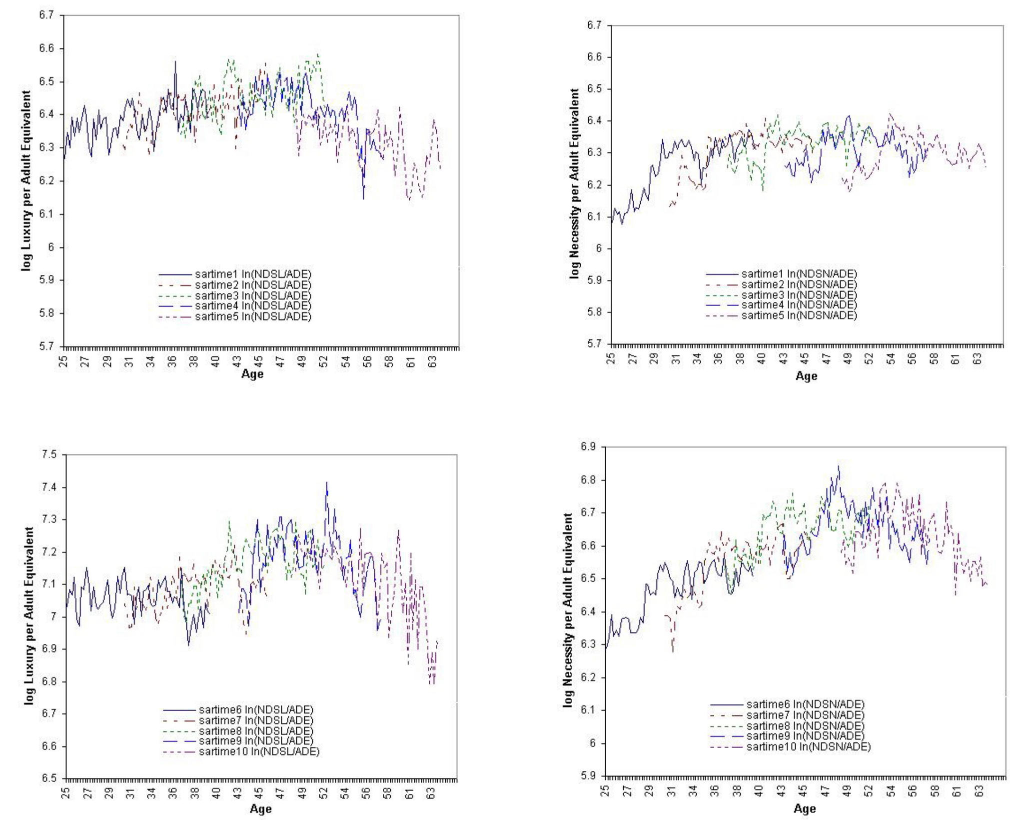


Source: Consumer Expenditure Survey 1984-1998

of consumption expenditure per adult equivalent have been laid out in Figure 2.

As can be seen from Figure 2, the adjustment by adult equivalent scale have considerably dissipated a hump shape in the age profile. Now, the age profiles for both education cohorts reveal a fair magnitude of consumption smoothing over the life-cycle. For the necessity goods, both high school and college cohorts tend to increase their expenditure over the life-cycle. Interestingly, for the luxury goods, both education cohorts tend to decrease their spending toward the end of the life-cycle. Of course, the adjustment of consumption expenditure by adult equivalent scale is rather ad-hoc. It is not our intention to claim this ad-hoc adjustment will solve the hump shape puzzle in consumption pattern, but rather wanted to illustrate that demographic adjustment is important whenever scrutinizing the consumption behavior, especially on the cohort level. In the later section, we will adopt a more sophisticated

Figure 2: Age Profile of Luxury and Necessity Goods (Demographic Adjusted)



Source: Consumer Expenditure Survey 1984-1998

method to account for hump shape in consumption by allowing demographic taste shift in the household's utility function. The bottom line is that it is only possible to test for the existence of liquidity constraint after taking into account for the demographic factors.

Finally, as for a preliminary step for cointegration analysis in the following section, we have conducted a unit root test for consumption expenditure of each cohort. Table 11 reports the results from ADF test[20]. For the case of HS3, HS5 and Col5, the test results have indicated that log luxury goods expenditures are trend stationary. For the case of HS2, the test gave a mixed results rejecting the unit root without trend, while not rejecting the unit root with trend. For the log necessity goods expenditure, the null of unit root were

[20]We have also conducted PP test and J(p,q) test, but the results were similar and will not be reported here.

Table 11: Unit Root Test of Luxury and Necessity Goods Expenditure

				High School		
		HS1	HS2	HS3	HS4	HS5
$\ln L_t$	cst.	-2.690	-3.084*	-1.421	0.006	-1.551
	cst. & trd.	-3.184	-2.983	-3.561*	-2.717	-3.613*
$\ln N_t$	cst.	-2.215	-1.793	-0.861	-0.455	-0.696
	cst. & trd.	-0.978	-0.951	-2.698	-2.097	-2.346
$\Delta Adult_t$	cst.	-7.151**	-6.378**	-6.974**	-7.167**	-5.657**
	cst. & trd.	-7.086**	-6.319**	-7.519**	-7.185**	-5.655**
$\Delta Child_t$	cst.	-6.232**	-7.306**	-7.471**	-5.851**	-6.750**
	cst. & trd.	-7.184**	-7.527**	-7.424**	-6.245**	-7.056**
				College		
		Col1	Col2	Col3	Col4	Col5
$\ln L_t$	cst.	-2.074	-2.399	-1.894	-0.574	-1.150
	cst. & trd.	-2.347	-1.913	-2.236	-2.369	-4.528**
$\ln N_t$	cst.	-1.881	-2.092	-1.580	0.012	-0.639
	cst. & trd.	-1.337	-1.597	-1.871	-1.273	-2.824
$\Delta Adult_t$	cst.	-8.450**	-5.779**	-6.460**	-6.138**	-6.701**
	cst. & trd.	-8.375**	-5.726**	-6.688**	-6.093**	-6.649**
$\Delta Child_t$	cst.	-5.540**	-4.570**	-6.867**	-6.640**	-6.676**
	cst. & trd.	-5.971**	-5.063**	-6.824**	-6.747**	-6.694**

Note: Results are based on ADF test. Lag order was fixed at 2 for all of the cases. * denotes rejection of the null of unit root non-stationarity at 5% level. ** denotes the rejection of the null at 1% level.

not rejected for any cases. Overall, the null of unit root non-stationarity for consumption expenditures were accepted, setting the ground for the cointegration analysis in the next subsection. In addition to consumption expenditures, we have also tested the stationarity of change in demographic factors (i.e. number of adults and children in household) over the quarter. As we will see later, the stationarity in change of demographic factors are necessary when conducting Panel cointegration analysis and Panel GMM estimation. As can be clearly seen from Table 11, the null of unit root non-stationarity has been rejected for any cases, supporting the stationarity of change in demographic factors. These preliminary results set the ground for estimation and testing in the following section.

