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Endogenous Technological Change and
Distribution with Inter-Class Conflict:
A Kaleckian Model of Growth

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Abstract

This paper presents a Kaleckian model of growth in which both technological change and income distribution are endogenously determined by inter-class conflict between capitalists and workers. Considering the adjustment speed of variables, we investigate the medium-run equilibrium and the long-run equilibrium. In the medium run, the rate of capital accumulation and the profit share are adjusted to the medium-run equilibrium. In the long run, the normal planned rate of capacity utilization and the growth rate of labor productivity are adjusted to the long-run equilibrium. In the analysis, two alternative investment functions are used: a Kalecki type investment function and a Marglin and Bhaduri (1990) type investment function. We show that different results are obtained both in the medium run and in the long run depending on which investment function is used.

Keywords: Kaleckian model; endogenous technological change; long-run equilibrium; wage-led; profit-led

JEL Classification: E12; E22; E25; O30; O41

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1 Introduction

The purpose of this paper is to present a Kaleckian growth model that incorporates endogenous technological change and to investigate its properties.[1] Although a large number of attempts to endogenize technical progress have been made in mainstream growth theory, relatively less attention has been paid in the post-Keynesian tradition. In mainstream growth models, much emphasis is placed on technical progress as an engine of growth because supply-side factors determine economic growth. In contrast, since demand-side factors decide economic growth in post-Keynesian growth models, supply-side factors have not been considered so much. This is not to say that there have been no attempts to endogenize technical progress in the Kaleckian model. You (1994) introduces into a Kaleckian model a technical progress such that the growth rate of the capital-labor ratio depends on the rate of capital accumulation. In Cassetti (2003), induced technical progress known as the Kaldor-Verdoorn law (Verdoorn, 1949; Kaldor, 1966) is incorporated into a Kaleckian growth model. Stockhammer and Onaran (2004) also use the Kaldor-Verdoorn law to build a model based on Marglin and Bhaduri's (1990) work, and they empirically test the model for the US, UK, and France by means of a structural VAR analysis. Lima (2004) develops a Kaleckian model in which endogenous technological innovation plays a significant role. In Lima's model, the rate of labor-saving technological innovation depends non-linearly on the wage share, which can generate limit cycles as to the wage share and the capital-effective labor ratio.[2]

In order to endogenize technological change, this paper adopts a technique such that a change in the growth rate of labor productivity depends positively on the difference between the growth rates of employment and labor supply. This formulation is proposed by Dutt (2006) and Bhaduri (2006).[3] According to Dutt (2006), this view of technological change differs from the mainstream endogenous growth theory in two respects. First, it draws attention on the demand side of the economy: technological change occurs in response to labor shortage caused by the growth of employment rather than supply side which focuses on the research and development process. Second, it stresses the process of diffusion of technological change among firms which are driven to adopt the technology by labor shortages, rather than the process of invention. To the extent that demand-side factors and diffusion are

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1) See Kalecki (1971) for his economic theory. For the fundamental Kaleckian model, see Rowthorn (1981). Lavoie (1992, ch. 6) explains various Kaleckian models.

2) Raghavendra (2006) also obtains limit cycles in a Kaleckian model, but the method is different from that in Lima (2004) mentioned above. In Raghavendra's (2006) model, stagnationism and exhilarationism can be repeated alternately, and this result greatly depends on the non-linearity of the investment function.

3) Indeed, Bhaduri (2006) proposes two specifications. One is what we employ in this paper, and the other is that the growth rate of labor productivity is adjusted through the gap between the growth rates of real wage and labor productivity.

important in the process of technological change, this approach can be argued to be a plausible one. Bhaduri (2006) states that this captures a view that technological change is driven by inter-class conflict over income distribution between workers and capitalists. Bhaduri's (2006) model is not a Kaleckian one because income distribution is not determined by mark-up pricing. However, it bears similarity to the Kaleckian model in that effective demand plays a crucial role in determining output. In contrast, Dutt's (2006) model can be said to be Kaleckian, but it does not deal with such issues as income distribution and inflation because its purpose is to present a simple growth model that integrates the roles of aggregate demand and aggregate supply.

Our specification of endogenous technological change has the following theoretical implication. Conventional Kaleckian growth models assume that labor supply is unlimited and that firms employ as many workers as they desire at given wages. If, however, the labor supply grows at an exogenously given rate, there is no guarantee that the endogenously determined growth rate of employment is equal to the labor supply growth. Thus, if the former exceeds the latter in the steady state, then the rate of unemployment will keep on rising, but this is not realistic.[4] In contrast, the steady state unemployment rate in our model remains constant because the two growth rates coincide in the long run. Stockhammer (2004) proposes and investigates a post-Keynesian growth model in which income distribution and the rate of unemployment are endogenously determined in the long run. In this respect, the model is similar to our model. In Stockhammer (2004), however, technological change is not considered, and consequently the long-run rate of capital accumulation is equal to the natural rate of growth (i.e., the growth rate of labor supply) that is given exogenously. In contrast, the log-run rate of capital accumulation in our model is endogenously determined.

The basic framework of our model is based on a series of Mario Cassetti's studies (Cassetti, 2002; 2003; 2006). In standard Kaleckian models, the level of money wage and mark-up are fixed and given exogenously, so that the price level is constant. Cassetti (2002, 2003, 2006) combine a Kaleckian growth model and the theory of conflicting-claims inflation, in which the rate of inflation is determined by negotiations between workers and capitalists (Rowthorn, 1977).[5] Let us explain the theory according to Dutt (1987), Lavoie (1992, ch. 7), and Cassetti (2002, 2003, 2006). Workers are assumed to attempt to change income distribution in their favor by negotiating for higher money wages, while capitalists are assumed to secure their profits by setting higher prices. At a steady state, wage and profit shares will be constant, and the growth rates of prices and money wages are equalized if the

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4) Cassetti (2002) also sees it as a problem that the long-run rate of employment in the conventional Kaleckian model is not constant.

5) Lima (2004), mentioned above, is also an attempt to integrate the theory of conflicting-claims inflation with a Kaleckian growth model.

labor productivity is constant. The resultant inflation rate depends on the bargaining power of workers and that of capitalists, and on the gap between the target profit share desired by workers and the target profit share desired by capitalists. This gap is called the "aspiration gap."

In what follows, we analyze the medium-run equilibrium and the log-run equilibrium of the model to consider the adjustment speed of endogenous variables. In the medium run, the rate of capital accumulation and the profit share are simultaneously adjusted to the medium-run equilibrium while, in the long run, both the normal planned rate of capacity utilization and the growth rate of labor productivity are adjusted to the long-run equilibrium.[6]

In addition to the comparison between the medium run and the long run, we use two alternative investment functions to compare results obtained from differences between the investment functions. In the conventional Kaleckian model, investment is assumed to depend positively on the profit rate and the rate of capacity utilization (hereafter the Kalecki type investment function). In contrast, Marglin and Bhaduri (1990) argue that investment should be an increasing function of the profit share and the rate capacity utilization (hereafter the MB type investment function).[7] In this regard, Cassetti (2006) conducts an analysis that treats the medium-run equilibrium and the long-run equilibrium using the Kalecki type investment function, but does not use the MB type investment function.

