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An Econometric Analysis of Fiscal Policy Budget
Constraints in Endogenous Growth Models

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Constraints
in Endogenous Growth Models*

by

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Abstract

　This paper provides an econometric theory for examining the predictions of the public-policy endogenous growth models of Barro (1990) and others that suggest that unlike distortionary taxation and productive expenditures, nondistortionary taxation and nonproductive expenditures have no direct effect on the rate of growth. We prove that the estimates of all other regressions can be produced using only the estimates of the regression equation originally chosen. This illustrates from a statistical point of view why we are indifferent to the choice of omitted variable. Hence, decisions regarding omitted variables must incorporate criteria from the endogenous growth models. A numerical example using Kneller et al. (1999) sheds light on how the econometric analysis works in practice, and clarifies its significance for empirical study.

1. Introduction

The public-policy endogenous growth models of Barro (1990) and Barro and Sala-i-Martin (1992; 1995) provide mechanisms by which fiscal policy can determine both the level of output and the steady-state growth rate. The endogenous growth models classify elements of the government budget into one of four categories: distortionary or nondistortionary taxation and productive or nonproductive expenditures. Nondistortionary taxation and nonproductive expenditures have no effect on the rate of growth, while distortionary taxation and productive expenditures have direct effects. Subsequently, many empirical studies have considered the predictions of endogenous growth models in both developed (Mendoza et al. 1997, Kneller et al. 1999, Miyakoshi et al. 2007) and developing (Devarajan et al. 1996, Gupta et al. 2005) countries.

Most of previous research employs linear regression. Because the fiscal variables of taxation and public expenditures are subject to a budget constraint, linear regression models suffer from multicollinearity, and we must omit one of the fiscal variables in order to carry out the standard estimation procedure. Kneller et al. (1999) pointed out that many studies either neglected the budget constraint or omitted some of the fiscal variables in an ad hoc manner. Accordingly, the results provide only crude tests of the empirical validity of endogenous growth models. However, Kneller et al. (1999) did not explicitly explore the implications of econometric methodology when specifying the regression models.

The purpose of this paper is to provide an econometric theory to analyze the evidence from endogenous growth models of Barro (1990) and others, and to clarify its significance for empirical studies. We prove that the estimates of all other regressions can be produced using only the estimates of the regression equation originally chosen. This implies that we are indifferent to the choice of omitted variable from a purely statistical point of view. Econometric theory alone does not provide any criteria to determine which variables to omit, although this issue is crucial for empirical research. On the other hand, Barro's (1990) endogenous growth models predict that nondistortionary taxation and nonproductive expenditures have no effect on the rate of growth. It is this prediction that provides a criterion. Because the coefficients for these variables are zero, the omission of these variables does not change the coefficients for the remaining variables. Furthermore, this paper reveals that the regression equation with two omitted variables provides estimates that are more efficient and more powerful test statistics than a regression with just one omitted variable when economic theory indicates that two different coefficients are simultaneously zero.

We apply the analysis to work by Kneller et al. (1999) as a numerical example to show how the econometric theory works in practice. Their study supports the predictions of endogenous growth models with respect to the effects of the structure of taxation and expenditure on growth. They also argue that misspecification of the government budget constraint leads to widely differing parameter estimates. This paper confirms their empirical study from the viewpoint of econometric analysis.

This paper is organized as follows. Section 2 briefly states a linear regression model with fiscal policy budget constraints, and provides some propositions for the effects of omitted variables on the estimation and testing results. Section 3 provides a numerical example to illustrate how econometric analysis works in practice. Section 4 concludes the paper.

2. Effects of omitted variables on a linear regression with fiscal policy budget constraints

2.1 A linear regression model with fiscal policy budget constraints

Let a panel data linear regression be[1]:

$$y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j Y_{jit} + \sum_{j=1}^{m} \gamma_{j,n} X_{jit} + u_{it}, \quad i = 1, \cdots, I; t = 1, \cdots N, \tag{1}$$

where y_{it}, X_{jit} and Y_{jit} respectively denote the per capita growth rate of GDP, fiscal variables and conditioning (nonfiscal) variables, and u_{it} are i.i.d. (independently and identically distributed) normal random variables with mean 0 and variance σ^2 denoted as $N(0, \sigma^2)$. Assuming that all elements of the budget (including the deficit/surplus) are included, the fiscal variables are subject to a linear constraint[2]:

$$\sum_{j=1}^{m} X_{jit} = 0, \qquad i = 1, \ldots, I, \quad t = 1, \ldots, N. \tag{2}$$

The standard estimation procedure is not applicable for equation (1) because of perfect collinearity. In order to avoid collinearity, one variable (say, the last variable X_{mit}) must be omitted from equation (1). The equation actually estimated is:

$$y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j Y_{jit} + \sum_{j=1}^{m-1} \gamma_{j,n} X_{jit} + u_{it}, \tag{3}$$

where $\gamma_{j,m} = \gamma_j - \gamma_m (j = 1, \ldots, m-1)$. We can only estimate $\gamma_{j,m}$ for $j = 1, \ldots, m-1$ but cannot estimate each parameter of γ_j for $j = 1, \ldots, m$.