5.3 Estimation and Test

5.3.1 Parametrization and Cross-Euler, Euler specification

In dealing with the cohort data, we slightly modify the shape of utility function to account for the socioeconomic factors - number of adults in the household and number of children in the household[21]. Specifically, each cohort's period-by-period utility is specified as follows:

$$U(L_t^h, N_t^h) = \exp(\theta_h' Z_t^h)\left[\frac{(N_t^h)^{1-\alpha^h}}{1-\alpha^h} + K\frac{(L_t^h)^{1-\gamma^h}}{1-\gamma^h}\right] \text{ for } h \in \{HS1, ..., HS5, Col1, ..., Col5\}$$

where θ_h is a vector of coefficients and Z_t^h represents the socioeconomic factors. To put it another way, we have modified the Houthakker's addi-log utility function allowing for "taste shifter." When the liquidity constraints not binding, the Euler equation for luxury and necessity goods are:

$$E_t\left[\beta^h\left(\frac{L_{t+1}^h}{L_t^h}\right)^{-\gamma^h}\exp(\theta_h'\Delta Z_{t+1}^h)(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L}\right] = 1 \tag{19}$$

$$E_t\left[\beta^h\left(\frac{N_{t+1}^h}{N_t^h}\right)^{-\alpha^h}\exp(\theta_h'\Delta Z_{t+1}^h)(1+r_{t+1})\frac{P_t^N}{P_{t+1}^N}\right] = 1 \tag{20}$$

and Cross-Euler equations can be derived as:

$$E_t\left[\beta K\frac{(L_{t+1}^h)^{-\gamma^h}}{(N_t^h)^{-\alpha^h}}\exp(\theta_h'\Delta Z_{t+1}^h)(1+r_{t+1})\frac{P_t^N}{P_{t+1}^L}\right] = 1 \tag{21}$$

$$E_t\left[\frac{\beta}{K}\frac{(N_{t+1}^h)^{-\alpha^h}}{(L_t^h)^{-\gamma^h}}\exp(\theta_h'\Delta Z_{t+1}^h)(1+r_{t+1})\frac{P_t^L}{P_{t+1}^N}\right] = 1. \tag{22}$$

Following the same procedures as in Section 3, by log-linearizing the Cross-Euler equations, we obtain the following cointegrating restrictions.

$$\ln\beta K - \gamma^h \ln L_{t+1}^h + \alpha^h \ln N_t^h + \theta_h'\Delta Z_{t+1} + \ln\left[(1+r_{t+1})\frac{P_t^N}{P_{t+1}^L}\right] \sim I(0) \tag{23}$$

$$\ln\frac{\beta}{K} - \alpha^h \ln N_{t+1}^h + \gamma^h \ln L_t^h + \theta_h'\Delta Z_{t+1} + \ln\left[(1+r_{t+1})\frac{P_t^L}{P_{t+1}^N}\right] \sim I(0). \tag{24}$$

But since constant terms and ΔZ_{t+1} are deemed stationary based on the unit root test, the above cointegration restrictions can be further transformed.

$$\ln\left[(1+r_{t+1})\frac{P_t^N}{P_{t+1}^L}\right] - \gamma^h \ln L_{t+1}^h + \alpha^h \ln N_t^h \sim I(0) \tag{25a}$$

$$\ln\left[\frac{1}{1+r_{t+1}}\frac{P_{t+1}^N}{P_t^L}\right] - \gamma^h \ln L_t^h + \alpha^h \ln N_{t+1}^h \sim I(0). \tag{25b}$$

In principle, the estimation of the preference parameters (i.e. γ^h, α^h and θ_h') can be done allowing for the heterogeneity among all cohorts. In that case, the estimation can be

[21] As Attanasio and Webber (????) claims, assuming that marginal utility of consumption is not independent from leisure, it is appropriate to account for husband and spouse job status in the socioeconomics factor. However, since we have assumed that leisure is separable from consumption, we have not included them.

implemented by single-by-single equation manner or by SUR method. However, in this paper, we are going to restrict the preference parameters to be the same among each education cohort. In other words, we restrict the preference parameters of High School cohort as $(\gamma^{HS}, \alpha^{HS}, \theta^{HS})' = (\gamma^{HS1}, \alpha^{HS1}, \theta^{HS1})' = ... = (\gamma^{HS5}, \alpha^{HS5}, \theta^{HS5})'$ and College cohort as $(\gamma^{Col}, \alpha^{Col}, \theta^{Col})' = (\gamma^{Col1}, \alpha^{Col1}, \theta^{Col1})' = ... = (\gamma^{Col5}, \alpha^{Col5}, \theta^{Col5})'$. This restriction can be motivated at least from two reasons. First, considering that time series observation per cohort is mere 61 observations, estimating preference parameters in a single equation context or even in a SUR context will be vulnerable to small sample distortion. By restricting the preference parameters to be equal among the education cohort, it will enable us to estimate the parameters in the (pseudo) panel setting offering a better small sample property. Second, among the same education cohort, the major portion of difference in the consumption pattern is probably generated by the life-cycle motives (i.e. age profile of the number of adults and children in the household). It is hard to believe that the difference in consumption pattern over time is coming from the shift in the taste parameters. Indeed, as we have already seen in Figure 2, the consumption profile adjusted by adult equivalent have revealed a considerable smoothness among the same education cohort, supporting the view that fundamental preference parameters are stable over the life-cycle.

5.3.2 Panel Dynamic OLS

As a preliminary step for the cointegration analysis, we test for the cointegration restriction implied in eq.(25a). We conducted Park's H(p,q) test for each cohort[22] and the results are presented in Table 12.

As can be seen from Table 12, the null of deterministic cointegration was not rejected for most of the test, except for older college cohorts. It is likely that the rejections are due to the large sampling errors under Col4 and Col5 whose average cohort size were mere 120 and 98, respectively. For the rest of cohorts, where average cohort size are over 200, H(p,q) tests were not able rejected the null of deterministic cointegration. Based on this result, there is good reason to believe that implications of deterministic cointegration by eq. (25a) and (25b)are holding. We proceed to estimate the cointegrating vectors, assuming that the deterministic cointegration restriction is holding.

In estimating the preference parameters from cointegrating restrictions implied in eq. (25a) and (25b), we adopt Mark and Sul's (2001) Panel Dynamic OLS (PDOLS) estimation method. PDOLS estimation method are in principle equivalent to Panel FM-OLS method, but it has an advantage of computational simplicity. Obviously, by the virtue of larger degrees of freedom in estimating the parameters, PDOLS enjoys a better small sample property compared to CCR estimation in the single equation context.

[22] Here we have applied H(p,q) test in the single-equation context. For the sake of power, it is more desirble to test the cointegrating restriction in the panel context - i.e. panel cointegration test. (Cite literature HERE.)

Table 12: Test of Cointegrating Restriction under the Null of Cointegration

	High School				
	HS1	HS2	HS3	HS4	HS5
H(0,1)	0.527	0.021	0.368	0.126	0.366
	[0.468]	[0.884]	[0.544]	[0.722]	[0.545]
H(0,2)	1.153	0.068	0.376	0.144	0.453
	[0.572]	[0.966]	[0.828]	[0.930]	[0.797]
H(0,3)	4.149	2.186	1.950	2.244	3.917
	[0.245]	[0.534]	[0.582]	[0.523]	[0.270]
	College				
	Col1	Col2	Col3	Col4	Col5
H(0,1)	1.393	0.155	0.257	4.188*	1.572
	[0.237]	[0.693]	[0.612]	[0.040]	[0.209]
H(0,2)	1.404	0.247	0.378	4.358	6.040*
	[0.495]	[0.883]	[0.827]	[0.113]	[0.046]
H(0,3)	3.282	2.375	0.947	6.040	6.270
	[0.350]	[0.498]	[0.814]	[0.109]	[0.099]

Note: Numbers in the brackets represent p-values. * denotes rejection of the null of cointegration at 5% level.

