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Abstract

This paper presents a Kaleckian growth model that considers elements from Goodwin and Marx. The model has a system of differential equations for the rate of utilization, the profit share, and the rate of employment. We show that cyclical fluctuations occur depending on the sizes of the reserve-army effect and the reserve-army-creation effect. Moreover, we show that if the stable long-run equilibrium corresponds to the profit-led growth regime, an increase in the bargaining power of workers increases the rate of unemployment; on the other hand, if the equilibrium corresponds to the wage-led growth-regime, an increase in the bargaining power of workers decreases the rate of unemployment.

Keywords: Goodwin-Kalecki-Marx model; cyclical growth; reserve-army-creation effects; Hopf bifurcation

JEL Classification: E12; E24; E25; E32; O41

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1 Introduction

This paper presents a Kaleckian model of cyclical growth that introduces elements from Goodwin and Marx. Using the model, we investigate how the rate of capacity utilization, the profit share, and the rate of employment are determined, and also analyze the stability of the long-run equilibrium.

Thus far, a number of models of cyclical growth have been developed. The point of departure is Goodwin (1967), in which closed orbits around the equilibrium are obtained with regard to the rate of employment and the wage share. Goodwin's model is considered to capture the class-conflict over the income distribution between workers and capitalists. The structure of Goodwin's model is relatively simple, and as such, the model has been modified in many different ways.[1)]

Shah and Desai (1981) introduce endogenous technical change into Goodwin's model. They assume that the growth rate of labor productivity and that of the capital-output ratio are endogenously determined by induced technical change. Shah and Desai's model is a three-dimensional model with regard to the rate of employment, the wage share, and the capital-output ratio. Unlike the original Goodwin model, the equilibrium is locally stable and cyclical fluctuations are not obtained. van der Ploeg (1987) modifies Shah and Desai's (1981) model to allow for different savings rates between wages and profits, and the effects of productivity growth on the wage-bargaining equation. van der Ploeg's model is a three-dimensional model with regard to the rate of employment, the wage share, and the capital-output ratio. Using the Hopf bifurcation theorem, he shows that there exists a stable limit cycle.

The abovementioned models, like Goodwin's model, are classical in the sense that savings determine investment, and accordingly, do not consider the problem of effective demand. In addition, the determination of income distribution is open to question. In these

1) Modifications of Goodwin's (1967) model are as follows. Pohjola (1981) presents a discrete-time version of Goodwin's model, in which a non-linearfirst-order difference equation of the rate of employment is derived,and shows that chaotic behavior occurs. Wolfstetter (1982) presentsa three-dimensional model with respect to the rate of employment,the wage share, and the rate of capacity utilization. In Wolfstetter'smodel, the equilibrium is either stable or unstable, and cyclicalfluctuations are not obtained. Sato (1985) develops a two-sector Goodwin's model with a consumption goods producing sector and a capital goods producing sector. Sportelli (1995) presents a modified Goodwin model that generates a stable limit cycle. Choi (1995) introduces the efficiency wage hypothesis into Goodwin's model, and shows that if the effort level depends positively on the real wage rate, the equilibrium is stable, whereas if the effort level depends negatively on the real wage rate, cyclical fluctuations occur. Foley's (2003) model can be said to be a modified Goodwin model, in which endogenous technical change and factor substitution are considered. Ryzhenkov (2009) constructs a three-dimensional Goodwin model to analyze the Italian economy.

models, the equilibrium wage share is determined by only the shape of the innovation possibility frontier—only technological factors determine income distribution. Hence, other factors such as the bargaining between workers and capitalists never affect income distribution.

In contrast to these models, there are several models of cyclical fluctuations that consider effective demand, especially investment demand. Yoshida (1999) presents a modified Harrodian model to analyze the dynamics of the expected real wage rate, the efficient factor intensity, and the rate of capital accumulation, and using the Hopf bifurcation theorem, shows that there exists a limit cycle. The model of Sportelli (2000) is also a modified Harrodian model, which considers the dynamics of the natural rate of growth, the warranted rate of growth, and the savings rate. Using a Šil'nikov scenario, he shows that the model generates a chaotic motion. Skott (1989) combines a Kaldorian business cycle model with a Goodwin model and using the Poincare-Bendixon theorem, shows that cyclical fluctuations occur.

As stated earlier, we extend the Kaleckian growth model.[2] In the Kaleckian framework, several models consider cyclical fluctuations. Lima (2004) develops a Kaleckian growth model in which the rate of utilization is adjusted in the short run, while the wage share and the capital-effective labor ratio are adjusted in the long run. In Lima's model, the rate of labor-saving technological change depends non-linearly on the wage share, which can generate the limit cycle. Raghavendra (2006) builds a Kaleckian model that considers the dynamics of the rate of capacity utilization and the profit share, and shows that a stable limit cycle occurs. However, this model cannot determine the rate of employment. Moreover, the model treats the level of output, and not the growth of output. Therefore, the dynamics obtained are not indicative of cyclical growth. Flaschel, Franke, and Semmler (2008) consider the three-dimensional dynamics of the output-capital ratio, the real wage rate, and the employment rate.

The basic framework of our model is based on a Kaleckian model with the theory of conflicting-claims inflation.[3] In the usual Kaleckian model, the rate of capacity utilization and the rate of capital accumulation are determined with the income distribution given exogenously. In the Kaleckian model with the theory of conflicting-claims inflation, in contrast, the income distribution is also determined endogenously (Cassetti, 2003, 2006). However, even such extended models do not consider the labor market satisfactorily. Existing Kaleckian growth models assume that the labor supply is unlimited and that firms employ as many workers as they desire at the given wages. If, however, the labor supply

2) See Kalecki (1971) for his economic theory. For the basic framework of the Kaleckian growth model, see Rowthorn (1981), Lavoie (1992), Foley and Michl (1999, ch. 10), Blecker (2002), and Taylor (2004, ch. 5).

3) The theory of conflicting-claims inflation is developed by Rowthorn (1977).

grows at an exogenously given rate, there is no guarantee that the endogenously determined employment growth is equal to the exogenous labor supply growth. If labor supply grows faster than employment, then the rate of unemployment continues to rise; this is unrealistic. Hence, a model such as this cannot investigate long-run unemployment. Kaleckian models with the theory of conflicting-claims inflation consider the effect of a class conflict between workers and capitalists on income distribution, but do not consider its effect on employment. It is interesting to investigate how the changes in the bargaining power of both classes affect employment.

Therefore, we present a Kaleckian model in which the rate of employment is endogenously determined. For this purpose, we endogenize the growth rate of labor productivity, which is zero or given exogenously in the usual Kaleckian model. In particular, we assume that the growth rate of labor productivity depends positively on the rate of employment. This assumption is based on Bhaduri (2006) and Dutt (2006).[4)] Given the level of output, an increase in labor productivity lowers employment and accordingly, creates the reserve-army of labor. Because, under our formulation, the growth rate of labor productivity increases with the rate of employment, there exists a counterbalancing effect that lowers the increased rate of employment. We call this effect the "reserve-army creation effect." Marx emphasizes the role of labor-saving technological progress in creating the reserve-army of labor (Marx, 1976, ch. 10, 13, 23). We use this term to distinguish it from the "reserve-army effect." As will be explained later, the reserve-army effect relates to the rate of employment and wage rate. We show that the interaction between the reserve-army effect and the reserve-army-creation effect produces cyclical fluctuations of the endogenous variables.

As state above, our model considers the elements from Goodwin (the dynamics of the rate of employment and income distribution), Kalecki (an investment function independent of savings, and mark-up pricing in oligopolistic goods markets), and Marx (the reserve-army effect and the reserve-army-creation effect). For this reason, we call our model a Goodwin-Kalecki-Marx model.

The remainder of the paper is organized as follows. Section 2 presents the basic framework of the model and derives the fundamental equations for the analysis. Section 3 analyzes the existence and the local stability of the long-run equilibrium, and analytically shows that a limit cycle can occur. Section 4 shows the occurrence of cyclical fluctuations by using numerical simulations. Section 5 conducts a comparative statics analysis. Section 6 concludes

4) Based on the idea of Marx, Bhaduri (2006) states that this formulation captures the view that technological change is driven by inter-class conflict over income distribution between workers and capitalists. Dutt (2006) says that as the labor market tightens and labor shortage becomes clearer, the bargaining power of workers increases, which exerts an upward pressure on wages, leading capitalists to adopt labor-saving technical changes.

the paper.