The hypothesis of a zero coefficient of X_{jit} for equation (3) is:

$$H_0: \gamma_{j,m} = 0 \text{ vs } H_1: \gamma_{j,m} \neq 0 \text{ for } j = 1, \ldots, m-1. \tag{4}$$

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[1] This formulation allows for a two-way fixed effects model by including dummy variables for both a time-specific and a cross section-specific intercept into the conditioning variables in equation (1).

[2] For simplicity of exposition, we restrict ourselves to a zero-sum constraint. However, a constraint where the fiscal variables sum to a nonzero constant does not essentially change the following arguments.

We should note that the null is $\gamma_j - \gamma_m = 0$ rather than $\gamma_j = 0$. A standard test statistic is given by:

$$t_{j,m} = \frac{\hat{\gamma}_{j,m}}{\left\{\hat{Var}(\hat{\gamma}_{j,m})\right\}^{1/2}}, \tag{5}$$

where $\hat{\gamma}_{j,m}$ denotes the OLS (ordinary least squares) estimator of $\gamma_{j,m}$, and $\hat{Var}(\hat{\gamma}_{j,m})$ is its estimated variance. The statistic has a t-distribution with degrees of freedom $\nu_{j,m} = \text{NI} - (\text{k} + 1) - (\text{m} - 1)$ under the null.

Though the choice of fiscal variables to be omitted from the regression equation is an important issue for empirical studies, econometric theory itself cannot provide solutions to this problem. The recommendation is to choose a neutral category where economic theory suggests that $\gamma_m = 0$ if we wish to test the null $\gamma_j = 0$ of against $\gamma_j \neq 0$. The practical significance of determining the omitted variables is examined in detail with the numerical example in Section 3.

2.2 Effects of the choice of an omitted variable on the estimated coefficients

If we omit another fiscal variable (say X_{nit} $(n \neq m)$) instead of the last variable, the equation to be estimated is:

$$y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j Y_{jit} + \sum_{j=1(j\neq n)}^{m} \gamma_{j,n} X_{jit} + u_{it}, \quad n = 1, \cdots, m - 1, \tag{6}$$

where $\gamma_{j,n} = \gamma_j - \gamma_n; j = 1, \ldots, m\, (j \neq n)$. The coefficients of (6) are completely determined by those of (3); conversely, the coefficients of (3) are also completely determined by those of (6) as indicated in the following proposition. The proofs of all propositions are given in the Appendix.

Proposition 1: *The parameters of equations (3) and (6) hold the following relationships for n = 1, ..., m – 1:*

$$\gamma_{j,n} = \gamma_{j,m} - \gamma_{n,m} \quad j = 1, \ldots, m - 1\, (j \neq n) \ ; and\ \gamma_{m,n} = -\gamma_{n,m}, \tag{7}$$

or equivalently,

$$\gamma_{j,m} = \gamma_{j,n} - \gamma_{m,n} \quad j = 1, \ldots, m - 1\, (j \neq n) \ ; and\ \gamma_{n,m} = -\gamma_{m,n}. \tag{8}$$

We can apply OLS to estimate the parameters of either equation (3) or equation (6), while OLS is not applicable for equation (1). The following proposition indicates that the OLS estimates for the coefficients of equations (3) and (6) carry the same relations as in (7) and (8).

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Proposition 2 : *Let the OLS estimates of (3) and (6) be $\hat{\gamma}_{j,m}$ and $\hat{\gamma}_{j,n}$ respectively. Then, the estimates $\hat{\gamma}_{j,m}$ and $\hat{\gamma}_{j,n}$ hold the following relationships for $n = 1, ..., m-1$:*

$$\hat{\gamma}_{j,n} = \hat{\gamma}_{j,m} - \hat{\gamma}_{n,m}; j = 1, ..., m-1 (j \neq n) \text{ and } \hat{\gamma}_{m,n} = -\hat{\gamma}_{n,m}, \tag{9}$$

or equivalently:

$$\hat{\gamma}_{j,m} = \hat{\gamma}_{j,n} - \hat{\gamma}_{m,n}; j = 1, ..., m-1 (j \neq n) \text{ and } \hat{\gamma}_{n,m} = -\hat{\gamma}_{m,n}. \tag{10}$$

Proposition 2 shows that all coefficient estimates for the regression with any other single omitted variable are completely determined by the estimates of the coefficients for the equation (3) originally chosen. In this sense, any additional use of the regression equation with an alternative omitted variable cannot extract further information from the given data set.

