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Abstract

Sen's capability approach has been studied by many researchers including not only economists but also political philosophers. Almost all researchers consider capabilities as a static concept. However, D'Agata (2007) points out a need for consideration within a dynamic concept in the capability approach. D'Agata (2007) develops a model with utilization functions determined endogenously through the adaptive dynamics. Motivated by his work, we also consider the adaptive dynamics of capabilities. Our model formalized below focuses on a relationship between goods and capability for simplification. In this respect, our model seems to differ from D'Agata's one which focuses an evolution of utilization functions themselves. However, we assume implicitly that the evolution of capabilities depends on that of utilization functions. By applying Rosenbaum's formula (Rosenbaum, 2000; D'Agata, 2009a) to the dynamic context, to define well-being *freedom*, we show the existence of a distribution of goods equalizing well-being freedom of each individual, total amount of goods being fixed. Furthermore, the value of freedom is non-decreasing as the total amount of goods increases. On the other hand, by means of counter examples, we exemplify that one's well-being *achievement*, defined as in the dynamic context, *can* decrease even if he/she is given more (or equal) goods. In terms of the two-way evaluation of well-being *à la* Sen, the fact that a distribution of goods can negatively affect some individuals characterizes a normative quality of the adaptive process. Our model shows that economic distributive measures may have a limitation in improving individuals' well-being.

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1 Introduction

Since Sen's article of the title of "Equality of what ?," the capability approach, which is one of the evaluating systems of individual's well-being, has been studied by many researchers including not only economists but also political philosophers[1]. In economics, usually, individual's well-being is represented by a numerical value such as utility or income. However, Sen criticizes the utility-based or income-based approach in order to propose the concept of functionings. Furthermore, the capability approach focuses on not only his/her achievement of functionings but also freedom to choose alternatives.

Sen (1985b) evaluates an individual's advantage in terms of the four aspects which are summarized by the following table.

	well-being	agency
achievement	well-being achievement	agency achievement
freedom	well-being freedom	agency freedom

Table 1: The four aspects of an individual's advantage.

The first and second rows of table 1 represent achievement and freedom aspect of an individual, respectively. Achievement is the concept which reflects "what he/she is." Freedom is the concept which represents "what he/she can do or be." The first and second columns of table 1 are well-being and agency aspect of an individual, respectively. The agency aspect of an individual reflects an ethical point of view in a sense that an individual as agency does not always act as a well-being or utility maximizer. According to our interpretation, the agency is an advantage of the individual necessary to implement his well-being as the overall advantage. In this paper, we confine our argument to the well-being without explicitly considering the process of implementation.

Next, we refer to Sen's capability approach where an individual well-being is measured by his capability. In Sen's original formula, capability is defined as a set of functioning vectors which can be chosen by an individual. The functioning is not a mono-dimensional concept of well-being such

[1]Debates on distributive justice have been discussed since Rawls' work "A Theory of Justice." (See e.g., Arneson, 1989; Cohen, 1989; Dworkin, 1981a, 1981b; Nozick, 1974; Sen, 1980, 1985a, 1992; Rawls, 1971, and Roemer, 1996).

as utility or income but a multi-dimensional one in the form of a functioning vector. As an example of functioning, one might think of the Human Development Index (HDI) which consists of three dimensions such as life expectancy, the literacy rate and GDP per capita.

The capability is defined by the following three stages. The first stage is the commodity space. Consider an individual endowed with a set of goods. When he chooses a bundle of goods, he goes to the second stage, that is, the characteristic space by transforming the bundle of goods into a vector of characteristics by means of a characteristic function which maps goods into their desirable features (Lancaster, 1966). For example, a bicycle has transportation as its characteristic. An important point is that the characteristics of goods are determined in an objective manner. Whether the owner of a bicycle is a disable or not, it has transportation as a feature in itself. Then, he goes to the third stage, which is called the functioning space in Sen's term, by transforming a vector of characteristics into a vector of functionings by means of a utilization function which is different from person to person. According to his life plan, he chooses a utilization function and then obtains corresponding functionings. Lastly, the capability can be seen as a set of possible vectors of functionings to be chosen by an individual. Following Sen (1996), we interpret the value of chosen functionings as well-being achievement and that of the capability as well-being freedom, which are depicted in table 1.

Almost all researchers consider the capability as a static concept. Their models do not take the time aspect into consideration. As the first attempt to consider the capability approach in a dynamic context, D'Agata (2007) develops a model with utilization functions determined endogenously through the adaptive dynamics. In the adaptive dynamics, the capability at time $t+1$ depends on the chosen functionings at time t. Our model formalized below builds on D'Agata's pioneering model, where we pay more attention to a relationship between goods and capability.