2 Basic framework of the model

Consider an economy with workers and capitalists. Suppose that workers spend all their wages and capitalists save a constant fraction s of their profits. Let r and K be the rate of profit and capital stock, respectively. Then, the real savings are given by $S = srK$, and accordingly, the ratio of real savings to capital stock, $g_s = S/K$, yields

$$g_s = sr. \tag{1}$$

We ignore capital depreciation for simplicity.

Suppose that the firms operate with the following fixed coefficients production function:

$$Y = \min\{aL, (u/k)K\}, \tag{2}$$

where Y denotes real output; L, employment; and $a = Y/L$, the level of labor productivity.[5)] The rate of capacity utilization is defined as $u = Y/Y^*$, where Y^* denotes the potential output. The coefficient $k = K/Y^*$ denotes the ratio of capital stock to potential output, which is assumed to be constant. This assumption implies that both K and Y^* grow at the same rate. Moreover, when the rate of capacity utilization is constant, the growth rates of capital stock and actual output are the same. Accordingly, the actual output and potential output grow at the same rate in the long-run equilibrium. To simplify the analysis, we assume $k = 1$ in what follows. From this, we have $r = mu$, where m denotes the profit share.

Following the argument of Marglin and Bhaduri (1990), we specify the ratio of real investment (I) to capital stock (K), $g_d = I/K$, as follows:

$$g_d = \psi m^\beta u^\gamma, \quad \psi > 0,\ \beta \in (0,1),\ \gamma \in (0,1), \tag{3}$$

where ψ denotes a constant; β, the elasticity of the investment rate with respect to the profit share; and γ, the elasticity of the investment rate with respect to the rate of capacity utilization. Equation (3) implies that the desired investment rate of firms is increasing in both the profit share and the rate of capacity utilization. In conventional Kaleckian models, the investment function is assumed to depend positively on both the rate of profit and the rate of capacity utilization. Marglin and Bhaduri (1990), in contrast, argue that the profit share

5) Given the fixed coefficients production function, a cost-minimizing firm operates at a point on isoquant curves such that $aL = (u/k)K$, which yields $a = Y/L$.

and not the rate of profit should be a variable in the investment function.[6] Our specification of the investment function is based on Blecker (2002). Because the investment function is not linear but Cobb-Douglas, as will be shown later, different regimes can be produced by changing the sizes of β and γ. When $\beta < \gamma$, the economy is in the wage-led growth regime, whereas when $\beta > \gamma$, it is in the profit-led growth regime.

An equation of motion for the rate of capacity utilization is given by

$$\dot{u} = \alpha(g_d - g_s), \quad \alpha > 0, \tag{4}$$

where α denotes the speed of adjustment of the goods market. Equation (4) shows that excess demand leads to a rise in the rate of capacity utilization, while excess supply leads to a decline in the rate of capacity utilization.

From the definition of profit share, we have $m = 1 - (wL/pY)$, from which we obtain the following relation:[7]

$$\frac{\dot{m}}{1-m} = \frac{\dot{p}}{p} - \frac{\dot{w}}{w} + \frac{\dot{a}}{a}, \tag{5}$$

where w denotes the money wage and p, the price. To know the dynamics of m, we have to specify the dynamics of p, w, and a.

We specify the dynamics of the money wage and price by using the theory of conflicting-claims inflation. First, suppose that the growth rate of the money wage that workers manage to negotiate depends on the discrepancy between their target profit share and the actual profit share. Second, suppose that the firms set their price to close the gap between their target profit share and the actual profit share. From these considerations, the dynamics of the money wage and price can be described, respectively, as follows:

$$\frac{\dot{w}}{w} = \theta_w(m - m_w), \quad \theta_w > 0,\ m_w \in (0, 1), \tag{6}$$

$$\frac{\dot{p}}{p} = \theta_f(m_f - m), \quad \theta_f > 0,\ m_f \in (0, 1), \tag{7}$$

6) The reason for this is as follows. The rate of profit is equal to the product of the profit share and the rate of capacity utilization, that is, $r = mu$. Thus, it is plausible that a combination of high capacity utilization and a low profit share and a combination of low capacity utilization and a high profit share will produce different levels of investment even when the rate of profit is held constant at a given level.

7) Cassetti (2002, 2003, 2006) derives an equation of motion for the profit share by specifying a price-setting equation of firms and differentiating it with respect to time. However, this procedure is unnecessary for deriving the dynamics of the profit share, and our procedure is easier than his. Under the conflicting-claims inflation theory, the price-setting equation in Cassetti's model plays the role of determining the mark-up rate, and not the price level.

where θ_w and θ_f denote the speed of adjustment; m_w, the target profit share set by workers; and m_f, the target profit share set by firms.

We can interpret θ_w and θ_f as the bargaining powers of the workers and firms, respectively.[8] We assume $\theta_f + \theta_w = 1$ and define $\theta_f \equiv \theta$ because bargaining power is a relative concept. Then, we have $\theta_w = 1 - \theta$, where $0 < \theta < 1$.[9] For example, we consider an increase in the unionization rate as a factor in increasing the bargaining power of workers (i.e., a decrease in θ), and an increase in the market power of oligopolistic firms as a factor in increasing the bargaining power of firms (i.e., an increase in θ).

We assume that the workers' target m_w depends negatively on the rate of employment, e.

$$m_w = m_w(e), \quad m_w' < 0, \tag{8}$$

where $e = L/N$ and N denotes the exogenous labor supply. As the rate of employment increases, workers' demands in the bargaining are likely to increase, which leads workers to set a higher target wage share, and accordingly, set a lower target profit share. We consider equation (8) as expressing the "reserve-army effect."[10] On the other hand, for simplicity, we consider the firms' target profit share m_f as exogenously given.[11] Notice the difference between θ and equation (8). The parameter θ represents the relative bargaining power of firms (workers) and reflects the power to realize their demands. In contrast, equation (8) reflects their demands in the bargaining. To what extent their demands can be realized depends on θ.

From equation (2), the rate of employment is given by $e = uK/(aN)$, and hence, the rate of change in the rate of employment yields

$$\frac{\dot{e}}{e} = \frac{\dot{u}}{u} + g_d - g_a - n, \tag{9}$$

where n is the growth rate of N and given exogenously, and $g_a = \dot{a}/a$.

As stated above, we assume that the growth rate of labor productivity depends positively on the rate of employment.[12]

$$g_a = g_a(e), \quad g_a' > 0, \; g_a > 0. \tag{10}$$

8) This interpretation is also adopted in Lavoie (1992, p. 393), Cassetti (2002, p. 192), and Cassetti (2003, p. 453).

9) The constraints $0 < \theta_w, \theta_f < 1$ are also adopted by Dutt and Amadeo (1993), who, however, do not assume $\theta_f + \theta_w = 1$. Even if we impose only $0 < \theta_w, \theta_f < 1$ and not $\theta_f + \theta_w = 1$, we can obtain similar results.

10) Cassetti (2002, 2003) also considers such a reserve-army effect in the Kaleckian framework.

11) We can endogenize the target profit share of firms. Cassetti (2002) and Lima (2004) assume that m_f is an increasing function of u.

12) Flaschel and Skott (2006, p. 328) also use a similar specification.

This equation includes the reserve-army-creation effect. As the labor market tightens and labor shortage becomes clearer, the bargaining power of workers increases, which exerts an upward pressure on wages, leading the firms to adopt labor-saving technical changes.

In general, the natural rate of growth is defined as a sum of the growth rates of labor productivity and labor supply. Although the growth rate of labor supply in our model is exogenously given, the growth rate of labor productivity is endogenously determined. Under our specification, therefore, the natural rate of growth increases when business is good (i.e., when the employment rate is high) and it decreases when business is bad (i.e., when the employment rate is low). The assumption that the natural rate of growth is endogenously determined is consistent with the empirical studies of León-Ledesma and Thirlwall (2002) and Libânio (2009).

Using equation (10), we can show that in the long-run equilibrium, the growth rate of output per capita (Y/N) coincides with that of labor productivity (Y/L). In this respect, Sedgley and Elmslie (2004) empirically show the cointegration between the log of output per capita and the log of labor productivity. This evidence suggests that these two variables move together in the long run.