Next, we consider the effects of alternative omitted variable on the test for a zero coefficient of X_{jit} for equation (6):

$$H_0: \gamma_{j,n} = 0 \text{ vs } H_1: \gamma_{j,n} \neq 0 \text{ for } j = 1, ..., m (j \neq n). \tag{11}$$

The test statistic is:

$$t_{j,n} = \frac{\hat{\gamma}_{j,n}}{\left\{\hat{V}ar(\hat{\gamma}_{j,n})\right\}^{1/2}}, \tag{12}$$

where $\hat{V}ar(\hat{\gamma}_{j,n})$ is the estimated variance of $\hat{\gamma}_{j,n}$. The following relations between the test statistics of (5) and (12) hold.

Proposition 3: *The formula of (12) is written in terms of the quantities used only for estimating equation (3) as:*

$$t_{j,n} = \left[\frac{\hat{V}ar(\hat{\gamma}_{j,m})}{\hat{V}ar(\hat{\gamma}_{j,n})}\right]^{1/2} t_{j,m} - \left[\frac{\hat{V}ar(\hat{\gamma}_{n,m})}{\hat{V}ar(\hat{\gamma}_{j,n})}\right]^{1/2} t_{n,m}, \tag{13}$$

where:

$$\hat{V}ar(\hat{\gamma}_{j,n}) = \hat{V}ar(\hat{\gamma}_{j,m}) - 2\hat{C}ov(\hat{\gamma}_{j,m}, \hat{\gamma}_{n,m}) + \hat{V}ar(\hat{\gamma}_{n,m}), \tag{14}$$

and $\hat{C}ov(\hat{\gamma}_{j,m}, \hat{\gamma}_{n,m})$ is the estimate of covariance between $\hat{\gamma}_{j,m}$ and $\hat{\gamma}_{n,m}$.

We note that knowledge of the covariances $Cov(\hat{\gamma}_{j,m}, \hat{\gamma}_{n,m})$ for $j = 1, \cdots, m - 1$ $(j \neq n)$ is necessary for producing $\hat{V}ar(\hat{\gamma}_{j,n})$ from the regression results of equation (3).

The analysis in this subsection reveals the effects of omitted variables on the estimation and testing procedure. The issue of what particular variables should be omitted remains unsettled.

2.3 Effects of two simultaneously omitted variables

Suppose it is true that two different coefficients are simultaneously zero (say, $\gamma_m = 0$ and $\gamma_n = 0$), in equation (1). This assumption is justified by economic theory on the basis of the public-policy endogenous growth models in Barrow (1990) and Barrow and Sala-i-Martin (1992; 1995). This is not a statistical hypothesis to be tested by empirical data. The true regression equation is:

$$y_{it} = \beta_0 + \sum_{j=1}^{k} \beta_j Y_{jit} + \sum_{j=1(j\neq n)}^{m-1} \gamma_j X_{jit} + u_{it}, \quad n = 1, \cdots, m - 1. \qquad (15)$$

The regression equations (3) and (6) are misspecified because one of the zero coefficient restrictions is ignored. The parameters to be estimated are equal among the three equations, i.e. $\gamma_j = \gamma_{j,m} = \gamma_{j,n}$ for $j = 1, ..., m - 1$ $(j \neq n)$.

Proposition 4: *Let* $\hat{\gamma}_j$, $\hat{\gamma}_{j,m}$ *and* $\hat{\gamma}_{j,n}$ *be the OLS estimates of equations (15), (3) and (6) respectively. Then, we have for $j = 1, ..., m - 1$ $(j \neq n)$:*

$$(i) \qquad E\{\hat{\gamma}_{j,m}\} = E\{\hat{\gamma}_j\} = E\{\hat{\gamma}_{j,n}\} = \gamma_j, \qquad (16)$$

$$(ii) \qquad Var\{\hat{\gamma}_j\} < Min\{Var\{\hat{\gamma}_{j,m}\}, Var\{\hat{\gamma}_{j,n}\}\}. \qquad (17)$$

All three estimators of the coefficients are unbiased. The estimator of the true model is the most efficient among the three in the sense that estimator ($\hat{\gamma}_j$) has the smallest variance. Proposition 4 analytically implies the claim that when economic theory suggests that there is more than one neutral category, more precise parameter estimates can be obtained by omitting both categories.

We consider the testing of a zero coefficient for X_{jt} in equation (15):

$$H_0: \gamma_j = 0 \text{ vs } H_1: \gamma_j \neq 0 \text{ for } j = 1, ..., m - 1 \text{ } (j \neq n), \qquad (18)$$

where the test statistic is given by:

$$t_j = \frac{\hat{\gamma}_j}{\{\hat{V}ar(\hat{\gamma}_j)\}^{1/2}}. \qquad (19)$$

Both (5) and (12) can be used for testing the hypothesis (18) in addition to (19).