The aim of this paper is to assess critically the concept of distributive justice and a normative quality of the adaptive process. Before entering into a main argument, in section 2, we review briefly D'Agata (2007) in order to contrast with our model and add some remarks to justify adopting Rosembaum's measure. In section 3, we define the well-being freedom which is the concept proposed by Sen (1985b), by applying Rosenbaum's formula to the dynamic context. We show the existence of a distribution of goods equalizing well-being freedom of each individual with the total amount of goods being fixed. Furthermore, the value of freedom is non-decreasing as the total amount of goods increases. In section 4, we define well-being achievement, contrary to section 3, by means of counter examples, we exemplify that one's well-being achievement can decrease even if he/she is given more (or equal) goods under the similar conditions. Section 5 is concluding remarks.

2 D'Agata's adaptive dynamic model of capability

Let us start by briefly reviewing D'Agata's article in 2007, where he provides a formal treatment of the capability approach within a dynamic framework. According to D'Agata (2007), this attempt is justified by Sen himself. He points out that, in Sen's original formulation of capabilities (Sen, 1985a), utilization functions are function of time-dependent variables of functionings such as age, health (see D'Agata, 2007, p.181, footnote 4). Thus, it is said to be natural to assume that not only utilization functions but also a characteristic function change over time. In D'Agata's model, he assumes that, under a given bundle of commodity, utilization functions are determined endogenously through the adaptive dynamics, but the characteristic function does not change over time. The adaptive dynamics is formalized as follows. At the beginning of time 0, one chooses a utilization function which maximizes his valuation of each functioning over a given initial capability set. During time 0, by employing the utilization function, he develops his ability (i.e., new utilization functions) "around" the used utilization function. At the beginning of time 1, he obtains a new capability set and then chooses a utilization function in the same manner. And so on. As time goes by, we obtain a sequence of utilization functions chosen each time. D'Agata investigates mathematical properties of the sequence of utilization functions; he shows the existence of limit points of the sequence of utilization functions under certain conditions and provides conditions ensuring their uniqueness. Furthermore, he critically investigates Sen's following view: if two individuals are exactly the same, their equality in one focal variable space coincides with their equality in other focal variable spaces. By means of numerical examples, he argues that Sen's view is not always valid. That is, even if two individuals are exactly the same, their functioning may be different according to their own choices; an important insight we are starting from (apart from his negative view of Sen).

Since D'Agata is devoted to an evolution of utilization functions, he does not explicitly focus on the concept of distributive justice (i.e., equality of capabilities). In order to consider distributive justice properly, our model pays attention to a relationship between goods and capability, as opposed to his model which focuses only on an evolution of utilization functions themselves. Although our model seems to differ from D'Agata's one at first sight, we assume implicitly as well that the evolution of capabilities depends on that of utilization functions. Our model can been seen rather as an extension of D'Agata's one in the following respects. Firstly, in order to focus on distributive justice, we consider n individuals in an economy. Secondly, while the amount of goods is fixed in his model, we view it as variable (see below corollary 1 in section 3, and section 4), to clarify a relationship between the well-being freedom and goods, and a relationship between the well-being achievement and goods. Thirdly, we focus on both the aspect of the well-being achievement and the aspect of the well-being freedom, while D'Agata considers only the former. Thus, our model provides a broader framework compared to D'Agata's one.

We would like to make some additional remarks here to justify adopting Rosenbaum's measure in our paper. As we mentioned above, the capability approach has been scrutinized by many researchers. While in empirical studies, capabilities have been measured by using statistical data (Kuklys, 2005; Anand and van Hees, 2006), in theoretical studies, especially the social choice theory, a strand of researches focus on a ranking rule of opportunity sets, in particular of capability sets, which is each deduced by certain plausible axioms (Pattanaik and Xu, 1990; Sen, 1991; Klemisch-Ahlert, 1993; Bossert, Pattanaik and Xu, 1994; Puppe, 1995; Pattanaik and Xu, 1998; Sugden, 1998; van Hees, 1998; Pattanaik and Xu, 2000; Barberà, Bossert and Pattanaik, 2004, and Xu, 2008). For example, we might introduce the number of elements of a set as such a measure. The measure is called the cardinality criterion, which is characterized axiomatically by Pattanaik and Xu (1990). It is very simple and easy to understand intuitively, but difficult to apply to an infinite set. D'Agata (2009) investigates four plausible measures including the cardinality criterion and then examines critically whether they lead to counter-intuitive results or not. The other three measures are the social freedom measure (Steiner, 1994; Carter, 1999), the social-cardinal freedom measure (Kramer, 2003) and Rosenbaum's measure (Rosenbaum, 2000). He interprets the first two measures of the three as corollaries of the cardinal criterion by pointing out that the unit of social freedom measure and that of social-cardinal freedom measure can be reduced to that of the cardinal criterion. This would imply that the three measures except Rosenbaum's measure are only applicable to a finite set. In this sense, we consider Rosenbaum's measure as a first candidate of the specific measure of freedom which can be defined on infinite sets.