We now focus on the derivation of the system of differential equations. First, substituting equations (1) and (3) in equation (4), we obtain the dynamics of u. Second, substituting equations (6) and (7) in equation (5), and substituting equations (8) and (10) in the resulting expression, we obtain the dynamics of m. Finally, substituting the dynamics of u, equation (3), and equation (10) in equation (9), we obtain the dynamics of e.

$$\dot{u} = \alpha(\psi m^{\beta} u^{\gamma} - smu), \tag{11}$$

$$\dot{m} = -(1-m)[m - \theta m_f - (1-\theta)m_w(e) - g_a(e)], \tag{12}$$

$$\dot{e} = e[\alpha(\psi m^{\beta} u^{\gamma-1} - sm) + \psi m^{\beta} u^{\gamma} - g_a(e) - n]. \tag{13}$$

We now provide an explanation with regard to the structure of our model. If we introduce $m_w = m_w(e)$ with g_a as exogenously given, we find that the rate of employment is endogenously determined whereas the natural rate of growth is exogenous. If we, on the other hand, introduce $g_a = g_a(e)$ with m_w as exogenously given, we find that both the rate of employment and the natural rate of growth are endogenously determined. Hence, to endogenize both the rate of employment and the natural rate of growth, we do not need to specify m_w as a function of e. Nevertheless, we use both $g_a(e)$ and $m_w(e)$. This is because we intend to capture the interaction between the reserve-army effect and the reserve-army-creation effect. Simply put, the reserve-army effect acts to destabilize the equilibrium, whereas the reserve-army-creation effect acts to stabilize the equilibrium. Therefore, this interaction plays an important role in the stability analysis of the equilibrium.

3 Existence and stability of the long-run equilibrium

3.1 Existence of the long-run equilibrium

The long-run equilibrium is a situation where $\dot{u} = \dot{m} = \dot{e} = 0$. Here, we have the following three equations:

$$\psi m^{\beta} u^{\gamma} - smu = 0, \tag{14}$$

$$m - \theta m_f - (1 - \theta) m_w(e) - g_a(e) = 0, \tag{15}$$

$$\psi m^{\beta} u^{\gamma} - g_a(e) - n = 0. \tag{16}$$

These equations show that the equilibrium values do not depend on the speed of adjustment (α). From equation (14), we obtain

$$u = \left(\frac{\psi}{s}\right)^{\frac{1}{1-\gamma}} m^{\frac{\beta-1}{1-\gamma}}. \tag{17}$$

Substituting equation (17) in equation (16), we find that the resulting expression will be an equation of m and e. Combining this equation with equation (15), we can obtain the equilibrium values of m and e, which are substituted in equation (17) to find the equilibrium value of u. In the following analysis, we assume that there uniquely exist long-run equilibrium values (u^*, m^*, e^*) that satisfy equations (14), (15), and (16) simultaneously. In addition, we assume $u^*, m^*, e^* \in (0, 1)$. Hereafter, the long-run equilibrium values are denoted with "$*$."

Here, we show that in the long-run equilibrium, the capital-labor ratio continues to increase. Let κ be the capital-labor ratio. Then, we have $\kappa = K/L$. The rate of change in κ is given by the difference between the rate of change in the capital stock and the rate of change in the employment. Using employment, we get $L = uK/a$, and consequently obtain

$$\frac{\dot{\kappa}}{\kappa} = g_a(e) - \frac{\dot{u}}{u}. \tag{18}$$

Hence, the rate of change in the capital-labor ratio is given by the difference between the growth rate of labor productivity and the rate of change in the capacity utilization. In the long-run equilibrium, we have $\dot{u} = 0$, and consequently, we obtain $\dot{\kappa}/\kappa = g_a(e)$. Because, in the long-run equilibrium, e is constant and hence, $g_a(e)$ is constant, the rate of change in the capital-labor ratio is also constant and equal to the growth rate of labor productivity. This property is similar to that of the neoclassical growth model with labor-augmenting technical progress. Note, however, that in the neoclassical growth model, full employment is assumed, while in our model, full employment is not assumed.

3.2 Local stability of the long-run equilibrium

To investigate the local stability of the long-run equilibrium, we linearize the system of differential equations (11), (12), and (13) around the equilibrium.

$$\begin{pmatrix} \dot{u} \\ \dot{m} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} & 0 \\ 0 & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} \begin{pmatrix} u - u^* \\ m - m^* \\ e - e^* \end{pmatrix}, \tag{19}$$

where the elements of the Jacobian matrix $\mathbf{J}$ are given by

$$J_{11} \equiv \frac{\partial \dot{u}}{\partial u} = -\alpha s(1-\gamma)m < 0, \tag{20}$$

$$J_{12} \equiv \frac{\partial \dot{u}}{\partial m} = -\alpha s(1-\beta)u < 0, \tag{21}$$

$$J_{22} \equiv \frac{\partial \dot{m}}{\partial m} = -(1-m) < 0, \tag{22}$$

$$J_{23} \equiv \frac{\partial \dot{m}}{\partial e} = (1-m)\underbrace{[(1-\theta)m_w'(e) + g_a'(e)]}_{\equiv \Gamma(e,\theta)} = (1-m)\Gamma(e,\theta) \gtrless 0, \tag{23}$$

$$J_{31} \equiv \frac{\partial \dot{e}}{\partial u} = sme\left[\frac{\alpha(\gamma-1)+\gamma u}{u}\right] \gtrless 0, \tag{24}$$

$$J_{32} \equiv \frac{\partial \dot{e}}{\partial m} = se[\alpha(\beta-1)+\beta u] \gtrless 0, \tag{25}$$

$$J_{33} \equiv \frac{\partial \dot{e}}{\partial e} = -eg_a'(e) < 0. \tag{26}$$

All elements are evaluated at the long-run equilibrium; we omit "$*$" to avoid troublesome notations.

The term $\Gamma(e,\theta) \equiv (1-\theta)m_w'(e) + g_a'(e)$ in equation (23) consists of the following three elements: the relative bargaining power of firms θ, the extent of the reserve-army effect m_w', and the extent of the reserve-army-creation effect g_a'. Because $m_w' < 0$ and $g_a' > 0$, Γ can be positive or negative. When the reserve-army effect is strong (i.e., the absolute value of m_w' is large), and the bargaining power of firms and the reserve-army-creation effect are weak (i.e., θ and g_a', respectively, are small), we have $\Gamma < 0$. On the other hand, when the reserve-army effect is weak, and the bargaining power of firms and the reserve-army-creation effect are strong, we have $\Gamma > 0$. The sign of Γ plays an important role in both the stability of the equilibrium and the comparative statics analysis below.

The signs of equations (24) and (25) are indeterminate. When α is sufficiently large, these signs are likely to be negative.

The characteristic equation of the Jacobian matrix (19) is given by

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{27}$$

where λ denotes a characteristic root. Each coefficient of equation (27) is given by

$$a_1 = -\mathrm{tr}\,\mathbf{J} = -(J_{11} + J_{22} + J_{33}), \tag{28}$$

$$a_2 = \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & 0 \\ J_{31} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{12} \\ 0 & J_{22} \end{vmatrix} = J_{22}J_{33} - J_{23}J_{32} + J_{11}J_{33} + J_{11}J_{22}, \tag{29}$$

$$a_3 = -\det\mathbf{J} = -J_{11}J_{22}J_{33} + J_{11}J_{23}J_{32} - J_{31}J_{12}J_{23}, \tag{30}$$

where $-a_1 = \mathrm{tr}\,\mathbf{J}$ denotes the trace of $\mathbf{J}$; a_2, the sum of the principal minors' determinants; and $-a_3 = \det\mathbf{J}$, the determinant of $\mathbf{J}$.

The necessary and sufficient condition for the local stability is that all characteristic roots of the Jacobian matrix have negative real parts, which, from Routh-Hurwitz condition, is equivalent to[13)]

$$a_1 > 0,\ a_2 > 0,\ a_3 > 0,\ a_1a_2 - a_3 > 0. \tag{31}$$

Let us examine whether these inequalities hold. We arrange the coefficients with respect to α.

First, a_1 is a linear function of α.

$$a_1 \equiv a_1(\alpha) = \underbrace{s(1-\gamma)m}_{\equiv A>0}\,\alpha + \underbrace{(1-m) + eg_a'(e)}_{\equiv B>0} = \underset{+}{A}\alpha + \underset{+}{B}. \tag{32}$$

Therefore, we can confirm that $a_1 > 0$. This implies that $\mathrm{tr}\,\mathbf{J} < 0$, which is a necessary condition for the local stability of the equilibrium.