Proposition 5:

(i) *All three statistics for (5), (12) and (19) have a t-distribution under the null and a noncentral t-distribution with noncentrality parameters δ_i under the alternative where $\delta_i = \dfrac{\gamma_i}{\{Var(\hat{\gamma}_i)\}^{1/2}}$ for $i = j, (j,m), and\ (j,n)$; $\nu_j = NI - k - m + 1$, and $\nu_{j,m} = \nu_{j,n} = NI - k - m$, respectively;*

(ii) *The expected values of t_i are respectively given by:*

$$E\{t_i\} = \left(\frac{1}{2}\nu_i\right)^{1/2} \frac{\Gamma\left(\frac{1}{2}(\nu_i - 1)\right)}{\left[\Gamma\left(\frac{1}{2}\nu_i\right)\right]^{1/2}} \delta_i. \tag{20}$$

where $E(\bullet)$ denotes an expectation operator, and $\Gamma(p)$ is a Gamma function with p degree of freedom[3].

The expected value is zero under the null and an increasing function of the noncentrality parameter δ_i under the alternative.

Proposition 6: *The following inequality holds for any $\gamma_j \neq 0$ for $j = 1, ..., m - 1$ (j $\neq n$):*

$$\left|E\{t_j\}\right| \geq \max\left\{\left|E\{t_{j,m}\}\right|, \left|E\{t_j, n\}\right|\right\}, \tag{21}$$

up to the order of $O((NI)^{-2})$ when the number of observations (NI) increases.

The absolute value of the expectation of the t-statistic of equation (19) for testing a zero coefficient is always the highest among the three tests under the alternative hypothesis ($\gamma_j \neq 0$) when the sample size is large. This suggests that among the three tests, the test of (20) has the highest power for testing the hypothesis (18).

3   A numerical example

Kneller et al. (1999, p.180) summarize the basic results about the growth effects of fiscal policy for a panel of 22 OECD countries from 1970 to 1995 in their Table 3. In order to examine how the analysis presented in the previous section works in practice,

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[3] See, for example, Johnson and Kotz (1970, p.203) for the properties of a noncentral t-distribution.

we reproduce their results in Table 1 after adjusting for the sign of the fiscal taxation variables[4].

Table 1 around here

The *i-th* column in Table 2 shows the estimates of the coefficients for the regression with the *i-th* variable omitted. These are calculated by utilizing the relations in Propositions 2 and 3 based upon the original regression with the last fiscal variable (nonproductive expenditures) omitted. The parameter estimates in column 6 of Table 2 are the same as those in column 1 of Table 1. The *t*-values in column 6 are almost the same as those in column 1 of Table 1. Any differences between the corresponding *t*-values are from rounding errors[5].

Table 2 around here

Table 2 illustrates how the estimates of the other regression coefficients are produced using only the estimates of the originally chosen regression equation. This explains why running other regression equations does not provide additional information. The estimated coefficients in Table 2 are symmetric along the diagonal elements with opposite signs.

The estimated coefficients for each fiscal variable differ considerably column by column. For example, the coefficient for distortionary taxation in column 8 is 0.410 and significantly different from zero while the corresponding coefficient in column 4 is zero to the third decimal point and apparently insignificant. The issue of what particular fiscal variable we should omit from the regression equation is fundamentally important for correct empirical study. Econometric theory itself may not provide any criteria for determining the omitted variables. Instead, economic theory plays an essential role. According to Barro's (1990) public-policy endogenous growth models, a neutral

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[4] The coefficient for distortionary taxation in Table 3 of Kneller et al. (1999) is negative, indicating that an increase in the tax rate for the distortionary category induces a reduction in the growth rate. Though this way of treating the fiscal revenue variables is intuitively appealing, the fiscal variables do not sum to zero. Hence, the fiscal budget constraint of equation (2) is not satisfied. In order to avoid this inconsistency, we measure the fiscal revenue variables as negative values and the expenditure variables as positive values. This variable adjustment should change the signs of the coefficients for distortionary taxation and other revenues in Table 3.

[5] In general, we cannot apply Proposition 2 for calculating the *t*-values in columns 1 through 7 because Kneller et al. (1999) do not report $\hat{C}ov(\hat{\gamma}_{j,8}, \hat{\gamma}_{n,8})$. However, we can directly use $\hat{V}ar(\hat{\gamma}_{j,6})$ in column 2 of their Table 3 for calculating the *t*-values in column 6 of our Table 2.

category (nondistortionary taxation and nonproductive expenditures) has no effect on the rate of growth. Economic theory suggests that $\gamma_6 = 0$ and $\gamma_8 = 0$ in this example.