The regression forms for each education group are specified as follows..

$$\begin{aligned} & \text{High School Pooled Regression} \\ y_t^h &= const^h + \gamma^{HS} x_{1,t}^h - \alpha^{HS} x_{2,t}^h + I(0) \text{ for } h = HS1, ..., HS5 \quad (26) \\ & \text{College Pooled Regression} \\ y_t^{h'} &= const^{h'} + \gamma^{Col} x_{1,t}^{h'} - \alpha^{Col} x_{2,t}^{h'} + I(0) \text{ for } h' = Col1, ..., Col5 \quad (27) \end{aligned}$$

where $y_t^h = \ln\left[(1 + r_{t+1})P_t^N / P_{t+1}^L\right]$, $x_{1,t}^h = \ln L_{t+1}^h$ and $x_{2,t}^h = \ln N_t^h$. We pool the time series data of each cohort by education category. In order to allow for the cohort-specific fixed effect, we allow the constant term to differs among each cohort. As we have seen, in eq. (23) and (24), the constant term in each cohort's cointegrating restriction depend upon a discount factor. Thus, by fixed effect approach, we are in a sense allowing for young and old cohorts to have different discount rate depending upon their stage in life-cycle.

Table 13 shows the result of the PDOLS estimation. In this subsection, we will simply report the results of the estimates and defer interpretation until the end of this section. Estimated preference parameter on luxury goods were 0.003 for high school cohorts and 0.006 for college cohorts. The standard error for the estimator has been estimated based on Andrew's (1990) pre-whitening method. As can be inferred from the results, the parameter estimates on luxury goods are not significantly different from zero. Turning to the preference parameter on necessity goods, the estimates under high school cohorts was 0.172 and 0.169 for college cohorts. Both estimates for the necessity goods are significantly different from

Table 13: PDOLS Results for Cross-Euler Equations

$\ln\left[(1+r_{t+1})\frac{P_t^N}{P_{t+1}^L}\right]-\gamma^h \ln L_{t+1}^h+\alpha^h \ln N_t^h \sim I(0)$		
High School		
	γ^{HS}	α^{HS}
Pooled Estimates	0.003	0.172
S.E.	(0.015)	(0.022)
College		
	γ^{Col}	α^{Col}
Pooled Estimates	0.006	0.169
S.E.	(0.017)	(0.021)
$\ln\left[\frac{1}{1+r_{t+1}}\frac{P_{t+1}^N}{P_t^L}\right]-\gamma^h \ln L_t^h+\alpha^h \ln N_{t+1}^h \sim I(0)$		
High School		
	γ^{HS}	α^{HS}
Pooled Estimates	0.009	0.074
S.E.	(0.014)	(0.019)
College		
	γ^{Col}	α^{Col}
Pooled Estimates	0.013	0.073
S.E.	(0.017)	(0.020)

Note: Standard error based on Andrew's (1990) Pre-whitening method..

zero, showing a sharp contrast to the results in luxury goods.

5.3.3 Panel GMM

In this subsection, we conduct Panel GMM in estimating the preference parameters from standard Euler equations. For each of the education cohorts, standard Euler equations are specified for luxury and necessity goods. Preference parameters pertaining to each goods are estimated in the context of pooled conditional moment restrictions with fixed cross-sectional observations of 5 cohorts. Following the procedure in PDOLS estimation, we allow for the fixed effect in each cohort's discount factor. For convenience, conditional moment restrictions for each education group and goods are presented below.

High-School Conditional Moment Restrictions

Luxury Goods:

$$E_t\left[\beta^h\left(\frac{L_{t+1}^h}{L_t^h}\right)^{-\gamma^{HS}}\exp(\theta_{HS}'\Delta Z_{t+1}^h)(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L}-1\right]=0 \text{ for } h=HS1,...,HS5$$

Necessity Goods

$$E_t\left[\beta^h\left(\frac{N_{t+1}^h}{N_t^h}\right)^{-\alpha^{HS}}\exp(\theta'_{HS}\Delta Z_{t+1}^h)(1+r_{t+1})\frac{P_t^N}{P_{t+1}^N}-1\right]=0 \text{ for } h=HS1,...,HS5$$

College Conditional Moment Restrictions
Luxury Goods:

$$E_t\left[\beta^{h'}\left(\frac{L_{t+1}^{h'}}{L_t^{h'}}\right)^{-\gamma^{Col}}\exp(\theta'_{Col}\Delta Z_{t+1}^{h'})(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L}-1\right]=0 \text{ for } h'=Col1,...,Col5$$

Necessity Goods:

$$E_t\left[\beta^{h'}\left(\frac{N_{t+1}^{h'}}{N_t^{h'}}\right)^{-\alpha^{Col}}\exp(\theta'_{Col}\Delta Z_{t+1}^{h'})(1+r_{t+1})\frac{P_t^N}{P_{t+1}^N}-1\right]=0 \text{ for } h'=Col1,...,Col5$$

The choice of instrumental variables in Panel GMM were chosen according to the following principle.

- Constant term and lagged real interest rate are common for all goods and cohorts.
- For the moment restriction of goods X and cohort H, lagged growth rate of goods X of cohort H and lagged real price change in goods X are chosen as instrumental variables.

Since all cohort representative agents are forward looking and form the future expectation by exhausting all the information available at period t, life-cycle model implies that forecast errors are serially uncorrelated. Thus, when conducting panel GMM estimation, we have set the lag order of GMM disturbance terms to be zero. Although lagged instrumental variables of any order are considered to be valid since they are inside the information set at period t, we have simply adopted the lagged instrumental variables of order one[23]. Estimation of variance-covariance matrix of GMM disturbance terms was based on Andrews and Monahan's (1992) HAC estimator with truncated kernel. However, since the lag order of disturbance terms was set to zero, HAC estimator is equivalent to White's (1980) HC estimator.

The results of Panel GMM estimation is shown in Table 14. As can be seen from the results, preference parameter on luxury goods was not statistically significant for both education cohorts. Estimation from high school cohorts even showed a negative estimates. Turning to preference parameter on necessity goods, the estimates have been statistically significant for both education cohorts, though the estimated standard errors tend to be larger than those from PDOLS estimates. Probably this is partially due to the difference in the rate of convergence. Estimates of coefficient on socioeconomic factors turned out to be not robust in general. None of the coefficients were statistically significant and were

[23] This treatment is partially due to a consideration of weak correlation between the forcing terms and instruments. Our preliminary inspection (which is not reported here) revealed that correlation between forcing terms and instruments to be weaker, as lag order of instruments increased.