The remainder of the paper is organized as follows. Section 2 presents the basic framework of our model. Section 3 analyzes a case where capacity is fully utilized. Section 4 investigates the medium-run equilibrium and the long-run equilibrium using the Kalecki type investment function when there is excess capacity. Section 5 conducts the same analysis as in Section 4 using the MB type investment function. Section 6 offers results of comparative static analysis both in the medium-run equilibrium and in the long-run equilibrium. Section 7 summarizes the paper and provides additional comments on future directions.

2   Basic framework of the model

2.1   Adjustment in the rate of capital accumulation

Consider an economy in which there are two social classes, workers and capitalists. Suppose that workers consume all their wage income and capitalists save a constant fraction *s* of

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6) For the concept of the long-run equilibrium in the Kaleckian model, see Lavoie (1995, 2003). Cassetti (2006) conducts an analysis that treats the medium-run and long-run separately as in the present paper. The concept of the medium-run in Cassetti (2006) is equal to that in the present paper. In the long-run of Cassetti (2006) four key variables—normal capacity, the normal rate of profit, autonomous investment, and the drop-out ratio of capital equipment—are simultaneously adjusted.

7) For this issue, see also Blecker (2002).

their profits. Since savings $S_t = sr_tK_t$ equal investments I_t in equilibrium, the rate of capital accumulation g_t leads to

$$g_t = sr_t, \quad 0 < s \leq 1, \tag{1}$$

where K_t is the capital stock and r_t is the rate of profit.

Let us specify two alternative investment functions. In the Kalecki type investment function, the rate of capital accumulation desired by firms is assumed to depend on the rate of profit and the discrepancy between the actual rate of capacity utilization u_t and the normal planned rate of capacity utilization u_n.

$$\text{Kalecki type}: \ g_{dt} = \gamma + \delta r_t + \varepsilon(u_t - u_n), \quad \gamma > 0, \ \delta > 0, \ \varepsilon > 0, \ \gamma - \varepsilon u_n > 0, \tag{2}$$

where γ is the growth rate of autonomous investment, δ is the sensitivity of investment to the rate of profit, and ε is the sensitivity of investment to the rate of capacity utilization. In the medium run, u_n is considered to be constant. The constraint $\gamma - \varepsilon u_n > 0$ means that $g_{dt} > 0$ when $r_t = u_t = 0$.

In contrast, the MB type investment function can be described as follows:

$$\text{MB type}: \ g_{dt} = \gamma + \delta m_t + \varepsilon(u_t - u_n), \quad \gamma > 0, \ \delta > 0, \ \varepsilon > 0, \ \gamma - \varepsilon u_n > 0, \tag{3}$$

where m_t is the profit share.[8] The constraint $\gamma - \varepsilon u_n > 0$ means that $g_{dt} > 0$ when $m_t = u_t = 0$.

Suppose that firms operate with the following fixed coefficient production function:

$$Y_t = \min\{a_t E_t, (u_t/k)K_t\}, \tag{4}$$

where Y_t is real output, E_t employment, and $a_t = Y_t/E_t$ the level of labor productivity. The rate of capacity utilization is defined as $u_t = Y_t/Y_t^*$, where Y_t^* is the potential output. Define the ratio of the capital stock to the potential output as $k = K_t/Y_t^*$ and suppose that k is constant, that is, both K_t and Y_t^* grow at the same rate.[9] To simplify the analysis, let $k = 1$. Then, we obtain $u_t = Y_t/K_t$. Note that the relationship among the profit rate, the profit share,

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8) Note that the same parameters are used both in the Kalecki type and in the MB type investment functions. To be exact, we have to use $g_{dt} = \gamma_0 + \delta_0 r_t + \varepsilon_0(u_t - u_n)$ for the Kalecki type investment function and $g_{dt} = \gamma_1 + \delta_1 m_t + \varepsilon_1(u_t - u_n)$ for the MB type investment function. In the following analysis, however, we do not distinguish the parameters to avoid a troublesome notation.

9) Foley and Michl (1999) emphasize that, in capitalist economies, a combination of growing labor productivity and declining capital productivity (a rise in k in our model) is the typical pattern of technological change. They accordingly term the pattern Marx-biased technical change. When an economy experiences Marx-biased technical change, the rate of profit will decline as long as the profit share remains constant. In our model, such a biased technical change is not taken into consideration.

and the rate of capacity utilization is given by $r_t = m_t u_t$ and the relationship among the rate of capacity utilization, the rate of capital accumulation, and the profit share is given by $u_t = g_t/(s m_t)$.

An equation of motion for the rate of capital accumulation can be formulated by

$$\dot{g}_t = \alpha(g_{dt} - g_t), \quad \alpha > 0, \tag{5}$$

where α is the speed of adjustment. When the profit share is constant, there is a one-to-one relationship between the rate of capital accumulation and the rate of capacity utilization. Thus, equation (5) implies quantity adjustment by the rate of capacity utilization in the goods market: excess demand ($g_{dt} > g_t$) leads to a rise in the rate of capacity utilization whereas excess supply ($g_{dt} < g_t$) leads to a decline in the rate of capacity utilization.

2.2 Adjustment in the profit share

In the Kaleckian tradition, firms operate with excess capacity in oligopolistic goods markets and set their price p_t with a mark-up μ_t on unit labor costs.

$$p_t = (1 + \mu_t)\frac{w_t}{a_t}, \tag{6}$$

where w_t is the money wage rate. It should be noted that $m_t = \mu_t/(1 + \mu_t)$, that is, there exists a one-to-one relationship between the mark-up and the profit share, so that m_t and μ_t change in the same direction. Therefore, we can rewrite equation (6) as follows:

$$p_t = \left(\frac{1}{1 - m_t}\right)\frac{w_t}{a_t}. \tag{7}$$

Differentiating both sides of equation (7) with respect to time yields:

$$\frac{\dot{p}_t}{p_t} = \frac{\dot{w}_t}{w_t} + \frac{\dot{m}_t}{1 - m_t} - \frac{\dot{a}_t}{a_t}. \tag{8}$$

Let us specify the dynamics of the money wage and price using the theory of conflicting-claims inflation.

First, suppose that the growth rate of the money wage which workers manage to negotiate depends on the discrepancy between their target real wage rate and the actual real wage rate. Let ω_t be the actual real wage rate. Then, the wage share is given by $1 - m_t = \omega_t/a_t$. This means that, given the labor productivity, to determine the real wage means to determine the wage share. Therefore, setting the target real wage rate is equivalent to setting the target

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profit share. From this the dynamics of the money wage can be described as:

$$\frac{\dot{w}_t}{w_t} = \theta_w(m_t - m_w), \quad \theta_w > 0,\ 0 < m_w < 1, \tag{9}$$

where m_w is the target profit share set by workers, which is assumed to be given exogenously, and θ_w is the speed of adjustment constant, which reflects the bargaining power of workers.