Second, a_2 is a linear function of α.

$$\begin{aligned} a_2 \equiv a_2(\alpha) = &\underbrace{\{s(1-\gamma)m[1-m+eg_a'(e)] + s(1-\beta)e(1-m)\Gamma(e,\theta)\}}_{\equiv C \gtrless 0}\,\alpha \\ &+ \underbrace{e(1-m)[(1-\beta su)g_a'(e) - s\beta(1-\theta)um_w'(e)]}_{\equiv D>0} = \underset{+/-}{C}\,\alpha + \underset{+}{D}. \end{aligned} \tag{33}$$

If $\Gamma > 0$, we always have $C > 0$. Hence, we obtain $a_2 > 0$. If, however, $\Gamma < 0$, we do not always have $C > 0$. From this, we obtain the following proposition:

Proposition 1. *Suppose that $C < 0$. If the speed of adjustment of the goods market α is sufficiently large, then the long-run equilibrium is locally unstable.*

Proof. The constant term of a_2 is always positive, while the coefficient of a_2 can be either positive or negative. If C is positive, we have $a_2 > 0$. This is a necessary condition for the local stability of the equilibrium. However, if C is negative, we have $a_2 < 0$ when α is sufficiently large. In this case, $a_2 > 0$ is not satisfied, and hence, the long-run equilibrium becomes locally unstable. ■

13) See Gandolfo (1996) for details.

Third, a_3 is a linear function of α.

$$a_3 \equiv a_3(\alpha) = s(1-\gamma)em(1-m)\underbrace{\left[g_a'(e) - \frac{s(\beta-\gamma)}{1-\gamma}\,u\Gamma(e,\theta)\right]}_{\equiv\Theta}\alpha$$

$$= \underbrace{s(1-\gamma)em(1-m)\Theta}_{\equiv E}\,\alpha = E\alpha. \tag{34}$$

Here, we introduce the following assumption:

Assumption 1. $\Theta > 0$.

The sign of a_3 depends on the sign of Θ. If $\Theta > 0$, we have $E > 0$, and consequently, $a_3 > 0$. This implies that $\det \mathbf{J} < 0$, which is a necessary condition for the local stability of the equilibrium. When $\beta > \gamma$, we always have $\Theta > 0$ irrespective of the sign of Γ.[14)] However, when $\beta < \gamma$, we do not always have $\Theta > 0$. When $\beta < \gamma$, we always have $\Theta > 0$ if $\Gamma > 0$. Even if $\Gamma < 0$, we can have $\Theta > 0$. Numerical simulations, which are introduced later, will show that there exist closed orbits around the equilibrium when both $\Gamma < 0$ and $\Theta > 0$.

Finally, $a_1a_2 - a_3$ is a quadratic function of α.

$$a_1a_2 - a_3 \equiv \phi(\alpha) = (AC)\alpha^2 + (AD + BC - E)\alpha + \underbrace{BD}_{+}. \tag{35}$$

At this stage, we cannot confirm whether this parabola is convex upward or convex downward. However, when $\alpha = 0$, we have $\phi(0) = BD > 0$, which shows that there exists an α such that $a_1a_2 - a_3 > 0$ for $\alpha > 0$. From this, we obtain the following proposition:

Proposition 2. *Suppose that the speed of adjustment of the goods market α is sufficiently close to zero. Then, the long-run equilibrium is locally stable.*

Proof. From the above discussion, we have $a_1 > 0$ and $a_3 > 0$. When $\alpha = 0$, we have $\phi(0) = BD > 0$, that is, $a_1a_2 - a_3 > 0$. If $a_1 > 0$, $a_2 > 0$, and $a_1a_2 - a_3 > 0$, then $a_2 > 0$ is necessarily satisfied. Hence, if $\alpha = 0$, the necessary and sufficient conditions given by (31) are all satisfied. Because $\phi(\alpha)$ is a continuous function of α, even if $\alpha > 0$, $a_1a_2 - a_3 > 0$ is satisfied when α is sufficiently close to zero. Therefore, when α is sufficiently close to zero, the necessary and sufficient conditions given by (31) are all satisfied. ■

14) Expanding Θ, we obtain

$$\Theta = \left(1 - su\frac{\beta-\gamma}{1-\gamma}\right)g_a'(e) - su\frac{\beta-\gamma}{1-\gamma}(1-\theta)m_w'(e).$$

When $\beta > \gamma$, $(\beta-\gamma)/(1-\gamma)$ is larger than zero and smaller than unity. From this, the first term of the right-hand side is positive. The second term of the right-hand side is positive because $m_w' < 0$. Therefore, when $\beta > \gamma$, we have $\Theta > 0$ irrespective of the sign of Γ.

Proposition 2 is obtained regardless of whether $C > 0$ or $C < 0$. That is, if the speed of adjustment of the goods market is very slow, the long-run equilibrium is locally stable irrespective of the size of the relative bargaining power of firms, the reserve-army effect, and the reserve-army-creation effect.

3.3 Existence of closed orbits

As explained above, $\Gamma > 0$ implies that $C > 0$, which in turn implies that $a_2 > 0$. Because we know that $a_1 > 0$ and $a_3 > 0$, the long-run equilibrium is stable if $a_1 a_2 - a_3 > 0$.

Proposition 3. *Suppose that* $\Gamma > 0$. *Suppose also that the equilibrium profit share* m^* *is less than or equal to* $1/2$. *Then, the long-run equilibrium is locally stable irrespective of the size of* α.

Proof. If $\Gamma > 0$, and consequently $C > 0$, $\phi(\alpha)$ becomes a parabola that is convex downward. Here, we focus on the coefficient of α in $\phi(\alpha)$, that is, $AD + BC - E$. Expanding this coefficient, we have

$$AD + BC - E = \underbrace{s(1-\gamma)m[1-m+eg_a'(e)]^2}_{+} + \underbrace{s(1-\beta)e(1-m)\Gamma}_{+}[\underbrace{eg_a'(e)}_{+} + \underbrace{(1-m-s\gamma um)}_{\equiv\Lambda}]. \tag{36}$$

When s, γ, and u are close to zero, $\Lambda \equiv 1 - m - s\gamma um$ will be positive because $s\gamma um$ will be sufficiently small. On the other hand, when s, γ, and u are close to unity, Λ approaches $\Lambda = 1-2m$. From this, if $m^* \leq 1/2$, we have $\Lambda \geq 0$. When $\Lambda \geq 0$, we have $AD+BC-E > 0$, and consequently, we obtain $\phi(\alpha) > 0$ for $\alpha > 0$. Therefore, if $\Gamma > 0$ and if $m^* \leq 1/2$, then the necessary and sufficient conditions for local stability, that is, $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1 a_2 - a_3 > 0$, are all satisfied. ∎

Proposition 3 is obtained when the reserve-army effect is weak, and the relative bargaining power of firms and the reserve-army-creation effect are strong. In general, the profit share in the real world is considered to be less than $1/2$, and hence, condition $m^* \leq 1/2$ is plausible. Note, however, that $m^* \leq 1/2$ is a sufficient and not a necessary condition for $a_1 a_2 - a_3 > 0$. Moreover, m^* depends on the parameters of the model.

Proposition 4. *Suppose that* $C < 0$. *Then, cyclical fluctuations occur when the speed of adjustment of the goods market lies within some range.*

Proof. If $C < 0$, $a_2(\alpha)$ becomes a straight line whose slope is negative and intercept is positive. This implies that there exists $\alpha_1 > 0$ such that $a_2(\alpha_1) = 0$. Moreover, if $C < 0$, $\phi(\alpha)$

becomes a parabola that is convex upward. Then, the quadratic equation $\phi(\alpha) = 0$ has one negative real root and one positive real root. Hence, there exists $\alpha_2 > 0$ such that $\phi(\alpha_2) = 0$. Let us investigate which is larger, α_1 or α_2. From $a_2(\alpha_1) = 0$, we obtain $\alpha_1 = -D/C > 0$. Substituting α_1 in $\phi(\alpha)$, we obtain

$$\phi(\alpha_1) = \frac{DE}{C} < 0 \tag{37}$$

because $C < 0$. This implies that $\alpha_2 < \alpha_1$. From this, we get that $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 > 0$ within the range $\alpha \in (0, \alpha_2)$, while $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, and $a_1a_2 - a_3 < 0$ within the range $\alpha \in (\alpha_2, \alpha_1)$. Consequently, a Hopf bifurcation occurs at α_2. Indeed, at $\alpha = \alpha_2$, we obtain

$$a_1 > 0,\ a_2 > 0,\ a_3 > 0,\ a_1a_2 - a_3 = 0,\ \left.\frac{\partial(a_1a_2 - a_3)}{\partial\alpha}\right|_{\alpha=\alpha_2} \neq 0. \tag{38}$$

That is, all conditions for the occurrence of the Hopf bifurcation are satisfied.[15] Therefore, when $C < 0$, there exists a continuous family of non-constant, periodic solutions of the system around $\alpha = \alpha_2$. ■

We obtain $C < 0$ when g'_a and θ are small, and the absolute value of m'_w is large. These conditions are similar to the conditions for $\Gamma < 0$. Indeed, $\Gamma < 0$ is a necessary condition for $C < 0$. From this, we can obtain Proposition 4 when the reserve-army effect is strong, and the relative bargaining power of firms and the reserve-army-creation effect are weak.