Table 14: Panel GMM Results: Euler Equations for Luxury and Necessity Goods

	High School					
	Luxury Goods			Necessity Goods		
	γ^{HS}	θ_1^{HS}	θ_2^{HS}	α^{HS}	θ_1^{HS}	θ_2^{HS}
Estimates	-0.0154	-0.0136	-0.0021	0.2251	0.0002	0.0002
S.E.	(0.016)	(0.022)	(0.005)	(0.108)	(0.0004)	(0.0003)
J-statistics		13.500			9.627	
P-value		[0.333]			[0.648]	
	College					
	Luxury Goods			Necessity Goods		
	γ^{Col}	θ_1^{Col}	θ_2^{Col}	α^{Col}	θ_1^{Col}	θ_2^{Col}
Estimates	0.0047	0.0259	-0.0086	0.3094	-0.0009	0.0002
S.E.	(0.008)	(0.010)	(0.008)	(0.110)	(0.001)	(0.001)
J-statistics		20.729			6.474	
P-value		[0.054]			[0.890]	

Note: Degrees of Freedom for J-statistics was 12 for all cases. θ_1 stands for the coefficient on change in numbers of adults and θ_2 stands for the coefficient on change in number of children. * denotes rejection of the null hypothesis as 5% level.

in general very close to zero. Finally, Hansen's J-statistics were not able to reject any of the over-identifying restriction of the standard Euler equations, though the call was pretty close for college cohort's luxury goods. Often, researchers have claimed non-existence of liquidity constraint solely based upon the non-rejection of the over-identifying restrictions of the standard Euler equations. There at least two arguments against this interpretation. First of all, it should be borne in mind that the null hypothesis of Hansen's J-test is that conditional moment restrictions are valid, nothing less, nothing more. In other words, the null of the test is joint in nature. Thus, although it is appropriate to claim the overall validity of standard Euler equation based on the test, it is not logical to claim the non-existence of liquidity constraint directly from there. Second, in the finite sample, there is always an issue of size distortion. It may well be the case that J-test was simply lacking the power that the null was not rejected. Apprehensive of these issues, we will not hasten to give any inference regarding the existence of liquidity constraint based on the result of J-test, but rather we propose to compare the parameter estimates from Cross-Euler and standard Euler equation in making an inference regarding the existence of liquidity constraint.

5.3.4 Testing Liquidity Constraint

In this subsection, we report the results from Cooley and Ogaki's LR type test. Once again, the motivation of this test is to check whether the parameter estimates from cointegration restriction implied by log-linearized cross-Euler equation is close enough to the estimates from standard Euler equations. Under the null that two parameter estimates are equal, the LR type test statistics (denoted QLR in this paper) is asymptotically χ^2 distributed. Following the same procedure as in Section 4, we used same instruments were used for both restricted and unrestricted GMM. For the restricted GMM, the preference parameters were restricted based on the parameter estimates from PDOLS. Two null hypothesis were tested for each education cohorts - i.e. $H_0^1 : \hat{\gamma}_{PDOLS} = \hat{\gamma}_{GMM}$ and $H_0^2 : \hat{\alpha}_{PDOLS} = \hat{\alpha}_{GMM}$. The test results are reported in Table 15.

Table 15: LR-type Test Results: Luxury and Necessity Goods

	High School	
	Luxury Goods	Necessity Goods
	$H_0 : \hat{\gamma}^{HS}_{PDOLS} = \hat{\gamma}^{HS}_{GMM}$	$H_0 : \hat{\alpha}^{HS}_{PDOLS} = \hat{\alpha}^{HS}_{GMM}$
QLR statistics	3.683	3.846*
P-value	[0.054]	[0.048]
	College	
	Luxury Goods	Necessity Goods
	$H_0 : \hat{\gamma}^{Col}_{PDOLS} = \hat{\gamma}^{Col}_{GMM}$	$H_0 : \hat{\alpha}^{Col}_{PDOLS} = \hat{\alpha}^{Col}_{GMM}$
QLR statistics	1.066	3.371
P-value	[0.301]	[0.066]

Note: * denotes rejection of the null hypothesis at 5% level.

First, let us turn to the results under the null of $H_0^1 : \hat{\gamma}_{PDOLS} = \hat{\gamma}_{GMM}$. The LR type test was not able to reject the null hypothesis for both education cohorts, though the call was close for high school cohorts. QLR was 3.683 for high school cohorts and was 1.066 for college cohorts. Next, turning test the test results under the null of $H_0^2 : \hat{\alpha}_{PDOLS} = \hat{\alpha}_{GMM}$, the LR type test rejected the null hypothesis for high school cohorts at 5% level, but was not able to reject it for college cohorts. QLR was 3.846 for high school cohorts and was 3.371 for college cohorts.

5.4 Interpretation

Based on the above estimation and test results, some interpretations are in order. First of all, we compare the preference parameter estimates from aggregate data and cohort data. While the estimates from aggregate data were, for the majority of the cases, over or close to one, to the sharp contrast, the estimates from cohort data were very close to zero. Since the parametrization was almost identical - except for the treatment in socioeconomic factor - the discrepancy can be mostly attributed to the aggregation problem, though some portion of the discrepancy is, for sure, stemming from the difference in data collection methodology. These kind of discrepancy in the estimates from aggregate data and cohort data have been widely observed by the predecessors (See Attanasio and Webber (1993, 1997)). The estimation results of this paper can be considered as, yet, another case that exemplifies the presence of aggregation problem.

Second, we observed that preference parameter estimates for both luxury and necessity goods to be extremely close to zero. Literally interpreting, the estimates implies that IES of necessity goods to be around 5 or so, and IES of luxury goods to be around 200! Comparing with the past literatures, the IES estimates of this paper are way higher than those reported in Hall (1988), Campbell and Mankiw (1989), Cooley and Ogaki (1996) and Attanasio and Webber (1997). Considering the past evidence, the IES estimates of this paper are hard to believe. One reason that may have contributed to this unaccountable estimates can be attributed to the restrictive assumption of Houthakker's addi-log utility function. Addi-log utility function assumes additive separability among goods, rendering marginal utility of one good to be independent from another. It offers a virtue of parsimony in the number of parameters when the assumption is holding, but at the same, its restrictive parametrization can mar the estimation when the assumption is not holding[24]. Thus, in the context of this paper, if luxury and necessity goods are non-separable with each other or if they are not separable from durable goods and/or leisure, IES is no longer a simple inverse of γ or α, but will be a function of luxury and necessity goods. If this is the case, parameter estimates in Section 5 are merely parameters that affect the intertemporal substitution and should not be directly linked to the interpretation of IES for each goods.