Suppose it is true that nonproductive expenditures have no effect on the rate of growth ($\gamma_8 = 0$). This is a prediction of Barro's (1990) model. In statistical terminology, the constraint $\gamma_8 = 0$ is one of the maintained hypotheses, not a hypothesis to be tested. The hypothesis in (4) now turns out to be the null of $\gamma_j = 0$ against $\gamma_j \neq 0$ for $j = 1, ..., 7$. Tests of the coefficients for distortionary taxation, nondistortionary taxation and productive expenditures are consistent with the predictions of economic theory. In particular, the estimate of nondistortionary taxation is not significantly different from zero. As expected, the estimated values in column 6 are very similar to those in column 8 because nondistortionary taxation is classified into a neutral category.

However, if we omit distortionary taxation, for instance, the estimates in column 5 are drastically different from those in column 8. The coefficient for the budget surplus is zero to the third decimal point. This result superficially contradicts the predictions of economic theory, but the test in equation (4) is actually the null of $\gamma_j = \gamma_5$ against $\gamma_j \neq \gamma_5$ rather than the null of $\gamma_j = 0$ against $\gamma_j \neq 0$. The zero-coefficient for distortionary taxation really indicates that $\hat{\gamma}_4 = \hat{\gamma}_5$. This is exactly implied by the fourth and fifth coefficients in column 8. We emphasize that the discussion so far is justified only when $\gamma_8 = 0$ is true. The empirical evidence is consistent with $\gamma_8 = 0$ in the sense that hypothesis testing supports the predictions of the endogenous growth models[6].

On the other hand, if we chose the budget surplus as an omitted variable in an ad hoc manner without careful consideration of its significance for economic theory, then we would reach drastically different findings from those in the original regression. In fact, most estimates in column 4 are negative while those in column 8 are positive[7]. The coefficient for distortionary taxation is not significantly different from zero, and consequently we may conclude that the endogenous growth model is not supported. Nevertheless, this conclusion is completely irrelevant for the reasons outlined above. Kneller et al. (1999) pointed out that many earlier studies made this kind of mistake[8].

Suppose, as suggested by the fiscal policy endogenous growth model, that both nonproductive expenditures and nondistortionary taxation have no effect on the rate of

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[6] We can discuss the same sort of arguments for $\gamma_6 = 0$, although we do not repeat this in the current paper.

[7] Though exact t-values for the estimates in column 4 cannot be produced, approximate intervals for the t-statistics are given in Table 3 of Appendix B. As shown, the significance of the estimates for each parameter changes completely.

[8] See Kneller et al. (1998) for a review of earlier work using this perspective.

growth ($\gamma_6 = 0$ and $\gamma_8 = 0$). The true regression model is written as in equation (14). The estimated coefficients are presented in the first column of panel (c) in Table 1. The entries in the last column of panel (a) and (b) in Table 1, respectively, show the t-values for the case with a single omitted fiscal variable, while the last column of panel (c) presents the t-values with two simultaneously omitted variables. The numerical values in panel (c) are the highest of the three for every coefficient. Proposition 6 confirms the facts empirically observed in Table 1 from the viewpoint of econometric theory.

4   Concluding remarks

This paper investigates an econometric theory for the predictions of the public-policy endogenous growth models of Barro (1990) and others in a framework of linear regressions subject to multicollinearity. First, we showed that if the estimates of the regression equation with an omitted variable have been obtained, the estimates of all other regressions can be produced using only the estimates of the originally chosen regression equation. This constructs an analytical basis for evaluating the effects of the choice of omitted variables on the estimated coefficients. Though the issue of choosing the omitted fiscal variables is crucially important for empirical study, econometric analysis alone cannot provide the criteria. The economic theory underlying Barro's (1990) endogenous growth models provides a criterion in this instance.

Second, we showed that when two different coefficients are known to be simultaneously zero using economic theory, the regression equation with two omitted variables provides more efficient estimates and more powerful test statistics than the regression with just one omitted variable.

As a numerical example, we applied our approach to an empirical study by Kneller et al. (1999) to show how it works in practice. Kneller et al. (1999) found evidence consistent with the predictions of endogenous growth models with respect to the effects of the structure of taxation and expenditure on growth. This paper reconfirms the significance of their empirical work from the viewpoint of econometric theory.

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Appendix A: Proofs of propositions

Proof of Proposition 1: This is straightforward from the definitions of $\gamma_{j,m}$ *and* $\gamma_{j,n}$.