As explained above, the long-run equilibrium can be both stable or unstable. Let us briefly explain this mechanism. Here, we focus on the rate of capacity utilization. Suppose that the rate of capacity utilization exceeds its equilibrium value for some reason. Then, as long as the speed of adjustment of the goods market is not extremely large, the increase in the rate of capacity utilization induces the rate of employment to increase through equation (24). This increase in the rate of employment changes the profit share through equation (23). The direction of the change in the profit share depends on the sign of Γ.

If $\Gamma > 0$, that is, the power of firms is relatively strong, then the profit share increases. This increase in the profit share has two opposing effects. (1) The increase in the profit share stimulates the investment of firms, which increases the output. (2) The increase in the profit share increases the savings of capitalists, which decreases the output. Because the adjustment process of the goods market is stable, the latter negative effect on the output is

15) For the Hopf bifurcation theorem, see Gandolfo (1996). The last condition in equation (38), that is, $\partial(a_1a_2 - a_3)/\partial\alpha|_{\alpha=\alpha_2} \neq 0$, is equivalent to the condition that the derivatives of the real parts of the characteristic roots with respect to α are not zero when evaluated at $\alpha = \alpha_2$. For details, see Asada and Semmler (1995, p. 634–635).

stronger than the former positive effect, and as a result, the output and the rate of capacity utilization decrease (see equation (21)). Therefore, here, a negative feedback effect acts on the rate of capacity utilization, and accordingly, the long-run equilibrium will be stable.

On the other hand, if $\Gamma < 0$, that is, the power of workers is relatively strong, then the profit share declines, which increases the rate of capacity utilization through equation (21). Therefore, here, a positive feedback effect acts on the rate of capacity utilization, and accordingly, the long-run equilibrium will be unstable.

$$u \uparrow \Longrightarrow e \uparrow \text{ (when } \alpha \text{ is not so large)} \Longrightarrow \begin{cases} m \uparrow \quad (\Gamma > 0) \quad \Longrightarrow \quad u \downarrow \quad \text{(stabilizing)} \\ m \downarrow \quad (\Gamma < 0) \quad \Longrightarrow \quad u \uparrow \quad \text{(destabilizing)} \end{cases}.$$

Finally, we refer to the roles of the reserve-army and reserve-army-creation effects. When only the reserve-army-creation effect exists, that is, $m'_w = 0$, we always have $\Gamma = g'_a > 0$. In this case, from Proposition 3, the long-run equilibrium is locally stable given that $m^* \leq 1/2$, and accordingly, the Hopf bifurcation never occurs. Therefore, the reserve-army-creation effect has a stabilizing effect.

In contrast, when only the reserve-army effect exists, that is, $g'_a = 0$, we always have $\Gamma = (1-\theta)m'_w < 0$. In this case,

$$\Theta = -\frac{s(\beta - \gamma)}{1 - \gamma} u(1-\theta)m'_w. \tag{39}$$

If $\beta > \gamma$, then $\Theta > 0$ necessarily holds. However, if $\beta < \gamma$, then $\Theta < 0$ necessarily holds, which contradicts Assumption 1, and consequently, the long-run equilibrium becomes unstable. This implies that the long-run equilibrium in the wage-led growth regime is always unstable, and also that the Hopf bifurcation never occurs. Therefore, the reserve-army effect has a destabilizing effect.

From the above reasoning, the interaction between the reserve-army and reserve-army-creation effects plays an important role in stabilizing both the wage-led and profit-leg growth regimes, and in the occurrence of the Hopf bifurcation.

4 Numerical simulations

In this section, we present numerical examples to show that, under plausible settings, an economically meaningful long-run equilibrium actually exists and cyclical fluctuations really occur. Note, however, that the results of numerical simulations depend on the specification of the functional form and the numerical values of the parameters.

For the numerical simulation, we have to specify the functional forms of equations (8) and (10). We specify these functions as follows:

$$m_w(e) = \delta(1 - e), \quad \delta > 0, \tag{40}$$

$$g_a(e) = \eta e, \quad \eta > 0. \tag{41}$$

In this case, Γ and Θ are respectively given by

$$\Gamma = \eta - (1 - \theta)\delta, \tag{42}$$

$$\Theta = \eta - \frac{s(\beta - \gamma)}{1 - \gamma}[\eta - (1 - \theta)\delta]u. \tag{43}$$

The equilibrium profit share m^* satisfies the following equation:

$$\Gamma A^{\frac{1}{1-\gamma}} s^{-\frac{\gamma}{1-\gamma}} m^{\frac{\beta-\gamma}{1-\gamma}} - \eta m + \eta[\theta m_f + (1 - \theta)\delta] - \Gamma n = 0. \tag{44}$$

From this, we obtain m^*, which we substitute in equation (17) to determine u^*. Furthermore, we substitute m^* in the following equation to determine e^*:

$$e^* = \frac{m^* - [\theta m_f + (1 - \theta)\delta]}{\Gamma}. \tag{45}$$

We consider four cases depending on which is larger, β or γ, and whether Γ is positive or negative.

4.1 Case 1: $\beta < \gamma, \Gamma < 0$

Case 1 corresponds to the case where the elasticity of the investment rate with respect to the profit share is smaller than the elasticity of the investment rate with respect to the rate of capacity utilization, the reserve-army effect is strong, and the relative bargaining power of firms and the reserve-army-creation effect are weak. We set the parameters as follows:

$$\beta = 0.2,\ \gamma = 0.4,\ \psi = 0.15,\ s = 0.6,\ \eta = 0.1,\ \delta = 0.5,\ \theta = 0.25,\ m_f = 0.3,\ n = 0.015. \tag{46}$$

In this case, the long-run equilibrium values, Γ and Θ, yield the following:[16)]

$$u^* = 0.74642,\ m^* = 0.220134,\ e^* = 0.835875,\ \Gamma = -0.275 < 0,\ \Theta = 0.0589496 > 0. \tag{47}$$

16) From these numerical settings, we obtain two values of m within the range $m \in (0, 1)$: $m_1^* = 0.220134$ and $m_2^* = 0.0516332$. However, from m_2^*, we obtain $u^* = 5.16014$ and $e^* = 1.44861$, which are unrealistic. For this reason, we do not adopt m_2^*.

These equilibrium values are economically meaningful. In addition, $\Theta > 0$ is satisfied.

Figure 1 displays the solution path when the initial values and the speed of adjustment are set as $u(0) = 0.7$, $m(0) = 0.21$, $e(0) = 0.8$, and $\alpha = 4$. The figure shows a cyclical fluctuation. In Figure 1, we draw the solution path from $t = 100$ to $t = 200$, and upon performing the calculations further, we find that the solution path is not a perfect closed orbit and that it converges to the long-run equilibrium with rotation. Moreover, if we set the initial conditions further away from the long-run equilibrium, we find that the solution path diverges from the equilibrium. From these observations, we confirm that in this numerical example, the subcritical Hopf bifurcation occurs and the periodic solution is unstable.

Figure 2 projects the three-dimensional dynamics on the (u, e)-plane. The solution path starting from point P converges to the long-run equilibrium with rotation. In contrast, the solution path starting from point Q diverges from the long-run equilibrium with rotation. These phenomena correspond to the "corridor stability" of Leijonhufvud (1973).

Figure 3 shows the graphs of $a_2(\alpha)$ and $\phi(\alpha)$. We find that $\alpha_1 = 4.61097$ and $\alpha_2 = 4.02288$. In Figure 1, we use $\alpha = 4$, from which we have $4 < \alpha_2$. Therefore, the subcritical Hopf bifurcation certainly occurs in this case.[17] Moreover, if we choose α larger than α_2, we find that the solution path diverges irrespective of the initial conditions. This confirms that the subcritical Hopf bifurcation occurs at α_2.