D'Agata, on the other hand, points out that Rosenbaum's measure leads to a counter-intuitive result. Rosenbaum's measure is defined as the ratio between maximal distance between two elements belonging to the set of socially technical feasible actions and maximal distance between two elements elements belonging to the set of free actions. Consider any square set and another set which consists only of its diagonal. In this case, the value of the square set measured *à la* Rosenbaum is the same as the value of the set of its diagonal. This fact yields a counter-intuitive result because of violation of so-called Kramer's rejection rule, that is, the rule which rejects any measure that gives the same degree of freedoms to any two sets such that the one is a proper subset of the other. However, as we see below, in our model, such a counter-intuitive result does not occur due to assumptions of no sudden shrinkage and comprehensiveness of capabilities (see below assumption 1 (ii) and (vi)). Thus, the rejection rule does not hinder us from adopting Rosenbaum's formula, as first approach, to measure capability sets.

3 Freedom of well-being

To begin with, we introduce some preliminary notations and definitions. Let $N = \{1, \ldots, n\}$ be the finite set of individuals in an economy and the cardinality of N, $|N|$ be the number of individuals. The economy is endowed with a certain amount of goods. We denote the total amount of goods in the economy as non-negative real number $X \in \mathbf{R}_+$. In most (though not all) parts of this paper, we assume a real number X to be fixed. Let x_i be an amount of goods distributed to individual i. Then, the sum of each x_i is the total amount of goods in the economy, that is, $\sum_{i=1}^{n} x_i = X$. Let m be the number of functionings. A functioning space stated above is defined on the set of non-negative m-dimensional real space $\mathbf{R}_+^m$. Let $\mathbf{y} \in \mathbf{R}_+^m$ be a vector of functionings and $Y \subset \mathbf{R}_+^m$ be a set of technically possible functioning vectors. Let $Q_i \subset \mathbf{R}_+^m$ be a capability of individual i. In this paper, we introduce time into a capability. Let us put a superscript t on Q_i. Q_i^t represents a capability of individual i at time t. Following Sen (1985a), we define a valuation function of individual i as a function $v_i : \mathbf{R}_+^m \to \mathbf{R}$.

We consider the well-being freedom, in other words, a individual's capability, in the dynamic context. For this purpose, we provide a model which links the capability approach with the adaptive model (see e.g., Atkinson and Stiglitz, 1969; Day and Kennedy, 1970; Cherene, 1978; D'Agata, 2005, 2009b). Then, we investigate the existence of an initial distribution of goods equalizing each capability in the economy. To begin with, we formally define an initial capability as follows.

Definition 1. (Initial capability)
Firstly, we define a correspondence $q_i : \mathbf{R}_+ \to Y, x_i \mapsto q_i(x_i)$. The image of x_i, $q_i(x_i)$, is interpreted as the initial capability of individual i with an amount of goods x_i, that is, $Q_i^0 = q_i(x_i)$.

We assume the following properties of the initial capability.

Assumption 1.
The correspondence q_i has the following six properties:

(i) compact valued,

(ii) continuous,

(iii) $q_i(0) = \{\mathbf{0}\}$,

(iv) $q_i(x_i^{'}) \subset q_i(x_i)$ for $x_i^{'} < x_i$,

(v) $\exists x_i \in \mathbf{R}_+, q_i(x_i) = Y$,

(vi) $\forall x_i,\ q_i(x_i)$ is comprehensive.

Property (i) means that the initial capability is bounded and closed. Property (ii) means that the initial capability does not expand or shrink suddenly as the amount of goods changes. Property (iii) means that the initial capability contains nothing but null vector whenever an individual has no goods. Property (iv) means the monotonicity of the initial capability with respect to the amount of goods. Property (v) means that there is the amount of goods at which the individual can reach the technically feasible set. Property (vi) means that the initial capability contains segments between any point of boundary and the origin.

Next assumptions concern a set of technically possible functioning vector.

Assumtion 2.
Y is compact, $\mathbf{0} \in Y$ and $Y \setminus \{\mathbf{0}\} \neq \emptyset$.

The boundedness of Y means that no one grows limitlessly throughout one's life. The closedness of Y means that every goal of growth is feasible.

The literature on the adaptive dynamic model listed above emphasize bounded rationality of individuals rather than perfect foresight. Individuals behave locally with knowledge based on the past behavior. The following definition means such individuals.

Definition 2. (Evolution of capability)
For all i, the capability at time $t+1$ ($t = 0, 1, \cdots$), Q_i^{t+1}, depends on the

chosen functioning vector at time t, $\mathbf{y}_i^t$, in the manner of a correspondence $\psi : Y \to Y, \mathbf{y} \mapsto \psi(\mathbf{y})$, that is, $Q_i^{t+1} = \psi(\mathbf{y}_i^t)$.