Second, suppose that firms set their price to close the gap between their target mark-up and the actual mark-up. The target mark-up corresponds to the target profit share because the mark-up bears a one-to-one relation to the profit share.

$$\frac{\dot{p}_t}{p_t} = \theta_f(m_f - m_t), \quad \theta_f > 0,\ 0 < m_f < 1, \tag{10}$$

where m_f is the target profit share set by firms and θ_f denotes the speed of adjustment, which can be interpreted as the bargaining power of firms.

For ease of tractability, we impose the following constraints on θ_f and θ_w.[10]

$$\theta_f + \theta_w = 1, \quad 0 < \theta_f, \theta_w < 1. \tag{11}$$

Let us denote the bargaining power of firms by $\theta_f \equiv \theta$. Then, the bargaining power of workers is given by $\theta_w \equiv 1 - \theta$. The bargaining power of firms gets stronger as θ approaches unity while that of workers gets stronger as θ approaches zero.

Substituting equations (9) and (10) into equation (8), we obtain an equation of motion for the profit share.

$$\dot{m}_t = (1 - m_t)[\theta(m_f - m_t) - (1 - \theta)(m_t - m_w) + g_{at}], \tag{12}$$

where g_{at} is the growth rate of labor productivity. Let us now turn to the aspiration gap mentioned above. For ease of explanation, suppose that the growth rate of labor productivity is given exogenously as $g_{at} = \bar{g}_a$. The profit share will be constant when $\dot{m}_t = 0$, and the equilibrium profit share m^* is given by

$$m^* = A + \bar{g}_a, \quad \text{where } A \equiv \theta m_f + (1 - \theta)m_w,\ 0 < A < 1. \tag{13}$$

The parameter A denotes the weighted average of the target profit shares set by firms and workers. Substituting equation (13) into equations (9) and (10) yields the rates of change in

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10) The constraint $0 < \theta_f, \theta_w < 1$ is also adopted by Dutt and Amadeo (1993), who, however, do not assume $\theta_f + \theta_w = 1$.

the money wage and price, respectively.

$$\frac{\dot{w}}{w} = \theta(1-\theta)(m_f - m_w) + (1-\theta)\bar{g}_a, \tag{14}$$

$$\frac{\dot{p}}{p} = \theta(1-\theta)(m_f - m_w) - \theta\bar{g}_a. \tag{15}$$

If $\bar{g}_a = 0$, then $\dot{p}/p = \dot{w}/w$ and inflation results from the aspiration gap $m_f - m_w$.[11] When $m_f > m_w$, the inflation rate is positive. Note that the growth rate of the real wage rate $\dot{\omega}/\omega = \dot{w}/w - \dot{p}/p$ is given by $\bar{g}_a$. In what follows, g_a will be determined endogenously in the long-run equilibrium, and thus changes in parameters that affect g_a influence the inflation rate.

2.3   Adjustment in the growth rate of labor productivity

Let us specify endogenous technological change. Following Bhaduri (2006), we describe an adjustment in the growth rate of labor productivity as follows:

$$\dot{g}_{at} = \beta(g_{Et} - n), \quad \beta > 0, \tag{16}$$

where β is the speed of adjustment, $g_{Et} \equiv \dot{E}_t/E_t$ is the rate of change in employment, and n is the exogenously given growth rate of labor supply. Since $E_t = Y_t/a_t$, we have $g_{Et} = (\dot{u}_t/u_t) + g_t - g_{at}$. That is, the rate of change in employment is decomposed into the rate of change in capacity utilization, the rate of capital accumulation, and the growth rate of labor productivity. Note that in the following analysis g_a is assumed to be constant in the medium run and equation (16) is introduced in the long run. As shall be explained later, the rate of capacity utilization in the long run is constant, so that $\dot{u}_t/u_t = 0$. Therefore, in the long run, equation (16) can be rewritten as

$$\dot{g}_{at} = \beta(g_t - g_{at} - n). \tag{17}$$

3   Full-capacity equilibrium

This section analyzes a case where capacity is fully utilized, that is, $u_t = 1$ in order to compare the full-capacity case with demand-constrained cases introduced in Sections 4 and

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11) In a series of studies by M. Cassetti, m_f and m_w are determined endogenously. Cassetti (2002) assumes that m_f depends positively on the rate of capacity utilization while m_w depends negatively on the employment rate. Cassetti (2003) gives m_f exogenously and defines m_w as a decreasing function of the growth rate of employment. In Cassetti (2006), m_f is given exogenously and m_w is decreasing in the rate of capital accumulation.

5. We assume that the full-capacity equilibrium is stable and derive stability conditions. These conditions are also imposed on the demand-constrained cases.

3.1 Medium-run analysis

When capacity is fully utilized, capital accumulation is constrained by saving, so that an autonomous investment function does not exist. In this case, the rate of profit is given by $r_t = m_t$, from which we have $g_t = sm_t$ with equation (1). Thus, an equation of motion for the rate of capital accumulation leads to $\dot{g}_t = s\dot{m}_t$. The dynamics of the profit share m_t is given by equation (12).

The medium-run equilibrium is characterized by a situation in which $\dot{m}_t = \dot{g}_t = 0$. The equilibrium values are as follows:

$$m^* = A + g_a, \tag{18}$$

$$g^* = s(A + g_a), \tag{19}$$

where g^* is the rate of capital accumulation in the medium-run equilibrium. From equation (12), we obtain $d\dot{m}_t/dm_t < 0$. Therefore, the medium-run equilibrium is stable.

3.2 Long-run analysis

The long-run dynamics of the full capacity case is governed only by $\dot{g}_a = \beta(g^* - g_a - n)$ from equation (17), and the long-run equilibrium values are given by

$$g_a^* = \frac{sA - n}{1 - s}, \tag{20}$$

$$g^{**} = \frac{s(A - n)}{1 - s}, \tag{21}$$

where g_a^* and g^{**} are the long-run growth rate of labor productivity and the long-run rate of capital accumulation, respectively. For g_a^* to be positive and stable, we require $sA - n > 0$. We also need the condition $A - n > 0$ for g^{**} to be positive. In the following analysis, we assume that these two conditions are met.

4 Analysis with the Kalecki type investment function

4.1 Medium-run analysis

The medium-run equilibrium is defined as a situation in which both the profit share and the rate of capital accumulation stay constant. Substituting equations (1) and (2) into equation (5), we obtain an equation of motion for the rate of capital accumulation as follows:

$$\dot{g}_t = \alpha\left[\left(\frac{\varepsilon}{sm_t} + \frac{\delta}{s} - 1\right)g_t + (\gamma - \varepsilon u_n)\right]. \tag{22}$$

The dynamics of the profit share is given by equation (12).