Note, however, that the Hopf bifurcation that occurs in case 1 is not always the subcritical Hopf bifurcation. We can find a numerical example wherein the supercritical Hopf bifurcation occurs in case 1.[18]

[Figures 1, 2, and 3 around here]

4.2 Case 2: $\beta < \gamma, \Gamma > 0$

Case 2 differs from case 1 in that $\Gamma > 0$, which implies that the reserve-army effect is weak and the relative bargaining power of firms and the reserve-army-creation effect are strong. We set the parameters as follows:

$$\beta = 0.2,\ \gamma = 0.4,\ \psi = 0.3,\ s = 0.7,\ \eta = 0.3,\ \delta = 0.4,\ \theta = 0.3,\ m_f = 0.3,\ n = 0.015. \quad (48)$$

In this case, the long-run equilibrium values, Γ and Θ, yield the following:

$$u^* = 0.871,\ m^* = 0.385,\ e^* = 0.732,\ \Gamma = 0.02 > 0,\ \Theta = 0.304 > 0. \quad (49)$$

17) In this numerical example, the range of α is such that both $a_2(\alpha) > 0$ and $\phi(\alpha) < 0$ are narrow. However, we can widen the range by choosing different parameter settings.

18) Under the parameters $\beta = 0.4$, $\gamma = 0.41$, $\psi = 0.2$, $s = 0.6$, $\eta = 0.1$, $\delta = 1$, and $\theta = 0.25$, the Hopf bifurcation point leads to $\alpha_2 = 2.56626$, and there exists a stable limit cycle at $\alpha > \alpha_2$.

These equilibrium values are economically meaningful. In addition, $\Theta > 0$ is satisfied.

Figure 4 displays the solution path when $u(0) = m(0) = e(0) = 0.5$ and $\alpha = 1$. The figure shows that the solution path converges stably to the long-run equilibrium. Figure 5 shows the graphs of $a_2(\alpha)$ and $\phi(\alpha)$. For $\alpha > 0$, both graphs are always positive, which shows that the long-run equilibrium is locally stable irrespective of the size of α.

[Figures 4 and 5 around here]

4.3 Case 3: $\beta > \gamma$, $\Gamma < 0$

Case 3 corresponds to the case where the elasticity of the investment rate with respect to the profit share is larger than the elasticity of the investment rate with respect to the rate of capacity utilization, the reserve-army effect is strong, and the relative bargaining power of firms and the reserve-army-creation effect are weak. We set the parameters as follows:

$$\beta = 0.4,\ \gamma = 0.2,\ \psi = 0.2,\ s = 0.6,\ \eta = 0.1,\ \delta = 1,\ \theta = 0.25,\ m_f = 0.3,\ n = 0.016. \quad (50)$$

Here, the long-run equilibrium values, Γ and Θ, yield the following:

$$u^* = 0.741857,\ m^* = 0.238619,\ e^* = 0.902125,\ \Gamma = -0.65 < 0,\ \Theta = 0.172331 > 0. \quad (51)$$

These equilibrium values are economically meaningful. In addition, $\Theta > 0$ is satisfied.

Figure 6 displays the solution path when $u(0) = 0.7$, $m(0) = 0.2$, $e(0) = 0.9$, and $\alpha = 2$. The figure shows a cyclical fluctuation. Using various initial values, we find that in this case, a stable limit cycle emerges: any initial point converges to the limit cycle with rotation. Therefore, the supercritical Hopf bifurcation occurs in case 3.

Figure 7 shows the graphs of $a_2(\alpha)$ and $\phi(\alpha)$. We find that $\alpha_1 = 2.34506$ and $\alpha_2 = 1.95504$. In Figure 6, we use $\alpha = 2$, from which we have $\alpha_2 < 2$. Therefore, the supercritical Hopf bifurcation certainly occurs in this case.[19)]

[Figures 6 and 7 around here]

Note that we were unable to find a numerical example that produced subcritical Hopf bifurcation.

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19) In this case, as in case 1, the range of α is such that both $a_2(\alpha) > 0$ and $\phi(\alpha) < 0$ are narrow. However, we can widen the range by choosing different parameter settings.

4.4 Case 4: $\beta > \gamma, \Gamma > 0$

Case 4 differs from case 3 in that $\Gamma > 0$, which implies that the reserve-army effect is weak and the relative bargaining power of firms and the reserve-army-creation effect are strong. We set the parameters as follows:

$$\beta = 0.4,\ \gamma = 0.2,\ \psi = 0.33,\ s = 0.6,\ \eta = 0.3,\ \delta = 0.4,\ \theta = 0.3,\ m_f = 0.3,\ n = 0.015. \tag{52}$$

In this case, the long-run equilibrium values, Γ and Θ, yield the following:

$$u^* = 0.971,\ m^* = 0.384,\ e^* = 0.696,\ \Gamma = 0.02 > 0,\ \Theta = 0.297 > 0. \tag{53}$$

These equilibrium values are economically meaningful. In addition, $\Theta > 0$ is satisfied.

Figure 8 displays the solution path when $u(0) = m(0) = e(0) = 0.5$ and $\alpha = 1$. The figure shows that the solution path converges stably to the long-run equilibrium. Figure 9 shows the graphs of $a_2(\alpha)$ and $\phi(\alpha)$. For $\alpha > 0$, both graphs are always positive, which shows that the long-run equilibrium is locally stable irrespective of the size of α.

[Figures 8 and 9 around here]

5 Comparative statics analysis

This section investigates the effects of the shifts in the parameters on the long-run equilibrium. To conduct a comparative statics analysis, we need the stability of the equilibrium. For this reason, we assume $\Theta > 0$ in the following analysis. As has been discussed above, cases 2 and 4 are always stable but cases 1 and 3 can be unstable. Therefore, we confine ourselves to the case where the long-run equilibrium is stable.

Table 1 summarizes the results of comparative statics in the four cases.[20] In Table 1, the "+" sign indicates that the corresponding variable increases with the parameter, while the "−" sign indicates that the corresponding variable decreases with the parameter. The signs "+/−" and "−/+" indicate that an increase in the parameter either increases or decreases the corresponding variable. The mark "†" shows that the left-hand sign applies when $\Gamma < 0$, while the right-hand sign applies when $\Gamma > 0$.

Moreover, for the effect of θ, we assume that $m_f - m_w(e) > 0$. Firms attempt to set their target profit share as high as possible, whereas workers attempt to set their target profit share as low as possible. Therefore, this assumption is reasonable.

20) See Appendix for details.

[Table 1 around here]

Let us explain the results in Table 1. Because of the limitations of space, we focus especially on the rate of employment.

■ Savings rate

An increase in the savings rate of capitalists decreases the rate of capacity utilization and the rate of capital accumulation. This negative effect on the growth rate is known as the "paradox of thrift." A rise in the savings rate decreases the rate of employment. Stockhammer (2004) also investigates the rate of employment in a Kaleckian framework. In Stockhammer's model, the long-run equilibrium value of the rate of employment consists of the exogenous natural rate of growth and the parameters of the investment function and the income distribution function, and does not depend on the savings rate. Hence, a change in the savings rate never affects the rate of employment. In our model, on the other hand, the natural rate of growth is endogenously determined, and accordingly, the change in the savings rate affects the rate of employment.

■ Labor supply growth

Previous Kaleckian models cannot investigate the effect of supply side factors on equilibrium values. In contrast, our model can investigate these. In either case, an increase in n lowers the rate of employment. Because the relation $g_a^* = sm^*u^* - n$ holds in the long-run equilibrium, an increase in n has three different effects on g_a^* and consequently, on e^*: it directly decreases g_a^* with the coefficient of n being -1; it indirectly affects g_a^* through m^*, which is positive when $\Gamma < 0$ and negative when $\Gamma > 0$; and it indirectly affects g_a^* through u^*, which is negative when $\Gamma < 0$ and positive when $\Gamma > 0$. In total, the two negative effects outweigh the one positive effect, which leads to a decline in g_a^* and e^*. Stockhammer (2004) also concludes that an increase in labor supply growth leads to a decrease in the rate of employment in the profit-led growth regime ($\beta > \gamma$ in our model), in which the long-run equilibrium is stable. Yet, the wage-led growth regime in Stockhammer's model ($\beta < \gamma$ in our model) is unstable, and thus, one cannot conduct a comparative statics analysis. In contrast, even the wage-led growth regime can be stable in our model.

■ Relative bargaining power

An increase in the relative bargaining power of firms either increases or decreases the rate of employment depending on the size of the two elasticities of the investment function. The rate of employment decreases when $\beta < \gamma$, whereas it increases when $\beta > \gamma$. An increase in θ has two different effects on g_a and e: it indirectly increases g_a and e through its positive effect on m and it indirectly decreases g_a and e through its negative effect on u. Whether or not the increase in θ leads to an increase in g_a and e depends on which effect dominates, which in turn depends on the size of the elasticities of the investment function.