Assumption 3.
The correspondence ψ has the following four properties:

(i) compact valued,
(ii) continuous,
(iii) $\psi(\mathbf{0}) = \{\mathbf{0}\}$,
(iv) $\mathbf{y} \in \psi(\mathbf{y})$ for all $\mathbf{y} \in Y$.

Properties (i), (ii), and (iii) of assumption 3 correspond to those of assumption 1, respectively. Property (iv) means that, whichever we chose any functioning vector at time t of Y, the corresponding image at time $t+1$ necessarily contains the chosen functioning vector at time t.

Next definition concerns the principle of individuals' behavior. Each time, individuals with limited knowledge act as locally maximizer. Then, we get a path of the chosen functionings generated by locally maximizing behavior of each time. We call the path adaptive process.

Definition 3. (Adaptive process)
Adaptive process $\{\bar{\mathbf{y}}_i^t\}_{t \in \mathbf{N}_0}$ is defined by $\bar{\mathbf{y}}_i^t = \mathrm{argmax}_{\mathbf{y} \in Q_i^t} v_i(\mathbf{y})$ and $Q_i^{t+1} = \psi(\bar{\mathbf{y}}_i^t)$ for $t \in \mathbf{N}_0$.

Definition 4. (Feasible set of functionings)
Let $\psi^t(Q_i^0) = \psi^t(q_i(x_i)) := \cup_{\mathbf{y} \in \psi^{t-1}(q_i(x_i))} \psi(\mathbf{y})$ $(t = 1, 2, \cdots)$ with $\psi^0(q_i(x_i)) := q_i(x_i)$, then we define a correspondence, $Y_i^t : \mathbf{R}_+ \to \mathbf{R}_+^m, x_i \mapsto Y_i^t(x_i)$, in the following manner:

$$Y_i^t(x_i) := \overline{\psi^t(Q_i^0)} = \overline{\psi^t(q_i(x_i))},$$

where $\overline{\psi^t(Q_i^0)}$ means a closure of $\psi^t(Q_i^0)$. $Y_i^t(x_i)$ is called the feasible set of functionings of individual i at time t.

Remark 1.
Readers may be a little bit difficult to understand definition 4 intuitively. To understand definition 4 clearer, recall the principle of individuals' behavior in our model, locally maximizing behavior. The concept of a feasible set of functionings is not limited to this maximizing behavior. We suppose any possible principles of individuals' behavior whatever we think of, which is

indeed relevant to the concept of freedom. Given an initial capability and a principle of individuals' behavior, individuals would not need to choose a functioning vector maximizing their own valuations. Hence, we could get paths different from the adaptive process. Then, we would also get the sequence of capabilities associated with each of them at each time. By doing so, we could take a union of capabilities over all possible paths at each time. Thus, definition 4 means that the union generated by the above procedure is defined as a feasible set of functionings at each time. Therefore, Q_i^t is a subset of Y_i^t for each t.

Definition 5. (Temporal measure of freedom)
Applying Rosenbaum's formula to the dynamic context, we define the temporal measure of freedom of individual i at time t as:

$$\varphi_i^t = \varphi_i^t(x_i) := \frac{\max_{\mathbf{y}_1,\mathbf{y}_2 \in Y_i^t(x_i)} |\mathbf{y}_1 - \mathbf{y}_2|}{\max_{\mathbf{y}_1,\mathbf{y}_2 \in Y} |\mathbf{y}_1 - \mathbf{y}_2|}.$$

Definition 6. (Measure of freedom)
The measure of freedom of individual i is defined as:

$$\varphi_i = \varphi_i(x_i) := \lim_{t \to \infty} \varphi_i^t(x_i) = \frac{\lim_{t\to\infty} \max_{\mathbf{y}_1,\mathbf{y}_2 \in Y_i^t(x_i)} |\mathbf{y}_1 - \mathbf{y}_2|}{\max_{\mathbf{y}_1,\mathbf{y}_2 \in Y} |\mathbf{y}_1 - \mathbf{y}_2|}.$$

Remark 2.
The measure of freedom of individual i, $\varphi_i(x_i)$, is well defined. Clearly, the denominator in the above formula has a maximum value and is not zero (see assumption 2). For all $t \in \mathbf{N}_0, i$ and x_i, $Y_i^t(x_i)$ is compact. Especially, given i and x_i, a feasible set of functionings at time $t+1$ contains that at time t, that is, $Y_i^{t+1}(x_i) \supset Y_i^t(x_i)$ (see assumption 3 (iv) and definition 4). Thus, $\max_{\mathbf{y}_1,\mathbf{y}_2 \in Y_i^t(x_i)} |\mathbf{y}_1 - \mathbf{y}_2|$ is monotonously non-decreasing with respect to t. Also, it is bounded because of the compactness of Y. Therefore, $\max_{\mathbf{y}_1,\mathbf{y}_2 \in Y_i^t(x_i)} |\mathbf{y}_1 - \mathbf{y}_2|$ is convergent as $t \to \infty$.