Apart from the abnormally low estimates of the preference parameters, we made one important observation regarding the existence of liquidity constraints. As have argued in Section 3, assuming that aggregation problem is treated appropriately and each agent's preference is intertemporally additive-separable, the parameter estimates from Cross-Euler equation and standard Euler equation are expected to yield a same estimates. However, as we have seen in the LR type test results in Table 15, the null hypothesis $H_0^2 : \hat{\alpha}_{PDOLS} = \hat{\alpha}_{GMM}$ has been rejected for high school cohorts. Also, we have seen that the call for the null hypothesis $H_0^1 : \hat{\gamma}_{PDOLS} = \hat{\gamma}_{GMM}$ was pretty close for high school cohorts. Under the assumptions we have made earlier, the main culprit of the rejection is the presence of liquidity constraints. Taking education level as a proxy of life-time income, the results can be interpreted as an evidence that "poor" agents are more likely to be liquidity constrained, which conforms with the finding from the aggregate data in Section 4. The test results are also consistent with the past findings by Hall and Mishkin (1982), Zeldes (1989), Meghir and Webber (1996) who report some evidence of liquidity constraint for "poor" agents. However, one should not hasten to conclude that "poor" agents are liquidity constrained, while "rich" agents are not from the test results in Section 5. Turning to the test results for college cohorts (which can be thought of as a proxy of high life-time income agents), the call for the null hypothesis $H_0^2 : \hat{\alpha}_{PDOLS} = \hat{\alpha}_{GMM}$ was very close - p-value of 0.066 - while the null hypothesis of $H_0^1 : \hat{\gamma}_{PDOLS} = \hat{\gamma}_{GMM}$ was accepted, signaling the mixed evidence whether the "rich" agent are liquidity constrained or not. This observation is does not exactly conform with

[24] The robust remedy for this defects is to rely on more flexible functional form such as translog utility function adopted by Meghir and Webber (1996). This remedy seems promising and will be kept for the future research.

the evidence reported, for instance, in Meghir and Webber (1996) that liquidity constraints are not binding for the "rich" agents, which is the standard view in the literature.

One factor, which should be borne in mind whenever interpreting the test result of this paper, is the power of the test based on Cross-Euler equation approach. The approach taken in this paper is structural and therefore inherently parametric. Moreover, considering that utility function has been tightly parametrized by the addi-log function, some sort of misspecification in utility function will probably contribute to the higher value of QLR statistics. Thus, unless the utility function is correctly specified, the LR type test will probably be more powerful than one desire to be. The general observation of high value of QLR statistics both for high school and college cohorts, may well be stemming from the misspecification of the utility function. That said, still the *relative* difference in QLR statistics between high school cohorts and college cohorts cannot be satisfactorily explained by the misspecification of the utility function. Under auxiliary assumptions that aggregation problem is treated appropriately and the agent's preference is time-separable, as far as we are concerned, it seem reasonable to call for the liquidity constraint factor to account for this relative difference in QLR statistics between high school and college cohorts. To summarize, we interpret the result of the LR type test in this section as an evidence, albeit weak, that agents with low life-time income is liquidity constrained.

6 Conclusion

In this paper, we adopted standard two goods version of the life-cycle model to study the consumption behavior of necessity goods and luxury goods under addi-log utility function which allows for the non-homothetic preference. We proposed a new empirical method in testing for the existence of liquidity constraint utilizing the concept of Cross-Euler equation. The Cross-Euler equation represents the optimal consumption pattern of a good in the current period to another good at a future period. It can be interpreted as the composite optimal condition that embeds both intertemporal and intratemporal optimal consumption relationships into one equation. The Cross-Euler equation has an advantage over the standard Euler equation, in the sense that the cointegrating relationship is maintained even when the liquidity constraint is present in the agent's decision problem. Thus, by comparing the preference parameter estimates from the Cross-Euler equation to those from the standard Euler equation, it is possible to detect the existence of a liquidity constraint.

In testing for the existence of liquidity constraints, we applied the Cross-Euler equation approach to both aggregate data and synthetic panel data constructed from Consumer Expenditure Survey. For the aggregate data, by construction, significant portion of the luxury goods expenditure comes from the richer agents in the economy, while significant portion of the necessity goods expenditure comes from the poorer agents. Since the poorer agents tend to be more vulnerable to the liquidity constraint, we expect that the Euler equation for the necessity goods to be misspecified, but the Euler equation for the luxury goods to be specified. Indeed, the empirical results presented in Section 4 of this paper supported this view. We conducted LR type test on the null hypothesis that the IES parameter estimates from the Cross-Euler equations and the standard Euler equations are equal. We rejected null hypothesis for the necessity goods frequently, while that of luxury goods was not. This empirical results implies that the Euler equation for necessity goods is misspecified, but keeps the possibility open for the Euler equation of the luxury goods to be specified - empirical evidence that the liquidity constraint is a serious factor in rendering the Euler equation to be misspecified, but only for the poor agents. This result can be thought of as empirical evidence from the aggregated time series data that supports the existence of liquidity constraint in the U.S. economy.

However, the empirical results in Section 4 were based on the aggregate time-series data, implicitly assuming the existence of the representative consumer. The existence of the representative agent requires that all the agents in the economy to share the identical and homothetic preferences, which is obviously a too strong assumption. In order to control for this aggregation problem and to legitimately test for the liquidity constraints in the economy, we conducted cohort analysis in Section 5. Based on Consumer Expenditure Survey from 1984 to 1998, the households have been classified into 10 cohorts by their age and educational attainment. Age classification was necessary because the households revealed the heterogeneous consumption patterns (i.e. life-cycle in consumption) depending on their age. Classification by education attainments was adopted as a mean to classify the households by their life-time income (i.e. permanent income). In order to compare the result with the empirical evidence from aggregate data, we aggregated the consumption goods into necessity and luxury goods. Then we estimated the preference parameters for each education cohorts exploiting the cointegrating restriction implied by the Cross-Euler equations using (pseudo) Panel Dynamic OLS estimation method. Further, preference parameters has been estimated from standard Euler equations using (pseudo) Panel GMM.

Two major features emerged from this cohort analysis. First, the preference parameter estimates from the Consumer Expenditure Survey were significantly lower than those based on aggregate time-series data. Difference in estimates can be interpreted, in some extent, as the severity of aggregation problem in the aggregate data sets. Second, the LR type test rejected the null hypothesis for high school cohorts for some cases, while the null hypothesis for college cohorts were not. Taking the educational attainment as proxy for permanent income, the test results were consistent with the view that poorer agents are more likely to be liquidity constrained. Thus, despite the large difference in estimates stemming possibly from the aggregation problem, the hypothesis that the poorer agents are more liquidity constrained was both supported by the evidence from aggregate data and from disaggregated data.

Obviously, the main drawback in this paper was an adoption of tightly parametrized addi-log utility function. It is interesting to see whether the empirical results found in this paper will also be confirmed under other functional form such as trans-log utility function in Meghir and Webber (1996) or flexible functional form proposed by Attanasio and Webber (1997). The Cross-Euler equation approach to testing for the liquidity constraints under more flexible functional form will be left for future agenda.