From $\dot{m}_t = 0$, the profit share in the medium-run equilibrium is given by $m^* = A + g_a$. The rates of capital accumulation and capacity utilization in the medium-run equilibrium are given by

$$g^* = \frac{(\gamma - \varepsilon u_n)sm^*}{(s - \delta)m^* - \varepsilon}, \quad u^* = \frac{\gamma - \varepsilon u_n}{(s - \delta)m^* - \varepsilon}. \tag{23}$$

For the medium-run equilibrium to be positive, the following condition is needed.

$$(s - \delta)m^* - \varepsilon > 0. \tag{24}$$

This condition has the following meaning. Using $r = mu$, we obtain $g_d = \delta(mu) + \varepsilon(u - u_n)$ from the investment function and $g = smu$ from the saving function. The partial derivative of g_d with respect to u is given by $\partial g_d/\partial u = \delta m + \varepsilon$ and that of g with respect to u is given by $\partial g/\partial u = sm$. Accordingly, equation (24) states that $\partial g/\partial u > \partial g_d/\partial u$, that is, saving reacts to the rate of capacity utilization more strongly than investment. Rearranging equation (24) gives $(s - \delta)m^* > \varepsilon$, which implies $s > \delta$ because $\varepsilon > 0$. The condition $s > \delta$ means that saving responds to the rate profit more strongly than investment. In addition to these constraints, $\gamma - \varepsilon u_n < (s - \delta)m^* - \varepsilon$ is needed for $u_t < 1$.

Let us examine the stability of the medium-run equilibrium. Let $\mathbf{J} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}$ be the

Jacobian matrix for this dynamical system. Then, we have

$$J_{11} = \frac{\partial \dot{m}_t}{\partial m_t} = -(1 - m^*) < 0, \tag{25}$$

$$J_{12} = \frac{\partial \dot{m}_t}{\partial g_t} = 0, \tag{26}$$

$$J_{21} = \frac{\partial \dot{g}_t}{\partial m_t} = -\frac{\alpha \varepsilon}{s} \frac{g^*}{(m^*)^2} < 0, \tag{27}$$

$$J_{22} = \frac{\partial \dot{g}_t}{\partial g_t} = -\frac{\alpha(\gamma - \varepsilon u_n)}{g^*} < 0. \tag{28}$$

All the elements are evaluated at m^* and g^*.

To know the stability, we have to know the sign of the trace of $\mathbf{J}$ ($\mathrm{tr}\,\mathbf{J}$) and that of the determinant of $\mathbf{J}$ ($\det \mathbf{J}$), which are respectively given by

$$\mathrm{tr}\,\mathbf{J} = J_{11} + J_{22} = -(1 - m^*) - \frac{\alpha(\gamma - \varepsilon u_n)}{g^*} < 0, \tag{29}$$

$$\det \mathbf{J} = J_{11}J_{22} - \underbrace{J_{12}J_{21}}_{=0} = \alpha(1 - m^*)\frac{\gamma - \varepsilon u_n}{g^*} > 0. \tag{30}$$

We have both $\mathrm{tr}\,\mathbf{J} < 0$ and $\det \mathbf{J} > 0$, which imply the local stability of the medium-run equilibrium. Moreover, the discriminant of the characteristic equation for the Jacobian matrix is given by $\mathcal{D} = (J_{11} - J_{22})^2 > 0$, which shows that two roots are real and distinct. Therefore, the medium-run equilibrium leads to a stable node.
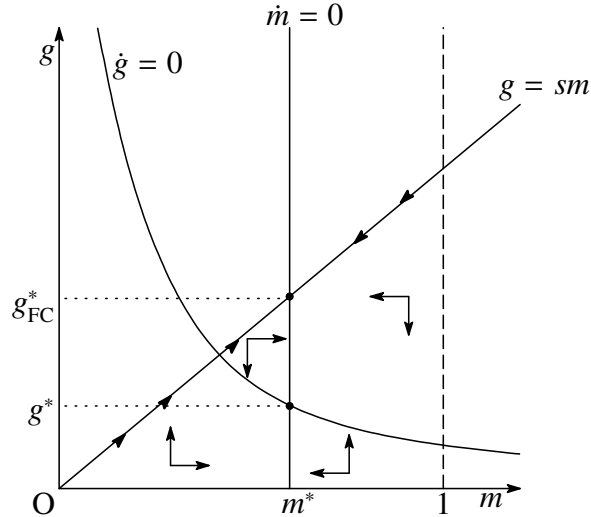

Figure 1: Convergence to the medium-run equilibrium—the Kalecki case

Let us draw the medium-run phase diagram on a (m, g)-plane (Figure 1). The $\dot{m} = 0$ locus is a straight line and vertical to the horizontal line because it does not depend on g.

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The $\dot{g} = 0$ locus is a downward-sloping curve and convex to the origin because

$$\frac{dg}{dm} = -\frac{s\varepsilon(\gamma - \varepsilon u_n)}{[(s - \delta)m - \varepsilon]^2} < 0, \tag{31}$$

$$\frac{d^2g}{dm^2} = \frac{2s\varepsilon(s - \delta)(\gamma - \varepsilon u_n)}{[(s - \delta)m - \varepsilon]^3} > 0. \tag{32}$$

The intersection of the two lines is the medium-run equilibrium, which is stable from the analysis above. The straight line from the origin $g = sm$ represents a combination of m and g when $u = 1$, that is, capacity is fully utilized. The intersection of this straight line and $\dot{m} = 0$ denotes the full-capacity equilibrium in the medium run and g^*_{FC} stands for the rate of capital accumulation in the full-capacity equilibrium. Excess capacity, that is, $u < 1$ corresponds to the region below $g = sm$. Note that equations of motion for m_t and g_t differ between the case where $u = 1$ and the case where $u < 1$. The full capacity equilibrium must lie on $g = sm$ and is not allowed to deviate from this line.

4.2 Long-run analysis

In the long run, the normal planned rate of capacity utilization and the growth rate of labor productivity will be adjusted.[12] Let us specify this adjustment mechanism as follows:

$$\dot{u}_{nt} = \phi(u^* - u_{nt}), \quad \phi > 0. \tag{33}$$

where ϕ is the speed of adjustment. From equation (17) the dynamics of the growth rate of labor productivity follows $\dot{g}_{at} = \beta(g_{Et} - n) = \beta(g^* - g_{at} - n)$. It should be noted that from the result of the medium-run equilibrium we have

$$u^* = \frac{\gamma - \varepsilon u_{nt}}{(s - \delta)(A + g_{at}) - \varepsilon}, \quad g^* = \frac{s(\gamma - \varepsilon u_{nt})(A + g_{at})}{(s - \delta)(A + g_{at}) - \varepsilon}, \tag{34}$$

which are inserted into $\dot{u}_{nt}$ and $\dot{g}_{at}$. The long-run equilibrium is a situation where $\dot{u}_{nt} = \dot{g}_{at} = 0$, and thus the equilibrium values u_n^* and g_a^* fulfill the following system of equations.

$$\frac{\gamma - \varepsilon u_n^*}{(s - \delta)(A + g_a^*) - \varepsilon} - u_n^* = 0 \tag{35}$$

$$\frac{s(\gamma - \varepsilon u_n^*)(A + g_a^*)}{(s - \delta)(A + g_a^*) - \varepsilon} - g_a^* - n = 0. \tag{36}$$

From equations (35) and (36), the following solutions are obtained:[13]

$$g_a^* = \frac{s\gamma - n(s - \delta)}{s - \delta}, \tag{37}$$

$$u_n^* = \frac{\gamma}{s\gamma + (s - \delta)(A - n)}, \tag{38}$$

$$g^{**} = \frac{s\gamma}{s - \delta}. \tag{39}$$

We need $s\gamma - n(s - \delta) > 0$ for g_a^* to be positive. From the analysis above, $s > \delta$ and $A - n > 0$, so that g^{**} and u_n^* are positive. For u_n^* to be smaller than unity, the following condition is required:

$$\underbrace{(s - \delta)}_{+} \underbrace{(A - n)}_{+} - \gamma \underbrace{(1 - s)}_{+} > 0, \tag{40}$$

which is assumed to be satisfied in the following analysis.