Before proving proposition 1, we prove the following lemmata.

Assumption 4
For all i, function φ_i^t is *uniformly* convergent to φ_i.

Lemma 1.
Function φ_i is continuous.

proof.
As is known, if $f : A \to B, g : B \to C$ are two continuous correspondences, then the composite correspondence $g \circ f : A \to C, a \mapsto g \circ f(a), g \circ f(a) := \cup_{b \in f(a)} g(b)$ is also continuous. By definitions, correspondences q_i and ψ as well as the function $|\cdot| : \mathbf{R}^m \times \mathbf{R}^m \to \mathbf{R}_+, (\mathbf{y}_1, \mathbf{y}_2) \mapsto |\mathbf{y}_1 - \mathbf{y}_2|$ are each continuous, therefore for all i and t, $\{|\mathbf{y}_1 - \mathbf{y}_2| | \mathbf{y}_1, \mathbf{y}_2 \in Y_i^t(x_i)\}$ is continuous with respect to x_i, which implies that $\varphi_i^t(x_i)$ is a continuous function of x_i. Since φ_i^t is uniformly convergent to φ_i, φ_i is continuous. ■

Lemma 2.
$Y_i^t(x_i)$ and $\varphi_i(x_i)$ is monotonously non-decreasing with respect to x_i.

proof.
Obvious. ■

Proposition 1.
There exists an initial distribution of goods $(x_1, x_2, \ldots, x_n)$ so that $\varphi_1(x_1) = \varphi_2(x_2) = \cdots = \varphi_n(x_n) =: \varphi(X)$ with $\sum_{i=1}^n x_i = X$.

proof.
According to lemma 1, for all i, $\varphi_i(x_i)$ is continuous with $\varphi_i(0) = 0$ and $\varphi_i(x_i) = 1$ for sufficiently large x_i because of Assumption 1 (v). Therefore, for every $\varphi \in [0, 1]$, there exists at least one value of x_i fulfilling $\varphi_i(x_i) = \varphi$ because of the intermediate value theorem. Define $x_i^+(\varphi) := \max\{x_i | \varphi_i(x_i) = \varphi\}$, $x_i^-(\varphi) := \min\{x_i | \varphi_i(x_i) = \varphi\}$, $X^+(\varphi) := \sum_{i=1}^n x_i^+(\varphi)$ and $X^-(\varphi) := \sum_{i=1}^n x_i^-(\varphi)$. Note that $X^+(\varphi)$ and $X^-(\varphi)$ are monotonously increasing with $X^-(0) = 0$ and $X^+(1) = \infty$. Then, (i) for any $\bar{X} \in [0, X^-(1))$, there exist a unique $\bar{\varphi} \in [0, 1]$ and a unique $\bar{\lambda} \in [0, 1]$ so that $\bar{\lambda} X^-(\bar{\varphi}) + (1 - \bar{\lambda}) X^+(\bar{\varphi}) = \bar{X}$ is valid. Choose $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n$ so that $\bar{x}_i = \bar{\lambda} x_i^-(\bar{\varphi}) + (1 - \bar{\lambda}) x_i^+(\bar{\varphi})$ for $i = 1, 2, \ldots, n$. Then, we obtain $\varphi_1(\bar{x}_1) = \cdots = \varphi_n(\bar{x}_n) = \bar{\varphi} = \varphi(\bar{X})$ with $\sum_{i=1}^n \bar{x}_i = \bar{X}$. (ii) In case of $\bar{X} \geqq X^-(1)$, there exists a unique $\bar{\mu} \geqq 1$ so that $\bar{\mu} X^-(1) = \bar{X}$ is valid. Choose $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n$ so that $\bar{x}_i = \bar{\mu} x_i^-(1)$ for $i = 1, 2, \ldots, n$. Then, we obtain $\varphi_1(\bar{x}_1) = \cdots = \varphi_n(\bar{x}_n) = 1 = \varphi(\bar{X})$ with $\sum_{i=1}^n \bar{x}_i = \bar{X}$. ■

Note that $x_i^-(\bar{\varphi}) = x_i^+(\bar{\varphi})$ implies $\bar{x}_i = x_i^-(\bar{\varphi}) = x_i^+(\bar{\varphi})$.

Corollary 1.
$\varphi(X)$ is monotonously non-decreasing.

proof.
Since $X^+(\varphi)$ and $X^-(\varphi)$ are monotonously increasing, $\varphi(X)$ is monotonously non-decreasing. ■

4 Achievement of well-being

In this section, we provide two examples of the well-being achievement. As we proved above, the well-being freedom is monotonously non-decreasing with respect to the amount of goods. Likewise, we are likely to speculate on the well-being achievement in the manner that. That is to say, the well-being achievement of each individual cannot decrease as the quantity of goods distributed to them increases. However, under the same conditions as section 3, our examples suggest that there can exist an individual whose well-being achievement decreases suddenly at one point of distribution. Some people may take our examples as arbitrary ones. We do not have to consider them to be the expression of a reality, but a logically counter-example to the intuition that one's well-being achievement increases if he/she is given more goods.