When $\beta < \gamma$, the negative effect of the capacity utilization dominates the positive effect of the profit share, thereby leading to a decrease in the growth rate of labor productivity and the employment rate. When $\beta > \gamma$, in contrast, the positive effect of the profit share dominates the negative effect of the capacity utilization, thereby leading to an increase in the growth rate of labor productivity and the employment rate.

Stockhammer (2004) also investigates the relationship between bargaining power and unemployment. He concludes that in the profit-led growth regime, a decrease in the bargaining power of workers leads to higher employment and lower unemployment. This result is consistent with our results. However, in the wage-led growth regime of Stockhammer's model, the long-run equilibrium is unstable, and consequently, one cannot investigate the relationship between bargaining power and unemployment. In our model, in contrast, thelong run equilibrium of the wage-led growth regime can be stable. In this case, we reach the opposite conclusion that an increase in the bargaining power of workers leads to higher employment and lower unemployment. This result is consistent with the empirical result of Storm and Naastepad (2007). Using data for 20 OECD countries during 1984–1997, they show that an increase in the bargaining power of firms because of labor market deregulation increases the unemployment rate; this is in contrast to the view of the mainstream theory.

■ Autonomous investment

We can regard the parameter ψ of the investment function as expressing a demand policy. Setterfield (2009), for instance, relates a constant term of the investment function to a fiscal policy and discusses the effectiveness of output targeting and inflation targeting. An increase in ψ in our model increases all equilibrium values: stimulating effective demand lowers the unemployment rate even in the long run. This implication makes a marked contrast to the implication of the mainstream theory.

6 Concluding remarks

In this paper, we have developed a demand-led growth model that considers elements from Goodwin, Kalecki, and Marx. In the model, we have considered the two opposing effects caused by an increase in the rate of employment. One is the reserve-army effect: as the labor market tightens and labor shortage becomes clearer, the bargaining power of workers increases, which exerts an upward pressure on wages. The other is the reserve-army-creation effect: such an upward pressure on wages leads firms to adopt labor-saving technical changes to intentionally create the reserve-army of labor.

We have presented the four cases according to the size of parameters of the investment function, the relative bargaining power of firms, the reserve-army effect, and the reserve-army-creation effect. We obtain cyclical fluctuations in two of the four cases. These two cases correspond to the case where the relative bargaining power of firms is weak, the reserve-army effect is strong, and the reserve-army-creation effect is weak.

Using the model, we have analyzed how the relative bargaining power of workers and firms affects the long-run equilibrium rate of employment. The relationship between the bargaining power and the employment rate differs depending on the regime of the long-run equilibrium. If the long-run equilibrium is characterized as the wage-led growth regime, a rise in the relative bargaining power of workers increases the employment rate. If, on the other hand, the long-run equilibrium is characterized as the profit-led growth regime, a rise in the relative bargaining power of workers decreases the employment rate. The latter result is also obtained in the mainstream theory, but the former result is never obtained in the mainstream theory.

Note, however, that in the wage-led growth regime, an increase in the workers' bargaining power leads to higher employment, but it simultaneously leads to lower profit share: the workers' interests interfere with firms' interests. For this reason, it may be difficult to implement an economic policy intended to adjust the bargaining power of both classes. Even in this case, nonetheless, the demand stimulation policy is effective. As discussed in the text, the stimulation of effective demand brings about higher employment and accordingly, lower unemployment. This policy implication is never obtained in the mainstream theory.

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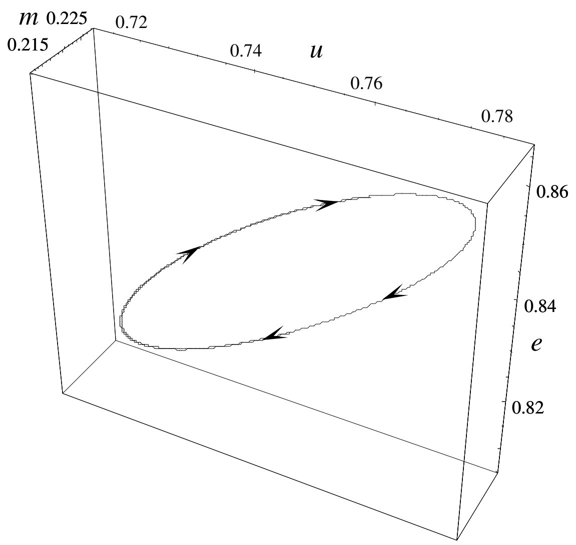

Figure 1: Solution path in case 1 ($\alpha = 4$)

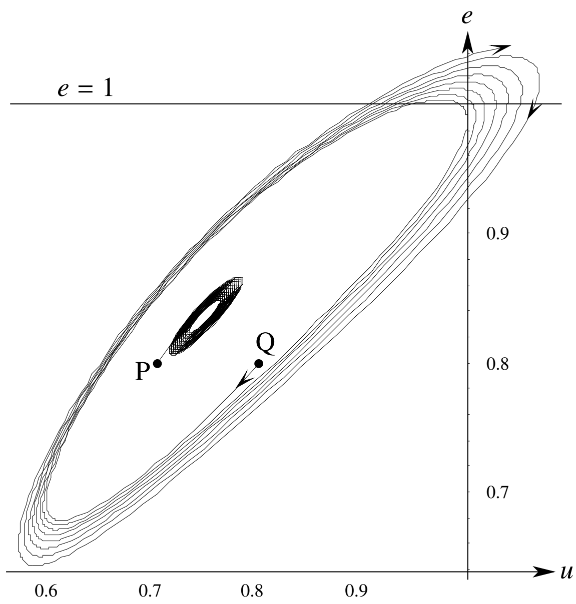

Figure 2: Solution paths starting from different initial values in case 1 ($\alpha = 4$)

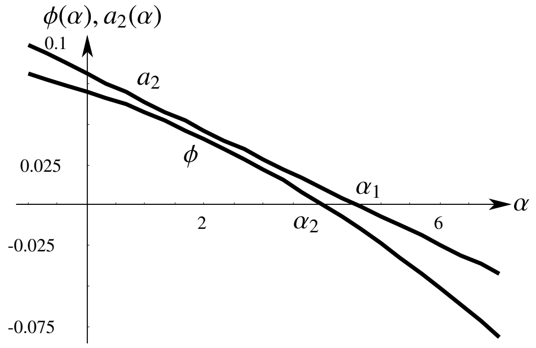

Figure 3: Graphs of $a_2(\alpha)$ and $\phi(\alpha)$ in case 1

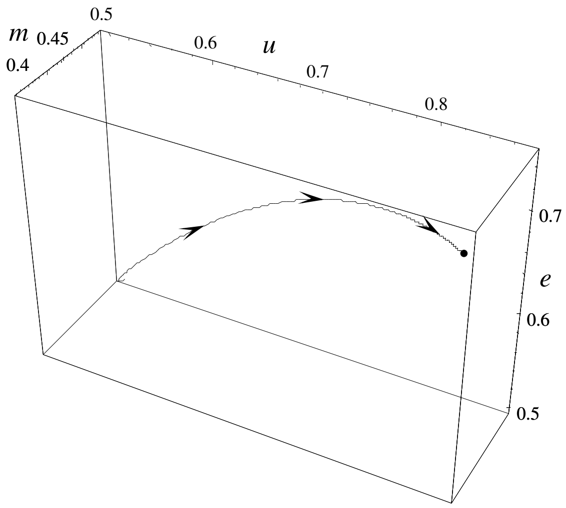

Figure 4: Solution path in case 2 ($\alpha = 1$)