Proof of Proposition 2: We assume $n = m - 1$ without loss of generality by arranging the order of variables. Writing equations (3) and (6) in a matrix form is useful for proving the propositions. Equation (3) is expressed as:

$$y_{it} = Y'_{\Delta,it}\beta_\Delta + X'_{\Delta,it}\gamma_{\Delta,m} + X_{m-1,it}\gamma_{m-1,m} + u_{it}\, ; \quad i = 1,...,I,\ \ t = 1,...,N\, , \qquad (A.1)$$

where $Y_{\Delta,it} = (1, Y_{1,it}, ..., Y_{k,it})'$ is a $(k+1) \times 1$ vector of the conditioning variables and a constant term, $X_{\Delta,it} = (X_{1,it}, ..., X_{m-2,it})'$ an $(m-2) \times 1$ vector of the first $(m-2)$ fiscal variables, and $\beta_\Delta = \left(\beta_0, \beta_1, ..., \beta_k\right)'$ and $\gamma_{\Delta.m} = \left(\gamma_{1,m}, \gamma_{2,m}, ..., \gamma_{m-2,m}\right)'$ are respectively $(k+1) \times 1$ and $(m-2) \times 1$ vectors of parameters. Stacking the observations through $i = 1$, ..., I, and $t = 1, ..., N$, we have:

$$y = Y_\Delta \beta_\Delta + X_\Delta \gamma_{\Delta,m} + X_{m-1}\gamma_{m-1,m} + u, \qquad (A.2)$$

where $y = \left(\left(y_{1,1}, y_{2,1}, ..., y_{I,1}\right), ..., \left(y_{1,N}, y_{2,N}, ..., y_{I,N}\right)\right)'$; $\text{NI} \times 1$,

$$Y_\Delta = \left(\left(Y_{\Delta,1,1}, ..., Y_{\Delta,I,1}\right), ..., \left(Y_{\Delta,1,N}, ..., Y_{\Delta,I,N}\right)\right)'\ ; \text{NI} \times (k+1),$$

$$X_\Delta = \left(\left(X_{\Delta,1,1}, ..., X_{\Delta,I,1}\right), ..., \left(X_{\Delta,1,N}, ..., X_{\Delta,I,N}\right)\right)'; \text{NI} \times (m-2),$$

$$X_{m-1} = \left(\left(X_{m-1,1,1}, ..., X_{m-1,I,1}\right), ..., \left(X_{m-1,1,N}, ..., X_{m-1,I,N}\right)\right)'; \text{NI} \times 1,$$

$$u = \left(\left(u_{1,1}, ..., u_{I,1}\right), ..., \left(u_{1,N}, ..., u_{I,N}\right)\right)'; \text{NI} \times 1.$$

Then, the regression equation (3) is written in a standard matrix form:

$$y = W\beta + u, \quad u \sim N(0, \sigma^2), \qquad (A.3)$$

where $W = \left(Y_\Delta, X_\Delta,\ X_{m-1}\right)$; $\text{NI} \times ((k+1) + (m-2) + 1)$, and $\beta = \left(\beta'_\Delta, \gamma'_{\Delta,m}, \gamma_{m-1,m}\right)'$; $((k+1) + (m-2) + 1) \times 1$. Similarly, the regression equation (6) is written as:

$$y = Y_\Delta \beta_\Delta + X_\Delta \gamma_{\Delta,m-1} + X_m \gamma_{m,m-1} + u\, , \qquad (A.4)$$

where $\gamma_{\Delta,m-1} = \left(\gamma_{1,m-1}, \gamma_{2,m-1}, \ldots, \gamma_{m-2,m-1}\right)'$; $(m-2) \times 1$, or alternatively as:

$$y = Z\delta + u, \tag{A.5}$$

where $Z = \left(Y_\Delta, X_\Delta, X_m\right)$; $\mathrm{NI} \times ((k+1) + (m-2) + 1)$, and $\delta = \left(\beta'_\Delta, \gamma'_{\Delta,m-1}, \gamma_{m,m-1}\right)'$;

$((k+1) + (m-2) + 1) \times 1$. The independent variables Y_Δ and X_Δ both appear in equations (A.3) and (A.5).

We have the relation among the matrices of the independent variables W and Z:

$$W = ZQ \ , \tag{A.7}$$

where:

$$Q = \begin{pmatrix} I_{k+1} & 0 & 0 \\ 0 & I_{m-2} & -J_{m-2} \\ 0 & 0 & -1 \end{pmatrix} \ ; \ ((k+1) + (m-2) + 1) \times ((k+1) + (m-2) + 1),$$

$$J_{m-2} = (1, \cdots, 1)' \ ; (m-2) \times 1.$$

The budget constraint $X_\Delta J_{m-2} + X_{m-1} + X_m = 0$ is used for deriving (A.7). From equations (A.3), (A.5) and (A.7), we have $W\beta = ZQ\beta = Z\delta$, which leads to:

$$\delta = Q\beta . \tag{A.8}$$

The normal equation of the OLS for (A.3) is:

$$W'W\hat{\beta} = W'y . \tag{A.9}$$

Plugging the relation of (A.7) into equation (A.9), we have $(ZQ)' (ZQ)\hat{\beta} = (ZQ)'y$. This implies that:

$$\hat{\delta} = \left(Z'Z\right)^{-1} Z'y = Q\hat{\beta} . \tag{A.10}$$

The component-wise expression in equation (A.10) includes the relations in proposition 2.