Assumption 5.
Valuation function $v_i(\mathbf{y})$ is continuous for all i.

Definition 7. (Global maximum)
Global maximum of individual i (GM_i) is defined as $GM_i := \max_{\mathbf{y}\in Y} v_i(\mathbf{y})$.

Definition 7 says that, given Y and v_i, the global maximum of individual i is defined as an achievable maximum value of v_i on Y.

Definition 8. (Well-being achievement)
The well-being achievement of individual i is measured by the adaptive process $\{\bar{\mathbf{y}}_i^t\}_{t\in\mathbf{N}_0}$ (see definition 3) as follows:

$$\alpha_i = \alpha_i(x_i) := \lim_{t\to\infty} v_i(\bar{\mathbf{y}}_i^t).$$

Given v_i, we can obtain a sequence of valuation of individual i associated with his/her adaptive process. We define the well-being achievement as a limit of this sequence.

The following examples are the simplest cases that satisfy the assumptions we stated above. The difference between two examples we intend to emphasize is the number of a point of contact between a valuation function and an initial capability; just one point in example 1 and an infinite point in example 2.

Example 1.
We consider a representative individual i and define Y, v_i, q_i and ψ as
$Y := \{(y_1, y_2)|\ 0 \leqq y_1 \leqq 12, 0 \leqq y_2 \leqq 12\}$,
$v_i(y_1, y_2) := 2y_1 + y_2$,
$q_i(x_i) := \{(y_1, y_2)|\ 0 \leqq y_1 \leqq \min\{\frac{3}{2}x_i, 12\}, 0 \leqq y_2 \leqq \min\{\frac{3}{4}x_i^2, 12\}\}$ and

$$\psi(\tilde{y}_1, \tilde{y}_2) := \begin{cases} \{(y_1, y_2)|\ 0 \leqq y_1 \leqq \min\{\tilde{y}_1 + |\tilde{y}_1 - \tilde{y}_2|, 12\}, 0 \leqq y_2 \leqq \min\{\tilde{y}_2, 12\}\} & \text{for } \tilde{y}_1 \geqq \tilde{y}_2, \\ \{(y_1, y_2)|\ 0 \leqq y_1 \leqq \min\{\tilde{y}_1, 12\}, 0 \leqq y_2 \leqq \min\{\tilde{y}_2 + |\tilde{y}_1 - \tilde{y}_2|, 12\}\} & \text{for } \tilde{y}_2 \geqq \tilde{y}_1, \end{cases}$$

which are depicted in figure 1. The initial capability of individual i is given as a rectangle. As the amount of goods increases, the initial capability expands while it's northeast apex moving along $y_2 = \frac{3}{4}y_1^2$ up to the point (6,12) and then moving along the ceiling from (6,12) to (12,12). Clearly, for each x_i, v_i touches the northeast apex of the initial capabilities at one point. In case of a point where the first component is greater than the second, the northeast apex of the initial capabilities evolves to the right side until achieving the east side of Y. In case of the opposite, the point of apex evolves upward until achieving the ceiling of Y. So, the set of every achievable point by starting from the initial capability consists of the thick parts of the ceiling and the right side of Y , the origin and the point (3,3) which do not move. We can summarize these observations as figure 2. According to this figure, the well-being achievement of individual i increases as the amount of goods increases up to the neighborhood of 2. Yet, her well-being achievement decreases suddenly when she gets 2. Furthermore, even though she gets more on the interval (2,5), her well-being achievement is less than 27 at $2 - \varepsilon$ (see table 2).

x_i	0	$\cdots$	$2-\varepsilon$	2	$2+\varepsilon$	$\cdots$	5	$\cdots$	8	$\cdots$
α_i	0	$\nearrow$	27	9	18	$\nearrow$	27	$\nearrow$	36	$\rightarrow$

Table 2: The relationship between x_i and α_i.

Insert figure 1 and 2 here.