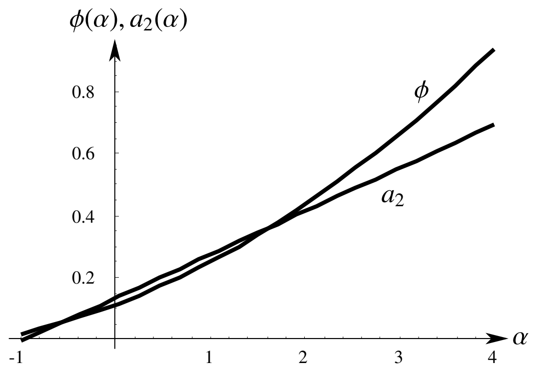

Figure 5: Graphs of $a_2(\alpha)$ and $\phi(\alpha)$ in case 2

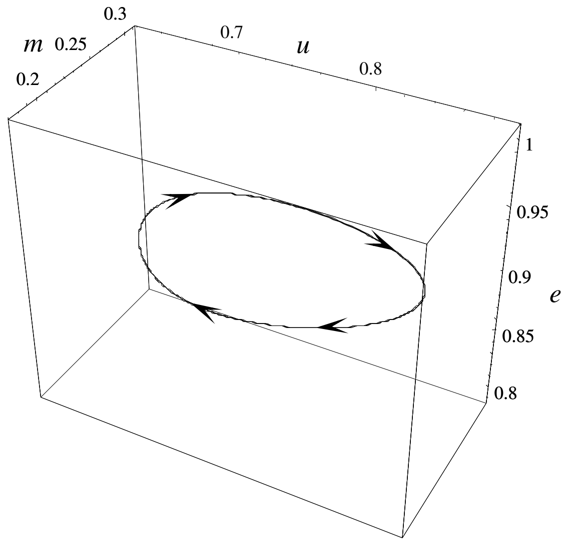

Figure 6: Solution path in case 3 ($\alpha = 2$)

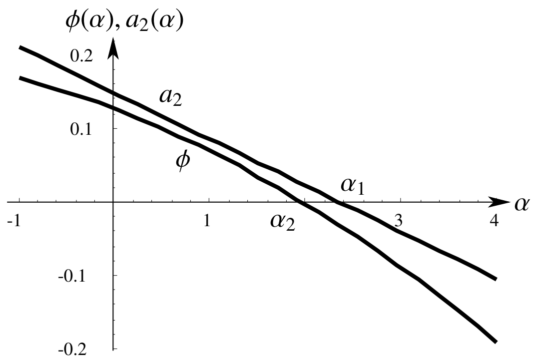

Figure 7: Graphs of $a_2(\alpha)$ and $\phi(\alpha)$ in case 3

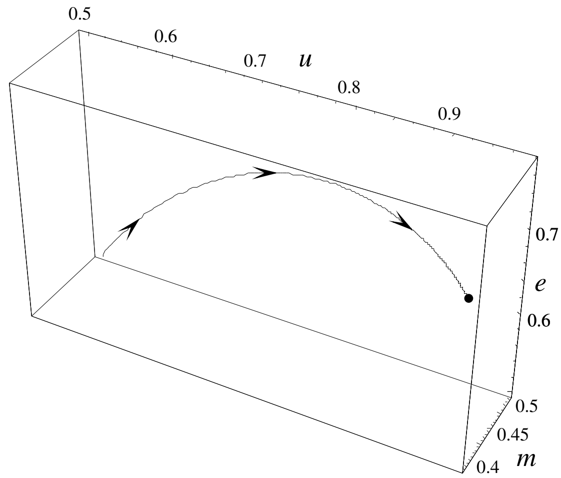

Figure 8: Solution path in case 4 ($\alpha = 1$)

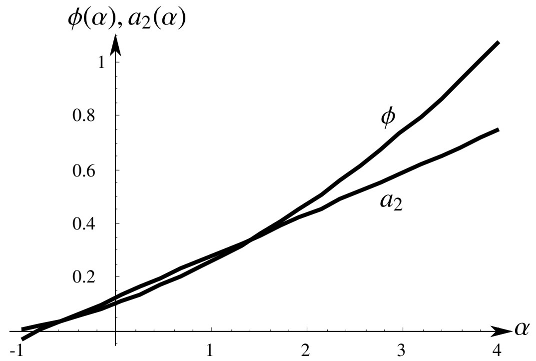

Figure 9: Graphs of $a_2(\alpha)$ and $\phi(\alpha)$ in case 4

Table 1: Results of comparative statics analysis

$\beta < \gamma$	s	n	θ	m_f	ψ
u^*	$-$	$-/+^{\dagger}$	$-$	$-$	$+^{\ddagger}$
m^*	$+/-^{\dagger}$	$+/-^{\dagger}$	$+$	$+$	$-/+^{\dagger}$
e^*, g_a^*	$-$	$-$	$-$	$-$	$+$
g^*	$-$	$-/+^{\dagger}$	$-$	$-$	$+$
$\beta > \gamma$	s	n	θ	m_f	ψ
u^*	$-$	$-/+^{\dagger}$	$-$	$-$	$+^{\ddagger}$
m^*	$+/-^{\dagger}$	$+/-^{\dagger}$	$+$	$+$	$-/+^{\dagger}$
e^*, g_a^*	$-$	$-$	$+$	$+$	$+$
g^*	$-$	$+/-^{\dagger}$	$+$	$+$	$+$

† When $\Gamma < 0$, the left-hand sign applies, and when $\Gamma > 0$, the right-hand sign applies.

‡ These signs are obtained by numerical calculations.

A Appendix

The effects of a rise in parameters on the long-run equilibrium values are as follows.

■ The rate of capacity utilization

$$\frac{du^*}{ds} = \frac{u^*[s\beta u^*(1-\theta)m_w'(e^*) - (1-s\beta u^*)g_a'(e^*)]}{s(1-\gamma)\Theta} < 0, \tag{A-1}$$

$$\frac{du^*}{dn} = \frac{(1-\beta)u^*\Gamma}{(1-\gamma)m^*\Theta} \gtrless 0, \tag{A-2}$$

$$\frac{du^*}{d\theta} = -\frac{(1-\beta)u^* g_a'(e^*)[m_f - m_w(e^*)]}{(1-\gamma)m^*\Theta} < 0, \tag{A-3}$$

$$\frac{du^*}{dm_f} = -\frac{\theta(1-\beta)u^* g_a'(e^*)}{(1-\gamma)m^*\Theta} < 0, \tag{A-4}$$

$$\frac{du^*}{d\psi} = \frac{u^*[(1-\gamma)\Theta - s(1-\beta)m^*\Gamma]}{\psi(1-\gamma)^2\Theta}. \tag{A-5}$$

■ The profit share

$$\frac{dm^*}{ds} = -\frac{\gamma u^* m^*\Gamma}{(1-\gamma)\Theta} \gtrless 0, \tag{A-6}$$

$$\frac{dm^*}{dn} = -\frac{\Gamma}{\Theta} \gtrless 0, \tag{A-7}$$

$$\frac{dm^*}{d\theta} = \frac{g_a'(e^*)[m_f - m_w(e^*)]}{\Theta} > 0, \tag{A-8}$$

$$\frac{dm^*}{dm_f} = \frac{\theta g_a'(e^*)}{\Theta} > 0, \tag{A-9}$$

$$\frac{dm^*}{d\psi} = \frac{sm^*u^*\Gamma}{\psi(1-\gamma)\Theta} \gtrless 0. \tag{A-10}$$

■ The rate of employment

$$\frac{de^*}{ds} = -\frac{\gamma u^* m^*}{(1-\gamma)\Theta} < 0, \tag{A-11}$$

$$\frac{de^*}{dn} = -\frac{1}{\Theta} < 0, \tag{A-12}$$

$$\frac{de^*}{d\theta} = \frac{s(\beta-\gamma)u^*[m_f - m_w(e^*)]}{(1-\gamma)\Theta} \gtrless 0, \tag{A-13}$$

$$\frac{de^*}{dm_f} = \frac{\theta s(\beta-\gamma)u^*}{(1-\gamma)\Theta} \gtrless 0, \tag{A-14}$$

$$\frac{de^*}{d\psi} = \frac{sm^*u^*}{\psi(1-\gamma)\Theta} > 0. \tag{A-15}$$

■ The rate of capital accumulation

$$\frac{dg^*}{ds} = -\frac{\gamma m^* u^* g_a'(e^*)}{(1-\gamma)\Theta} < 0, \tag{A-16}$$

$$\frac{dg^*}{dn} = -\frac{s(\beta-\gamma)u^*\Gamma}{(1-\gamma)\Theta} \gtrless 0, \tag{A-17}$$

$$\frac{dg^*}{d\theta} = \frac{s(\beta-\gamma)u^* g_a'(e^*)[m_f - m_w(e^*)]}{(1-\gamma)\Theta} \gtrless 0, \tag{A-18}$$

$$\frac{dg^*}{dm_f} = \frac{\theta s(\beta-\gamma)u^* g_a'(e^*)}{(1-\gamma)\Theta} \gtrless 0, \tag{A-19}$$

$$\frac{dg^*}{d\psi} = \frac{s m^* u^* g_a'(e^*)}{\psi(1-\gamma)\Theta} > 0. \tag{A-20}$$