The local stability of the long-run equilibrium is analyzed by the Jacobian matrix of the two-variable system:

$$J_{11} = \frac{\partial \dot{u}_{nt}}{\partial u_{nt}} = \phi\left(\frac{\partial u^*}{\partial u_n} - 1\right) = -\phi\left[\frac{(s - \delta)m^{**}}{(s - \delta)m^{**} - \varepsilon}\right] < 0, \tag{41}$$

$$J_{12} = \frac{\partial \dot{u}_{nt}}{\partial g_{at}} = \phi\frac{\partial u^*}{\partial g_a} = -\phi\left[\frac{(s - \delta)(\gamma - \varepsilon u_n^*)}{[(s - \delta)m^{**} - \varepsilon]^2}\right] < 0, \tag{42}$$

$$J_{21} = \frac{\partial \dot{g}_{at}}{\partial u_{nt}} = \beta\frac{\partial g^*}{\partial u_n} = -\beta\left[\frac{\varepsilon s m^{**}}{(s - \delta)m^{**} - \varepsilon}\right] < 0, \tag{43}$$

$$J_{22} = \frac{\partial \dot{g}_{at}}{\partial g_{at}} = \beta\left(\frac{\partial g^*}{\partial g_a} - 1\right) = -\beta\left[\frac{s\varepsilon(\gamma - \varepsilon u_n^*)}{[(s - \delta)m^{**} - \varepsilon]^2} + 1\right] < 0, \tag{44}$$

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13) In fact, we can easily obtain the rate of capital accumulation in the long-run equilibrium g^{**} without solving equations (35) and (36) directly. Since $u = u_n$ in the long-run equilibrium, we have $g^{**} = \gamma + \delta r^{**}$ from the investment and saving functions, where r^{**} denotes the long-run rate of profit. Substituting $r^{**} = g^{**}/s$ into this equation, we obtain g^{**}. In addition, because $g_a^* = g^{**} - n$ from $g_a^* = g^{**} - n$, we obtain g_a^*.

where $m^{**} = A + g_a^*$ is the profit share in the long-run equilibrium. Necessary and sufficient conditions for local stability are given by both $\text{tr}\,\mathbf{J} < 0$ and $\det\mathbf{J} > 0$. It is obvious that $\text{tr}\,\mathbf{J} = J_{11} + J_{22} < 0$. $\det\mathbf{J}$ is given by

$$\det\mathbf{J} = J_{11}J_{22} - J_{12}J_{21} = \frac{\phi\beta(s - \delta)m^{**}}{(s - \delta)m^{**} - \varepsilon} > 0, \tag{45}$$

which is clearly positive. Therefore, the long-run equilibrium is locally stable, and either a stable node or a stable focus will be obtained. However, analysis with a phase diagram introduced below will show that the long-run equilibrium is stable node.

Let us draw on a (u_n, g_a)-plane two curves that correspond to $\dot{u}_{nt} = 0$ and $\dot{g}_{at} = 0$, respectively. The following analysis will be conducted by a linearized system around the long-run equilibrium. The slopes of these curves are respectively given by

$$\dot{u}_{nt} = 0 \implies \left.\frac{dg_a}{du_n}\right|_{\dot{u}_n=0} = -\frac{\frac{\partial u^*}{\partial g_a}}{\frac{\partial u^*}{\partial u_n} - 1} < 0, \tag{46}$$

$$\dot{g}_{at} = 0 \implies \left.\frac{dg_a}{du_n}\right|_{\dot{g}_a=0} = -\frac{\frac{\partial g^*}{\partial u_n}}{\frac{\partial g^*}{\partial g_a} - 1} < 0. \tag{47}$$

Thus, both the curves are downward-sloping. Since the long-run equilibrium is stable, as has been shown above, the slope of $\dot{u}_n = 0$ has to be steeper than that of $\dot{g}_a = 0$ (see Figure 2), otherwise the long-run equilibrium would be a saddle-point.
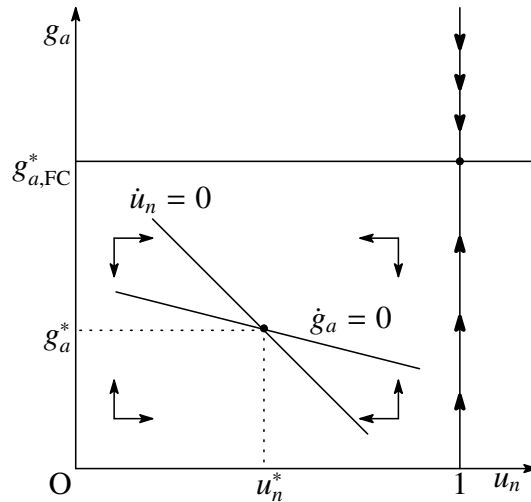

Figure 2: Convergence to the long-run equilibrium—the Kalecki case

In Figure 2, the horizontal line at $g_{a,\text{FC}}^*$ is the locus of $\dot{g}_a = 0$ when capacity is fully utilized. The intersection of this horizontal line and the line $u_n = 1$ corresponds to the full-capacity equilibrium in the long run. Note that as is the case with the medium-run analysis,

equations of motion for u_{nt} and g_{at} differ between the case where $u_n = 1$ and the case where $u_n < 1$. The full-capacity equilibrium must lie on the line $u_n = 1$. When $u_n < 1$, the long-run equilibrium is determined by the intersection of $\dot{u}_n = 0$ and $\dot{g}_a = 0$ as shown in Figure 2.

5 Analysis with the MB type investment function

5.1 Medium-run analysis

Substituting equations (1) and (3) into equation (5), we obtain an equation of motion for the rate of capital accumulation as follows:

$$\dot{g}_t = \alpha\left[\left(\frac{\varepsilon}{sm_t} - 1\right)g_t + \delta m_t + (\gamma - \varepsilon u_n)\right].\tag{48}$$

The dynamics of the profit share is given by equation (12) as before.