A Appendix 1

In this Appendix 1, we will prove the proposition that when some kind of an error is introduced to the intratemporal relationship eq. (?), then at least one of the Euler eq. (?) or (?) will be misspecified. For notational simplicity, let us define the terms in eq.(?), (?) and (?) as follows.

$$\begin{aligned} A_t &\equiv \frac{1}{K}\frac{(N_t)^{-a}}{(L_t)^{-\gamma}}\frac{P_t^N}{P_t^L} \\ B_{t+1} &\equiv \beta\left(\frac{N_{t+1}}{N_t}\right)^{-\alpha}(1+r_{t+1})\frac{P_t^N}{P_{t+1}^N} \\ \Gamma_{t+1} &\equiv \beta\left(\frac{L_{t+1}}{L_t}\right)^{-\gamma}(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L} \end{aligned}$$

Observe that under the optimization behavior of the representative agent,

$$\begin{aligned} A_t &= 1 \\ B_{t+1} &= 1+\varepsilon_{t+1}^B \\ \Gamma_{t+1} &= 1+\varepsilon_{t+1}^C \end{aligned}$$

where

$$\begin{aligned} \varepsilon_{t+1}^B &\equiv \beta\left(\frac{N_{t+1}}{N_t}\right)^{-\alpha}(1+r_{t+1})\frac{P_t^N}{P_{t+1}^N} - E_t\left[\beta\left(\frac{N_{t+1}}{N_t}\right)^{-\alpha}(1+r_{t+1})\frac{P_t^N}{P_{t+1}^N}\right] \\ \varepsilon_{t+1}^\Gamma &\equiv \beta\left(\frac{L_{t+1}}{L_t}\right)^{-\gamma}(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L} - E_t\left[\beta\left(\frac{L_{t+1}}{L_t}\right)^{-\gamma}(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L}\right] \end{aligned}$$

Before we prove the proposition, it is useful to prove the following lemma.

Lemma 1 *Let X and Y be any two random variables. Then, in general*[1],

$$E\left(\frac{X}{Y}\right) \neq \frac{E(X)}{E(Y)}.$$

Proof. Let us first observe that

$$E\left(\frac{X}{Y}\right) = E(X)\,E\left(\frac{1}{Y}\right) + Cov\left(X,\frac{1}{Y}\right).$$

We prove the lemma by way of contradiction. Suppose the statement $E(X/Y) = E(X)/E(Y)$ is true. Then the following relationship must be true.

$$E(X)\,E\left(\frac{1}{Y}\right) + Cov\left(X,\frac{1}{Y}\right) = \frac{E(X)}{E(Y)}$$

Dividing both side by $E(X)$, we get the following expression.

$$E\left(\frac{1}{Y}\right) + \frac{Cov(X,1/Y)}{E(X)} = \frac{1}{E(Y)}$$

[1] Except for the special case when $Cov\left(X,\frac{1}{Y}\right) = E(X)\left[\frac{1}{E(Y)} - E\left(\frac{1}{Y}\right)\right]$.

Now, this equation have to be true for any two random variables X and Y. In particular, let us choose the case when random variables X and Y are independent with each other. Then $Cov\,(X, 1/Y) = 0$. This implies the following equation.

$$E\left(\frac{1}{Y}\right) = \frac{1}{E\,(Y)}$$

But by Jensen's Inequality, the above equation cannot be true. A contradiction. ■

A.1 Case 1: Introducing Additive Error to A_t

Proposition 3 *When additive stationary error term is introduced to A_t, i.e. $A_t = 1 + e_t$ where $e_t \sim I(0)$, at least one moment condition of $E\,(B_{t+1}) = 1$ or $E(\Gamma_{t+1}) = 1$ will be violated.*

Proof. Suffice to show one inequality. Let us first note the following algebraic relationship

$$\begin{aligned} \Gamma_{t+1} &= \frac{A_t}{A_{t+1}} B_{t+1} \\ &= \frac{(1+e_t)}{(1+e_{t+1})}(1+\varepsilon^B_{t+1}) \end{aligned}$$

Applying conditional expectation operator $E_t\,(\cdot)$ on both side,

$$E_t\,(\Gamma_{t+1}) = (1+e_t)E_t\left[\frac{1+\varepsilon^B_{t+1}}{1+e_{t+1}}\right]$$

By lemma 1, the following inequality holds.

$$\begin{aligned} E_t\,(\Gamma_{t+1}) &\neq (1+e_t)\frac{E_t\left(1+\varepsilon^B_{t+1}\right)}{E_t(1+e_{t+1})} \\ &\neq \frac{1+e_t}{1+E_t(e_{t+1})} \end{aligned}$$

Applying unconditional expectation operator $E\,(\cdot)$ on both side,

$$\begin{aligned} E\,(\Gamma_{t+1}) &\neq E\left[\frac{1+e_t}{1+E_t(e_{t+1})}\right] \\ &\neq \frac{E(1+e_t)}{E\,[1+E_t(e_{t+1})]} = \frac{1+E(e_t)}{1+E(e_{t+1})} \end{aligned}$$

Now by the stationarity of e_t, $[1+E(e_t)]/[1+E(e_{t+1})] = 1$. Therefore, in general, $E(\Gamma_{t+1}) \neq 1$. ■

A.2 Case 2: Introducing Multiplicative Error[2] to A_t

Proposition 4 *When multiplicative error term is introduced to A_t, i.e. $A_t = e_t$ where $e_t \sim I(0)$, at least one moment condition of $E(B_{t+1}) = 1$ or $E(\Gamma_{t+1}) = 1$ will be violated.*

Proof. Suffice to show one inequality. Again, by the algebraic relationship,

$$\begin{aligned}\Gamma_{t+1} &= \frac{A_t}{A_{t+1}} B_{t+1} \\ &= \frac{e_t}{e_{t+1}}(1+\varepsilon^B_{t+1})\end{aligned}$$

Applying conditional expectation operator $E_t(\cdot)$ on both side,

$$\begin{aligned}E_t(\Gamma_{t+1}) &= e_t E_t\left[\frac{1+\varepsilon^B_{t+1}}{e_{t+1}}\right] \\ &\neq e_t \frac{E_t(1+\varepsilon^B_{t+1})}{E_t(e_{t+1})} = \frac{e_t}{E_t(e_{t+1})}\end{aligned}$$

Applying unconditional expectation operator $E(\cdot)$ on both side,

$$\begin{aligned}E(\Gamma_{t+1}) &\neq E\left[\frac{e_t}{E_t(e_{t+1})}\right] \\ &\neq \frac{E(e_t)}{E\left[E_t(e_{t+1})\right]} = \frac{E(e_t)}{E(e_{t+1})} = 1\end{aligned}$$

Thus, in general, $E(\Gamma_{t+1}) \neq 1$. ■

B Appendix 2

In this Appendix 2, we will show how to derive the Cross-Euler equations formally. For convenience, let us restate the representative agent's problem.

$$\max E_t \sum_{i=0}^{\infty} \beta^i U(N_{t+i}, L_{t+i})$$

$$s.t. \quad A_{t+1+i} = (1+r_t)A_{t+i} + Y_{t+i} - P^N_{t+i}N_{t+i} - P^L_{t+i}L_{t+i} \quad \text{for } \forall i \geq 0$$

By constructing a lifetime budget constraint from period-by-period budget constraints, we can reformulate the above optimization problem as follows.

$$\max E_t \sum_{i=0}^{\infty} \beta^i U(N_{t+i}, L_{t+i})$$

$$s.t. \quad A_t + \sum_{i=0}^{\infty}\left(\prod_{j=0}^{i}\frac{1}{1+r_{t+j}}\right)Y_{t+i} = \sum_{i=0}^{\infty}\left(\prod_{j=o}^{i}\frac{1}{1+r_{t+j}}\right)\left(P^N_{t+i}N_{t+i} + P^L_{t+i}L_{t+i}\right)$$

[2] The import demand model with preference shocks adopted by Clarida (1994) and Amano et al. (1996) is a special case of this multiplicative error.