Proof of Proposition 3: Equation (13) is easily derived from the relation in equation (9). The OLS of β in equation (A.9) is distributed as:

$$\hat{\beta} \sim N\left(\beta, Var\left(\hat{\beta}\right)\right), \tag{A.11}$$

where $Var\left(\hat{\beta}\right) = \sigma^2 \left(W'W\right)^{-1}$. In particular, defining $\theta = \left(\gamma_{j,m}, \gamma_{n,m}\right)'$, and $\hat{\theta} = \left(\hat{\gamma}_{j,m}, \hat{\gamma}_{n,m}\right)'$, we have $\hat{\theta} \sim N\left(\theta, Var\left(\hat{\theta}\right)\right)$, where:

$$Var\left(\hat{\theta}\right) = \begin{pmatrix} Var\left(\hat{\gamma}_{j,m}\right) & Cov\left(\hat{\gamma}_{j,m}, \hat{\gamma}_{n,m}\right) \\ Cov\left(\hat{\gamma}_{j,m}, \hat{\gamma}_{n,m}\right) & Var\left(\hat{\gamma}_{n,m}\right) \end{pmatrix}, \tag{A.12}$$

and:

$$\hat{\gamma}_{j,n} = a'\hat{\theta} \sim N\left(a'\theta, a'Var\left(\hat{\theta}\right)a\right). \tag{A.13}$$

where $a = (1, -1)'$. Then, the equality in (13) holds.

Proof of Proposition 4: We assume $n = m - 1$ without loss of generality. The true regression equation is written as:

$$y = Y_\Delta \beta_\Delta + X_\Delta \gamma_\Delta + u, \tag{A.14}$$

where $\gamma_\Delta = \left(\gamma_1, \cdots, \gamma_{m-2}\right)'$, and a misspecified regression equation (3) as:

$$y = Y_\Delta \beta_\Delta + X_\Delta \gamma_{\Delta,m} + X_{m-1} \gamma_{m-1,m} + u. \tag{A.15}$$

Let the residuals of regressing y, X_Δ and X_{m-1} on Y_Δ be respectively:
$R_0 = \bar{P}y$, $R_1 = \bar{P}X_\Delta$ and $R_2 = \bar{P}X_{m-1}$, where $\bar{P} = I - Y_\Delta \left(Y_\Delta' Y_\Delta\right)^{-1} Y_\Delta'$ is the projection operator to the orthogonal space of the space spanned by the columns of Y_Δ. Let the moment matrices be $S_{i,j} = R_i'R_j$ for $i, j = 0, 1$, and 2. The OLS estimators of (A.14) and (A.15) are then given by:

$$\hat{\gamma}_\Delta = S_{11}^{-1} S_{10} \sim N\left(\gamma_\Delta, \sigma^2 S_{11}^{-1}\right), \tag{A.16}$$

and:

$$\begin{pmatrix} \hat{\gamma}_{\Delta,m} \\ \hat{\gamma}_{m-1,m} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}^{-1} \begin{pmatrix} S_{10} \\ S_{20} \end{pmatrix} \sim N\left(\begin{pmatrix} \gamma_{\Delta,m} \\ \gamma_{m-1,m} \end{pmatrix}, \sigma^2 \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}^{-1}\right). \tag{A.17}$$

We obtain the relation between $\hat{\gamma}_\Delta$ and $\hat{\gamma}_{\Delta,m}$ as follows:

$$\hat{\gamma}_{\Delta,m} = \hat{\gamma}_\Delta - S_{11}^{-1}S_{12}\hat{\gamma}_{m-1,m} \sim N\left(\gamma_{\Delta,m}, \sigma^2\left[S_{11}^{-1} + S_{11}^{-1}S_{12}S_{22.1}^{-1}S_{21}S_{11}^{-1}\right]\right),$$

where $S_{22.1} = S_{22} - S_{21}S_{11}^{-1}S_{12}$. Hence:

$$Var\left(\hat{\gamma}_{\Delta,m}\right) - Var\left(\hat{\gamma}_\Delta\right) = \sigma^2\left\{S_{11}^{-1}S_{12}S_{22.1}^{-1}S_{21}S_{11}^{-1}\right\} > 0, \tag{A.18}$$

in the sense of the positive definiteness of a matrix. This implies that $Var\{\hat{\gamma}_j\} < Var\{\hat{\gamma}_{j,m}\}$ for $j = 1, ..., m-1$ $(j \neq n)$. Similarly, it can be shown that:

$$Var(\hat{\gamma}_{\Delta,n}) - Var(\hat{\gamma}_\Delta) > 0. \tag{A.19}$$

This completes the proof of inequality (17).