When $\dot{m}_t = \dot{g}_t = 0$, equilibrium values in the medium run are obtained as follows:

$$m^* = A + g_a,\tag{49}$$

$$g^* = \frac{sm^*[\delta m^* + (\gamma - \varepsilon u_n)]}{sm^* - \varepsilon},\tag{50}$$

$$u^* = \frac{\delta m^* + (\gamma - \varepsilon u_n)}{sm^* - \varepsilon}.\tag{51}$$

For g^* to be positive, we require

$$sm^* - \varepsilon > 0.\tag{52}$$

The partial derivative of g_d with respect to u is given by $\partial g_d/\partial u = \varepsilon$ and that of g with respect to u is given by $\partial g/\partial u = sm$. From this, equation (52) means that $\partial g/\partial u > \partial g_d/\partial u > 0$, that is, saving reacts to the rate of capacity utilization more strongly than investment. Moreover, for $u^* < 1$ we need $\gamma - \varepsilon u_n < (s - \delta)m^* - \varepsilon$, which is the same condition as in the analysis with Kalecki type investment function. This condition implies $s > \delta$ because the left-hand side of that condition is positive, that is, $\gamma - \varepsilon u_n > 0$.

Let us examine the stability of the medium-run equilibrium. All elements of the Jacobian

matrix $\mathbf{J} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}$ are as follows:

$$J_{11} = \frac{\partial \dot{m}_t}{\partial m_t} = -(1 - m^*) < 0, \tag{53}$$

$$J_{12} = \frac{\partial \dot{m}_t}{\partial g_t} = 0, \tag{54}$$

$$J_{21} = \frac{\partial \dot{g}_t}{\partial m_t} = \alpha \left[\frac{s\delta(m^*)^2 - \varepsilon g^*}{s(m^*)^2} \right], \tag{55}$$

$$J_{22} = \frac{\partial \dot{g}_t}{\partial g_t} = -\alpha \left(\frac{sm^* - \varepsilon}{sm^*} \right) < 0. \tag{56}$$

From these equations $\operatorname{tr} \mathbf{J}$ and $\det \mathbf{J}$ are respectively given by

$$\operatorname{tr} \mathbf{J} = -(1 - m^*) - \alpha \left(\frac{sm^* - \varepsilon}{sm^*} \right) < 0, \tag{57}$$

$$\det \mathbf{J} = \alpha(1 - m^*) \left(\frac{sm^* - \varepsilon}{sm^*} \right) > 0, \tag{58}$$

which satisfy necessary and sufficient conditions for local stability. The discriminant of the characteristic equation for the Jacobian matrix is given by $\mathcal{D} = (J_{11} - J_{22})^2 > 0$, which means that two roots are real and distinct. Therefore, the medium-run equilibrium is a stable node.
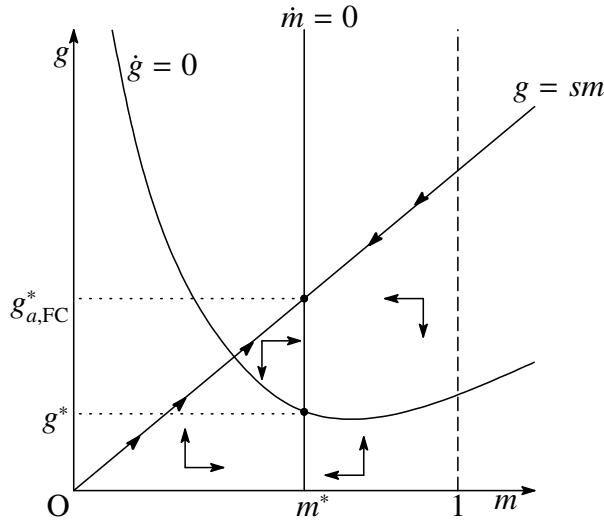

Figure 3: Convergence to the medium-run equilibrium—the MB case

Figure 3 shows the medium-run phase diagram in the MB case. A major difference

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between the MB case and the Kalecki case is the shape of $\dot{g} = 0$ curve.

$$\frac{dg}{dm} = \frac{s[\delta m \overbrace{(sm - \varepsilon)}^{+} - \varepsilon \overbrace{(\delta m + \gamma - \varepsilon u_n)}^{+}]}{(sm - \varepsilon)^2} \lesseqgtr 0, \tag{59}$$

$$\frac{d^2 g}{dm^2} = \frac{2s\varepsilon[\delta\varepsilon + s(\gamma - \varepsilon u_n)]}{(sm - \varepsilon)^3} > 0. \tag{60}$$

The right-hand side of equation (59) becomes positive or negative depending on the size of m, so that dg/dm also becomes positive or negative. Figure 3 is drawn on the assumption that $dg/dm < 0$ when m is small while $dg/dm > 0$ when m is large. As will be shown later, the downward-sloping part of $\dot{g} = 0$ curve corresponds to wage-led growth and the upward-sloping part of $\dot{g} = 0$ curve corresponds to profit-led growth.

5.2 Long-run analysis

Long-run dynamics is described by equations (17) and (33). Substituting equations (49), (50), and (51) into equations (17) and (33) and letting $\dot{u}_{n,t} = \dot{g}_{a,t} = 0$, we have

$$\frac{\delta(A + g_a^*) + (\gamma - \varepsilon u_n^*)}{s(A + g_a^*) - \varepsilon} - u_n^* = 0, \tag{61}$$

$$\frac{s(A + g_a^*)[\delta(A + g_a^*) + (\gamma - \varepsilon u_n^*)]}{s(A + g_a^*) - \varepsilon} - g_a^* - n = 0. \tag{62}$$

From these equations we obtain the following solutions:[14]

$$g_a^* = \frac{\gamma - n + \delta A}{1 - \delta}, \tag{63}$$

$$u_n^* = \frac{\gamma + \delta(A - n)}{s[\gamma + (A - n)]}, \tag{64}$$

$$g^{**} = \frac{\gamma + \delta(A - n)}{1 - \delta}. \tag{65}$$

Because $A - n > 0$ from the stability condition of full-capacity case, we need $1 - \delta > 0$ for g^{**} and u_n^* to be positive. The condition $1 - \delta > 0$ states that the sensitivity of investment to the profit share is smaller than unity. For g_a^* to be positive, we require $\gamma - n + \delta A > 0$.

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14) As in the Kalecki case, g_a^* is easily obtained as follows. Since $u = u_n$ in the long-run equilibrium, we have $g^{**} = \gamma + \delta(A + g_a^*)$ from the investment function. In addition, we have $g_a^* = g^{**} - n$ from $\dot{g}_{a,t} = 0$. Combining these two equations, we get $g_a^* + n = \gamma + \delta(A + g_a^*)$, from which g_a^* is obtained.

Moreover, For u_n^* to be smaller than unity, the following condition is needed.

$$\underbrace{(s - \delta)}_{+} \underbrace{(A - n)}_{+} - \gamma \underbrace{(1 - s)}_{+} > 0, \tag{66}$$

which is the same condition as in the Kalecki case.