Left-hand side of the constraint can be considered as the present value of a life-time wealth of the agent, while right-hand side of the constraint can be considered as the present value of a life-time consumption. Formulating the Lagrangian,

$$\begin{aligned}
\mathfrak{L} &= E_t \sum_{i=0}^{\infty} \beta^i U(N_{t+i}, L_{t+i}) \\
&\quad + \lambda \left[A_t + \sum_{i=0}^{\infty} \left(\prod_{j=0}^{i} \frac{1}{1+r_{t+j}} \right) Y_{t+i} - \sum_{i=0}^{\infty} \left(\prod_{j=o}^{i} \frac{1}{1+r_{t+j}} \right) \left(P_{t+i}^N N_{t+i} + P_{t+i}^L L_{t+i} \right) \right]
\end{aligned}$$

Then, the FOC's for N_t, L_t, N_{t+1}, and L_{t+1} will be as follows.

$$U_{N_t} = \lambda P_t^N \tag{B.1.}$$

$$U_{L_t} = \lambda P_t^L \tag{B.2.}$$

$$E_t \left(\beta U_{N_{t+1}} \right) = \lambda \frac{1}{1+r_{t+1}} P_{t+1}^N \tag{B.3.}$$

$$E_t \left(\beta U_{L_{t+1}} \right) = \lambda \frac{1}{1+r_{t+1}} P_{t+1}^L \tag{B.4.}$$

From eq.(B.1.) and (B.4.), we get,

$$E_t \left[\beta \frac{U_{L_{t+1}}}{U_{N_t}} (1+r_{t+1}) \frac{P_t^N}{P_{t+1}^L} \right] = 1$$

which is the Cross-Euler equation (11) in the paper. From eq.(B.2.) and (B.4.), we get,

$$E_t \left[\beta \frac{U_{N_{t+1}}}{U_{L_t}} (1+r_{t+1}) \frac{P_t^L}{P_{t+1}^N} \right] = 1$$

which is the Cross-Euler equation (12) in the paper.

C Appendix 3

In this Appendix 3, we will prove the (strict) stationarity of the forecast error embedded in the Cross-Euler equation (11). The strict stationarity of the forecast error from the Cross-Euler equation (12) is similar and therefore will be omitted.

Proposition 5 *Let N_{t+1}/N_t, r_t, and P_{t+1}^N/P_t^N be strictly stationary processes. The forecast error e_t is defined as*

$$e_t = \xi_t - E_{t-1}(\xi_t)$$

where

$$\xi_t \equiv \beta K \frac{(L_t)^{-\gamma}}{\left(C_{t-1}^N\right)^{-a}} (1+r_t) \frac{P_{t-1}^N}{P_t^L}$$

Then, $\ln(1+e_t)$ *is a strictly stationary process with* $E(e_t) = 0$ *and* $E(e_t e_{t-j}) = 0$ *for* $\forall j \neq 0$.

Proof. First, let us prove the proposition $E(e_t)$. Applying unconditional expectation operator $E(\cdot)$ on both side of $e_t = \xi_t - E_{t-1}(\xi_t)$,

$$\begin{aligned} E(e_t) &= E(\xi_t) - E\left[E_{t-1}(\xi_t)\right] \\ &= E(\xi_t) - E(\xi_t) = 0 \end{aligned}$$

Thus, $E(e_t) = 0$.
Next, let us prove the prove the proposition $E(e_t e_{t-j}) = 0$ for $\forall j \neq 0$. Without loss of generality, consider the case where $j \geq 1$. Since $E_{t-1}(e_t) = 0$ and e_{t-j} is inside the information set available at period $t-1$ for any $j \geq 1$, this will imply $E_{t-1}(e_t e_{t-j}) = 0$ for any $j \geq 1$. Applying unconditional expectation operator $E(\cdot)$ on both side of $E_{t-1}(e_t e_{t-j}) = 0$,

$$E\left[E_{t-1}(e_t e_{t-j})\right] = 0$$

$$\Rightarrow E(e_t e_{t-j}) = 0 \text{ for } \forall j \geq 1$$

Finally, let us prove the strict stationarity of $\ln(1 + e_t)$. Since the logarithmic function is a continuous and monotone function, it suffices to show the strict stationarity of e_t Recalling the definition of A_t and B_{t+1} from Appendix 1, we can observe the following algebraic relationship.

$$\xi_t = \frac{B_t}{A_{t-1}}$$

By the strict stationarity assumption of N_{t+1}/N_t, $(1 + r_t)$ and P_{t+1}^N/P_t^N, B_t is strictly stationary. Also, since $A_{t-1} = 1$ from Appendix 1, this implies the strict stationarity of ξ_t. Now, since $e_t = \xi_t - E_{t-1}(\xi_t)$ and by the strict stationarity of ξ_t, e_t will be strictly stationary. ■

References

[1] Altonji, J.G. and A. Siow (1987). "Testing the response of consumption to income change with (noisy) panel data," *Quarterly Journal of Economics* 102, 293-328.

[2] Andrews, Donald W.K (1990) "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation," *Econometrica* 59, 615-640.

[3] Andrews, Donald W.K. and J. Christopher Monahan (1992) "An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator," *Econometrica* 60, 953-966.

[4] Attanasio, O.P. (1995) "The intertemporal allocation of consumption: Theory and evidence," *Carnegie-Rochester Conference Series on Public Policy* 42.

[5] Attanasio, O.P (1999) "Consumption," *Handbook of Macroeconomics* Vol.1, J.B. Taylor and M. Woodford.eds., Amsterdam, Elsevier Science Publishers.

[6] Attanasio, O.P., J. Banks, C. Meghir and G. Webber (1999) "Humps and bumps in lifetime consumption," *Journal of Business & Economic Statistics* 17, 22-35.

[7] Attanasio, O.P. and M. Browning (1995) "Consumption over the life cycle and over the business cycle," *The American Economic Review* 85, 1118-1137.

[8] Attanasio, O.P. and G. Webber (1993) "Consumption growth, the interest rate and aggregation," *The Review of Economic Studies* 60, 631-649.

[9] Attanasio, O.P. and G. Webber (1995) "Is consumption growth consistent with intertemporal optimization? Evidence from the Consumer Expenditure Survey," *Journal of Political Economy* 103, 1121-1157.

[10] Banks, James, Richard Blundell and Arthur Lewbel (1997) "Quadratic Engel Curves and Consumer Demand," *The Review of Economics and Statistics* 129, 527-539.

[11] Blundell, R., M. Browning and C. Meghir (1994) "Consumer demand and the life-cycle allocation of household expenditures," *The Review of Economic Studies* 61, 57-80.

[12] Browning, M.A., A. Deaton and M. Irish (1985) "A profitable approach to labor supply and commodity demands over the life-cycle," Econometrica 53, 503-543.

[13] Campbell, John Y. and N. Gregory Mankiw (1989) "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence," *NBER Macroeconomics Annual* 4, 185-216.

[14] Campbell, J.Y. and N.G. Mankiw (1990) "Permanent income, current income, and consumption," *Journal of Business & Statistics* 8, 265-279.