Example 2.
This is another extreme case. Definitions of Y and $\psi(\tilde{y}_1, \tilde{y}_2)$ are the same

as those of example 1. We redefine v_i and q_i as
$v_i(y_1, y_2) := y_1 + y_2$,
$q_i(x_i) = Y \cap \{(y_1, y_2)|\ y_1 + y_2 \leqq x_i\}$,
which are depicted in figure 3. The initial capability of individual i is given as a southwest area of a dotted line for each x_i. Since a slope of v_i corresponds that of each dotted line, there exists an infinite number of maximum valuation points. Consider $q_i(16)$ and then choose any point on the corresponding dotted line. If the first component of the point is strictly greater than the second, the chosen point evolves to the right side until arriving at the east wall of Y according to ψ. In case of the opposite, the point evolves up to the ceiling of Y. If the first of the point equals the second, the chosen point stays here. Every achievable point from the dotted line of $q_i(16)$ consist of the fine dotted parts on the edge of Y and point (8,8). We can also consider cases of $q_i(12)$ and $q_i(4)$ in the same manner. Then, we obtain a mountain-shaped symmetrical curve for each initial capability depicted in figure 4. If we look at the highest mountain-shaped symmetrical curve, which corresponds to $q_i(16)$ where α_i is defined on the interval [4,12], we can find that it decreases suddenly at 8. We can also find out the same feature in the second highest mountain-shaped curve which corresponds to $q_i(12)$ and the lowest mountain-shaped curve which corresponds to $q_i(4)$. A point we would like to emphasize is that the well-being achievement with more amount of goods can be lower than that with less amount of goods. If one chooses $\mathbf{y}_i = (8, 8)$ under $q_i(16)$, then $\alpha_i = 16$. However, according to figure 4, under $q_i(12)$ which is associated with less amounts of goods compared to $q_i(16)$, she can obtain more well-being achievement by choosing any point in $\{(y_1, y_2)|y_1 + y_2 = 12, y_1 \in (4, 8) \setminus \{6\}\}$. Furthermore, this fact provides us with a remarkable matter. In our model, each individual is characterized by a triple (x_i, q_i, v_i). So, example 2 also suggests that, even if individuals are completely identical with respect to a triple (x_i, q_i, v_i), each well-being achievement can vary largely according to each functioning to be chosen by them; moreover, even the same individual can increase or decrease her well-being achievement depending on the chosen functioning even if she gets the same amount of goods.

Insert figure 3 and 4 here.

5 Concluding remarks

Freedom as a main constituent of the individual well-being has been widely acknowledged. Thus, when thinking about the individual well-being, we focus on both achievement aspect and freedom one. In this paper, on the basis of the two-way evaluation of well-being *à la* Sen, different aspects of the well-being are distinguished. Under these aspects, we constructed a model of the capability approach through the adaptive dynamics, as opposed to the conventional static treatment of the capability approach. Then, following Sen's original idea on distributive justice, we could show the existence of a distribution of goods equalizing well-being freedom (i.e., capabilities) in terms of Rosenbaum's measure. Furthermore, by dealing with the distributed amount of goods as variable, which is different from D'Agata's model, we presented the monotonicity of the well-being freedom with respect to the amount of goods. This fact is a very plausible consequence because it is difficult to understand that one's well-being freedom would not increase even though she gets more. What about the well-being achievement? Our examples lead to a counter-intuitive result. In other words, one's well-being achievement can decrease even if he/she gets more under the same conditions of the well-being freedom. Furthermore, even if individuals are completely identical with respect to a triple (x_i, q_i, v_i), each well-being achievement can change a lot depending on the course of functioning to be chosen.

What does it mean? The fact that an additional distribution of goods can negatively affect some individuals can characterize a normative quality of the adaptive process, that is, in the adaptive process, economic distributive measures may have a limitation in improving the individuals' well-being. It is necessary for the capability approach to be put into a broader perspective considering not only economic measure but also other supplementations such as communication, cooperation and so on. In this respect, our view is shared with Sen. "I shall consider the following types of instrumental freedoms: (1) *political freedom*, (2) *economic facilities*, (3) *social opportunities*, (4) *transparency guarantees* and (5) *protective sequlity.*"(Sen, 1999, p.38, italic in original.)

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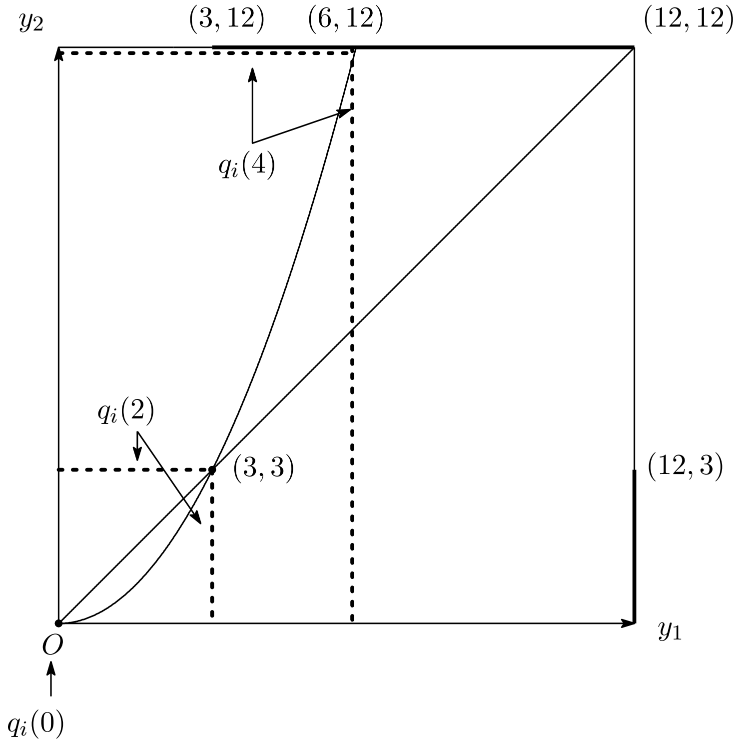

Figure 1: Dynamics of the well-being achievement in example 1.