Here, we examine the stability of the long-run equilibrium. We have

$$J_{11} = \frac{\partial \dot{u}_{nt}}{\partial u_{nt}} = -\phi \left(\frac{sm^{**}}{sm^{**} - \varepsilon} \right) < 0, \tag{67}$$

$$J_{12} = \frac{\partial \dot{u}_{nt}}{\partial g_{at}} = -\phi \left[\frac{\delta\varepsilon + s(\gamma - \varepsilon u_n^*)}{(sm^{**} - \varepsilon)^2} \right] < 0, \tag{68}$$

$$J_{21} = \frac{\partial \dot{g}_{at}}{\partial u_{nt}} = -\beta \left(\frac{s\varepsilon m^{**}}{sm^{**} - \varepsilon} \right) < 0, \tag{69}$$

$$J_{22} = \frac{\partial \dot{g}_{at}}{\partial g_{at}} = -\beta \left[\frac{s(1 - \delta)m^{**} \overbrace{(sm^{**} - 2\varepsilon)}^{+ \text{ or } -} + s\varepsilon(\gamma - \varepsilon u_n^*) + \varepsilon^2}{(sm^{**} - \varepsilon)^2} \right]. \tag{70}$$

If $sm^{**} - 2\varepsilon > 0$ in J_{22}, then $J_{22} < 0$. Since g_a^* is independent of ε and thus m^{**} is independent of ε, it is possible that $sm^{**} - 2\varepsilon > 0$ when ε is not large. This sufficient condition means that investment does not so strongly respond to the rate of capacity utilization. In the following analysis we assume it. Then, we have $\operatorname{tr} \mathbf{J} < 0$. The determinant of $\mathbf{J}$ is given by

$$\det \mathbf{J} = \frac{\phi\beta s(1 - \delta)m^{**}}{sm^{**} - \varepsilon} > 0. \tag{71}$$

Therefore, $\operatorname{tr} \mathbf{J} < 0$ and $\det \mathbf{J} > 0$, so that the long-run equilibrium is locally stable. The long-run phase diagram of the MB case is similar to that of the Kalecki case, and thus it is omitted (see Figure 3).

6 Effects of shifts in parameters

This section investigates the effects of shifts in parameters on the medium-run equilibrium and the long-run equilibrium. Results are summarized in Tables 1 and 2. The $+/-$ cases are treated in Appendix.

6.1 Comparative statics in the medium-run equilibrium

Table 1 shows the results of comparative statics in the medium-run equilibrium. In the analysis we assume $m_f > m_w$, which is relevant to the effects of θ. It is plausible to suppose that firms attempt to set m_f as high as possible whereas workers attempt to set m_w as low as possible. Therefore, the assumption $m_f > m_w$ is reasonable.

Table 1: Medium-run comparative statics

		s	γ	δ	ε	m_f	m_w	θ
Full capacity	m^*	0	0	0	0	+	+	+
	g^*	+	0	0	0	+	+	+
Kalecki case	m^*	0	0	0	0	+	+	+
	g^*	−	+	+	+/−	−	−	−
	u^*	−	+	+	+/−	−	−	−
MB case	m^*	0	0	0	0	+	+	+
	g^*	−	+	+	+/−	+/−	+/−	+/−
	u^*	−	+	+	+/−	−	−	−

A rise in the saving rate increases the rate of capital accumulation when capacity is fully utilized whereas decreases when there is excess capacity. The latter effect is known as "the paradox of thrift."

An increase in θ, that is, the bargaining power of firms leads to a rise in the profit share in every case. However, its effects on the rate of capital accumulation are different in the three cases: in the full-capacity case, g^* rises; in the Kalecki case, g^* falls; and in the MB case, g^* rises or falls depending on conditions. An increase in the bargaining power of firms leads to a decline in the rate of capacity utilization both in the Kalecki case and in the MB case.

6.2 Comparative statics in the long-run equilibrium

Table 2 shows the results of comparative statics in the long-run equilibrium. In the long run, all signs are definitely determined.

The effects of a rise in the saving rate on the rate of capital accumulation are different in the three cases. It has a positive effect on g^{**} when capacity is fully utilized. When there is excess capacity, its effects are different according to the investment functions. In the Kalecki case, a rise in the saving rate has a negative effect on g^{**}, which suggests the paradox of thrift as in the medium-run equilibrium. In the MB case, a rise in the saving rate has no effect on

Table 2: Long-run comparative statics

		s	γ	δ	ε	n	m_f	m_w	θ
Full capacity	m^{**}	+	0	0	0	−	+	+	+
	g_a^*	+	0	0	0	−	+	+	+
	g^{**}	+	0	0	0	−	+	+	+
Kalecki case	m^{**}	−	+	+	0	−	+	+	+
	g_a^*	−	+	+	0	−	0	0	0
	g^{**}	−	+	+	0	0	0	0	0
	u_n^*	−	+	+	0	+	−	−	−
MB case	m^{**}	0	+	+	0	−	+	+	+
	g_a^*	0	+	+	0	−	+	+	+
	g^{**}	0	+	+	0	−	+	+	+
	u_n^*	−	+	+	0	+	+	+	−

g^{**}, which is different from the MB case in the medium run.

The effect of a rise in the bargaining power of firms on the rate of capital accumulation is interesting. In the full-capacity case, it increases g^{**} as in the medium run. In the Kalecki case, it has no effect on g^{**}, which is different from the Kalecki case in the medium run. In the MB case, it has a positive effect on g^{**}.

Here, let us mention the differences between our model and the mainstream growth models. In endogenous growth models that possess "the scale effect," an increase in the saving rate leads to a rise in the growth rate of output on the balanced growth path.[15] In our model, on the other hand, an increase in the saving rate increases (the full-capacity case), decreases (the Kalecki case), or does not change (the MB case) the rate of capital accumulation in the long-run equilibrium. In semi-endogenous (non-scale) growth models, an increase in the growth rate of population leads to a rise in the growth rate of income per capita on the balanced growth path. In our model, on the other hand, an increase in the growth rate of labor supply leads to a decline in the growth rate of labor productivity in the long-run equilibrium.

6.3 Discussions

Here we discuss the effects of m_f and m_w, which are not mentioned in the preceding subsections. The profit share in our model is an endogenous variable and not an exogenous variable, and consequently we cannot treat the profit share as a parameter. For this reason,

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15) See Jones (1999) for the scale effect, endogenous growth, and semi-endogenous growth.

we instead pay attention to the parameters m_f and m_w. Because a rise in m_f (m_w) increases m^* and m^{**}, that is, m_f (m_w) and the profit share move in the same direction, our analysis is reasonable. Table 3 summarizes the results concerning m_f and m_w from Tables 1 and 2. Let us explain each item.

In the wage-led growth regime, a rise in the profit share declines the rate of capital accumulation.[16] In the profit-led growth regime, on the other hand, a rise in the profit share increases the rate of capital accumulation.

In the stagnationist regime (the wage-led aggregate demand regime), a rise in the profit share leads to a decline in the rate of capacity utilization. In the exhilarationist regime (the profit-led aggregate demand regime), a rise in the profit share increases the rate of capacity utilization.