[15] Carroll, Christopher D., (1997) “Death to the Log-Linearized Consumption Euler Equation! (And Very Poor Health to the Second-Order Approximation),” Unpublished Manuscript, Johns Hopkins University.

[16] Choi, W.G. (1999) “Asymmetric Monetary Effects on Interest Rates across Monetary Policy Stances,” *Journal of Money, Banking and Credit* 31, 386-416.

[17] Clarida, Richard H. (1994) ”Cointegration, Aggregate Consumption, and the Demand for Imports: A Structural Econometric Investigation,” *American Economic Review* 84, 298-308.

[18] Collado, M.D. (1997) “Estimating dynamic models from time series of independent cross-sections,” *Journal of Econometrics* 82, 37-62.

[19] Constantinides, George M. (1990) “Habit Formation: A Resolution of the Equity Premium Puzzle,” *Journal of Political Economy* 98, 519-543.

[20] Cooley, Thomas F. and Masao Ogaki (1996) ”A Time Series Analysis of Real Wages, Consumption, and Asset Returns: A Cointegration-Euler Equation Approach,” *Journal of Applied Econometrics* 11, 119-134.

[21] Deaton, A. (1985) “Panel data from time series of cross-sections,” *Journal of Econometrics* 30, 109-126.

[22] DeJuan, J.P. and J.J. Seater (1999) “The permanent income hypothesis: Evidence from the Consumer Expenditure Survey,” *Journal of Monetary Economics* 43, 351-376.

[23] Engle, Robert F. and C.W.J. Granger (1987) ”Co-integration and Error Correction: Representation, Estimation, and Testing,” *Econometrica* 55, 251-276.

[24] Flavin, M.A. (1981) “The adjustment of consumption to changing expectations about future income,” *Journal of Political Economy* 86, 974-1009.

[25] Gross, D.B. and N.S. Souleless (2001) “Do liquidity constraints and interest rates matter for consumer behavior? Evidence from credit card data,” NBER Working Paper No. 8314.

[26] Hall, R. and F. Mishkin (1982) “The sensitivity of consumption to transitory income: Estimates from panel data on households,“ *Econometrica* 50, 461-481.

[27] Hall, Robert E. (1988) “Intertemporal Substitution in Consumption,” *Journal of Political Economy* 96, 339-357.

[28] Hall, Alastair R. (1993) “Some Aspects Generalized Method of Moments Estimation,” *Handbook of Statistics* Vol.11, G.S. Maddala, C.R. Rao, and H.D. Vinod eds., Amsterdam, Elsevier Science Publishers.

[29] Hansen, Lars P. (1982) "Large Sample Properties of Generalized Method of Moments Estimator," *Econometrica* 50, 1029-1054.

[30] Hansen, Lars P. and K. J. Singleton (1982) "Generalized Instrumental Variable Estimation of Nonlinear Rational Expectations Models," *Econometrica* 50, 1269-1286.

[31] Hayashi, F. (1985) "The permanent income hypothesis and consumption durability: Analysis based on Japanese panel data," *Quarterly Journal of Economics* 100, 1083-1113.

[32] Heaton, John C. (1995) "An Empirical Investigation of Asset Pricing with Temporally Dependent Preference Specification," *Econometrica* 63, 681-717.

[33] Houthakker, H.S. (1957) "An International Comparison of Household Expenditure Patterns, Commemorating the Centenary of Engel's Law, " Econometrica 25, 532-551.

[34] Houthakker, H.S. (1960) "Additive Preferences," *Econometrica* 28, 244-256.

[35] Houthakker, H.S. and Lester D. Taylor (1970), *Consumer Demand in the United States: Analyses and Projections*, Second Enlarged Edition, Harvard University Press: Cambridge, MA.

[36] Kirman, A.P. (1992) "Whom or What Does the Representative Individual Represent," *The Journal of Econometric Perspective* 6, 117-136.

[37] Mankiw, N. Gregory, Julio J. Rotemberg and Lawrence H. Summers (1985) "Intertemporal Substitution in Macroeconomics," *Quarterly Journal of Economics* 100, 225-251.

[38] Mark, N.C. and D. Sul (2001) "A Computationally Simple Cointegration Vector Estimation for Panel Data," OSU Working Paper.

[39] Mariger, R.P. (1987) "A life-cycle consumption model with liquidity constraints: Theory and empirical results," *Econometrica* 55, 533-557.

[40] Meghir, C. and G. Webber (1996) "Intertemporal nonseparability or borrowing restrictions? A disaggregated analysis using a U.S. consumption panel," *Econometrica* 64, 1151-1181.

[41] Moffit, R. (1993) "Identification and estimation of dynamic models with a time series of repeated cross-sections," *Journal of Econometrics* 59, 99-123.

[42] Ogaki, Masao (1992) "Engel's Law and Cointegration," *Journal of Political Economy* 100, 1027-1046.

[43] Ogaki, Masao (1993) "Generalized Method of Moments: Econometric Applications," *Handbook of Statistics* Vol.11, G.S. Maddala, C.R. Rao, and H.D. Vinod eds., Amsterdam, Elsevier Science Publishers.

[44] Ogaki, Masao and Joon Y. Park (1998) "A Cointegration Approach to Estimating Preference Parameters," *Journal of Econometrics* 82, 107-134.

[45] Park, Joon Y. (1990) "Testing for Unit Roots and Cointegration by Variable Addition," *Advances in Econometrics* 8, 107-133.

[46] Park, Joon Y. (1992) "Canonical Cointegrating Regressions," *Econometrica* 60, 119-143.

[47] Phillips, Peter C.B. and Bruce H. Hansen (1990) "Statistical Inference in Instrumental Variables Regression with I(1) Processes," *Review of Economic Studies* 57, 99-125.

[48] Phillips, P.C.B. and H.S. Moon (2000) "Nonstationary Panel Data Analysis: An Overview of Some Recent Developments," Econometric Reviews 19, 263-286.

[49] Runkle, D.E. (1991) "Liquidity constraints and the permanent income hypothesis," *Journal of Monetary Economics* 27, 73-98.

[50] Shapiro, M. (1986) "The dynamic demand for capital and labor," *Quarterly Journal of Economics* 101

[51] Shea, J. (1995) "Union contracts and the life-cycle/permanent-income hypothesis," The American Economic Review 85, 186-200.

[52] Stoker, T.M. (1993) "Empirical approaches to the problem of aggregation over individuals," *Journal of Economic Literature* 31, 1827-1874.

[53] Verbeek, M. (1996) "Pseudo Panel Data," *The Econometrics of Panel Data*, Second Revised Edition, L. Matyas and P. Sevestre eds., Dordrecht, Kluwer Academic Publishers.

[54] Verbeek, M. and T. Nijman (1993) "Minimum MSE estimation of a regression model with fixed effects from a series of cross-sections," *Journal of Econometrics* 59, 125-136.

[55] Weise, C.L. (1999) "The Asymmetric Effects of Monetary Policy: A Nonlinear Vector Autoregression Approach," *Journal of Money, Banking and Credit* 31, 85-108.

[56] Zeldes, Stephen P. (1989) "Consumption and Liquidity Constraints: An Empirical Investigation," *Journal of Political Economy* 97, 305-346.