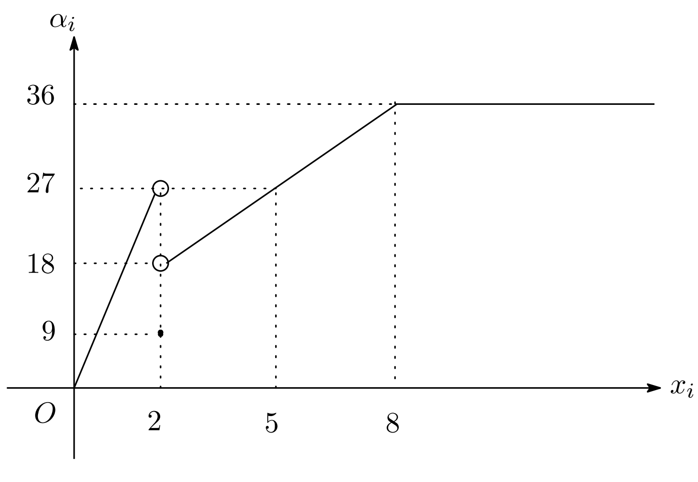

Figure 2: The relationship between x_i and α_i.

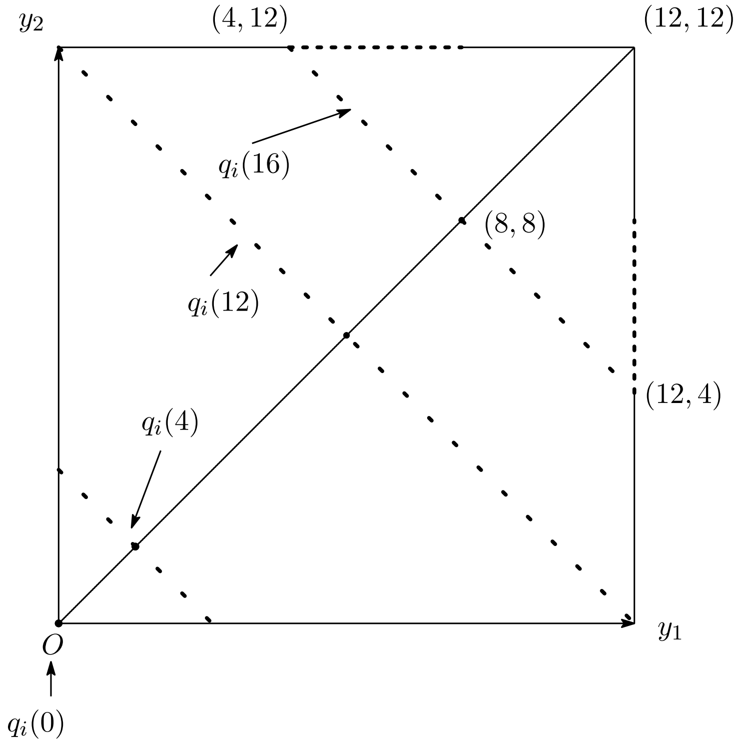

Figure 3: Dynamics of the well-being achievement in example 2.

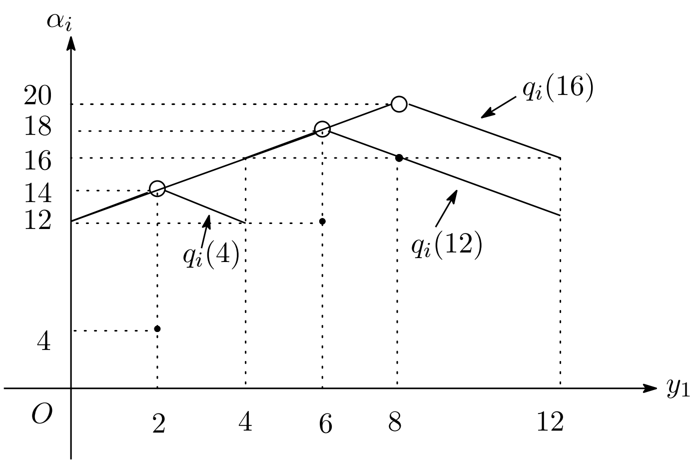

Figure 4: The relationship between y_1 and α_i.