Table 3: Classification of regime

	Kalecki case (Medium-run)	MB case (Medium-run)	Kalecki case (Long-run)	MB case (Long-run)
Wage-led growth	○	○		
Profit-led growth		○		○
Stagnationism	○	○	○	
Exhilarationism				○

The medium-run equilibrium in the Kalecki case corresponds to the wage-led growth regime and the stagnationist regime. This result is common when the Kalecki investment function is used. If the MB type investment function is used, either the wage-led growth regime or the profit-led growth regime is obtained. For the rate of capacity utilization, the stagnationist regime is obtained, which is the same as in the Kalecki case.

In the long-run equilibrium of the Kalecki case, neither the wage-led growth regime nor the profit-led growth regime is obtained. For the rate of capacity utilization, we have the stagnationist regime. The long-run equilibrium in the MB case corresponds to the profit-led growth regime and the exhilarationist regime.

In this way, different investment functions yield different regimes both in the medium run and in the long run.

Moreover, we emphasize that the medium-run equilibrium and the long-run equilibrium show different regimes both in the Kalecki case and in the MB case. In the Kalecki case, the medium-run equilibrium is the wage-led growth regime while the long-run rate of capital accumulation does not respond to the profit share. In the MB case, the medium-run

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16) Our classification of regimes is based on Blecker's (2002) classification.

equilibrium is characterized either as wage-led growth or as profit-led growth whereas the long-run equilibrium is only profit-led growth. For the rate of capacity utilization, we have stagnationist in the medium run whereas exhilarationist in the long run: a switch of regime occurs.

7   Concluding remarks

This paper has introduced endogenous technological change into a Kaleckian growth model. For the endogenization of technological change we assume that a change in the growth rate of labor productivity depends on the difference between the growth rates of employment and labor supply. This adjustment process makes the rate of employment stay constant in the long run. Using the model, we have analyzed the stability of the equilibrium and comparative statics. In addition, following the argument of Marglin and Bhaduri (1990), we use two alternative investment functions. If we use a Kalecki investment function, then we obtain stagnationism and wage-led growth in the medium run while we also obtain stagnationism but neither wage-led growth nor profit-led growth in the long run. If, instead, we use a MB type investment function, then we have stagnationism and either wage-led growth or profit-led growth in the medium run while we have exhilarationism and profit-led growth in the long run.

Our way of introduction of technological change is very simple. Rowthorn (1981) states that technical progress influences an economy in two ways. First, technical progress makes existing equipment obsolete, and thus it will affect the rate of depreciation. Second, technical progress stimulates firms that undertake innovations to invest more by bringing extra profits to them, and thus the form of investment function will be modified. In Cassetti (2003) these effects are taken into account, while in the present paper these issues are not dealt with for the purpose of emphasizing the role of endogenous technological change in the Kaleckian model of growth. For the same purpose target rates of workers and firms, m_w and m_f, are not endogenized. It is evident that technological change influences the target rates if these are endogenized. Taking these into account will be future research.

A Appendix

In the medium-run equilibrium of the Kalecki case, the following derivatives are worth mentioning.

$$\frac{dg^*}{d\varepsilon} = \frac{sm^*[\gamma - u_n m^*(s - \delta)]}{[(s - \delta)m^* - \varepsilon]^2}, \tag{A-1}$$

$$\frac{du^*}{d\varepsilon} = \frac{\gamma - u_n m^*(s - \delta)}{[(s - \delta)m^* - \varepsilon]^2}. \tag{A-2}$$

For $dg^*/d\varepsilon$, $dg^*/d\varepsilon > 0$ if $\gamma - u_n m^*(s - \delta) > 0$ while $dg^*/d\varepsilon < 0$ if $\gamma - u_n m^*(s - \delta) < 0$. The condition $\gamma - u_n m^*(s - \delta) > 0$ can be rewritten as

$$m^* < \frac{\gamma}{(s - \delta)u_n} \equiv m_0. \tag{A-3}$$

In the meantime, the rate of capital utilization has to be smaller than unity, from which we obtain

$$m^* > \frac{\gamma + \varepsilon(1 - u_n)}{s - \delta} \equiv m_1. \tag{A-4}$$

Comparing the size of m_0 with that of m_1, we have

$$m_0 - m_1 = \frac{(1 - u_n)(\gamma - \varepsilon u_n)}{(s - \delta)u_n} > 0, \tag{A-5}$$

which shows $m_0 > m_1$. Therefore, it is possible that $dg^*/d\varepsilon > 0$ or $dg^*/d\varepsilon < 0$ in the medium-run equilibrium. For $du^*/d\varepsilon$, the same reasoning holds.

In the medium-run equilibrium of the MB case, the following derivatives are worth mentioning.

$$\frac{dg^*}{d\varepsilon} = \frac{sm^*[(\delta m^* + \gamma - \varepsilon u_n) - u_n(sm^* - \varepsilon)]}{(sm^* - \varepsilon)^2}, \tag{A-6}$$

$$\frac{du^*}{d\varepsilon} = \frac{(\delta m^* + \gamma - \varepsilon u_n) - u_n(sm^* - \varepsilon)}{(sm^* - \varepsilon)^2}, \tag{A-7}$$

$$\frac{dg^*}{dm_f} = \frac{s\theta[\delta m^*(sm^* - \varepsilon) - \varepsilon(\delta m^* + \gamma - \varepsilon u_n)]}{(sm^* - \varepsilon)^2}, \tag{A-8}$$

$$\frac{dg^*}{dm_w} = \frac{s(1 - \theta)[\delta m^*(sm^* - \varepsilon) - \varepsilon(\delta m^* + \gamma - \varepsilon u_n)]}{(sm^* - \varepsilon)^2}, \tag{A-9}$$

$$\frac{dg^*}{d\theta} = \frac{s(m_f - m_w)[\delta m^*(sm^* - \varepsilon) - \varepsilon(\delta m^* + \gamma - \varepsilon u_n)]}{(sm^* - \varepsilon)^2}. \tag{A-10}$$

First, we turn to $dg^*/d\varepsilon$ and $du^*/d\varepsilon$. The signs of these derivatives are determined by the

sign of $(\delta m^* + \gamma - \varepsilon u_n) - u_n(sm^* - \varepsilon)$. From the assumptions, we have $\delta m^* + \gamma - \varepsilon u_n > 0$, $sm^* - \varepsilon > 0$, and $\delta m^* + \gamma - \varepsilon u_n < sm^* - \varepsilon$. For this reason, these derivatives can be positive or negative according to the size of u_n.

Second, we turn to dg^*/dm_f, dg^*/dm_w, and $dg^*/d\theta$. The signs of these derivatives are determined by the sign of $\delta m^*(sm^* - \varepsilon) - \varepsilon(\delta m^* + \gamma - \varepsilon u_n)$. Therefore, dg^*/dm_f, dg^*/dm_w, and $dg^*/d\theta$ can be positive or negative.

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