Proof of Proposition 5: The proof is straightforward from the definitions of the t-distribution under the null and noncentral t-distribution under the alternative.

Proof of Proposition 6: Using Stirling's formula:

$$\Gamma(p) \sim \sqrt{2\pi}e^{-p}p^{p-(1/2)}\left\{1 + \frac{1}{12p} + O\left(p^{-2}\right)\right\} \quad as \ p \to \infty, \tag{A.20}$$

from Proposition 5 we have:

$$E\{t_i\} = \frac{1}{\sqrt{2e}}\delta_i\left\{1 + O(NI)^{-2}\right\}, \text{ for r } i = j, (j,m), and\ (j,n). \tag{A.21}$$

Then:

$$\begin{aligned}&\left|E\{t_j\}\right| - \left|E\{t_{j,m}\}\right| \\ &\quad = \left|\delta_j\right| - \left|\delta_{j,m}\right| + O\left((NI)^{-2}\right) \\ &\quad = \frac{1}{\sqrt{2e}}\left[\frac{1}{\left\{Var(\hat{\gamma}_j)\right\}^{1/2}} - \frac{1}{\left\{Var(\hat{\gamma}_{j,m})\right\}^{1/2}}\right]\left|\gamma_j\right|\left(1 + O\left((NI)^{-2}\right)\right)\end{aligned} \tag{A.22}$$

Because $Var(\hat{\gamma}_j) < Var(\hat{\gamma}_{j,m})$ as shown in Proposition 4, the required relation holds.

Appendix B: Approximate range of *t*-values

This appendix provides approximate ranges of the *t*-values for the estimates of $\hat{\gamma}_{j,n}$ in columns other than the sixth and eighth columns of Table 2. This is despite the fact that the exact *t*-values for these estimates cannot be produced from the original regression results because Kneller et al. (1999) did not report the covariances between $\hat{\gamma}_{j,m}$ and $\hat{\gamma}_{n,m}$. From equation (14), the correlation coefficients are expressed as:

$$Corr(j, n; m) \equiv \hat{Corr}(\hat{\gamma}_{j,m}, \hat{\gamma}_{n,m})$$

$$= \frac{1}{2}\left\{\hat{Var}(\hat{\gamma}_{j,m}) + \hat{Var}(\hat{\gamma}_{n,m}) - \hat{Var}(\hat{\gamma}_{j,n})\right\} / \left\{\hat{Var}(\hat{\gamma}_{j,m})\hat{Var}(\hat{\gamma}_{n,m})\right\}^{1/2}, \quad (B.1)$$

where *m* indicates the omitted variable. Using the values in Table 1, the elements of $Corr(j, n; m)$ are calculated as (0.090, 0.019, 0.383, 0.288, 0.312, 1.000, 0.514, n. a.) for (n, m) = (6, 8), j = 1, ..., 8 ; and (0.633, 0.837, 0.740, 0.846, 0.817, n. a., 0.645, 1.000) for (n, m) = (8, 6), j = 1, ..., 8. All correlations coefficients lie within the range: $\rho_0 \equiv 0.019 \le Corr(j, n; m) \le 0.846 \equiv \rho_1$.

We roughly approximate the variance of $\hat{\gamma}_{j,n}$ with the following interval:

$$Var(j, n : 0) \le Var(\hat{\gamma}_{j,n}) \le Var(j, n : 1), \quad (B.2)$$

where $Var(j, n : i) = \hat{Var}(\hat{\gamma}_{j,m}) - 2\rho_i \left\{\hat{Var}(\hat{\gamma}_{j,m})\hat{Var}(\hat{\gamma}_{n,m})\right\}^{-1/2} + \hat{Var}(\hat{\gamma}_{n,m})$ for *i = 0* and *1*. Finally, we have the approximate intervals for the *t*-values:

$$|t(j, n : 1)| \le |t(\hat{\gamma}_{j,n})| \le |t(j, n : 0)|, \quad (B.3)$$

where $t(j, n : i) = \dfrac{\hat{\gamma}_{j,n}}{\left\{Var(j, n : i)\right\}^{1/2}}$.

We illustrate $t(j, n : i)$ for i = 0 and 1 only for the case of *n* = 4 in Table 3.

Table 3 around here

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Table 1. Regression results

(Table 3 from Kneller et al. (1999, p.180))

Omitted Fiscal Variable	Panel (a) Non-dis. Taxation			Panel (b) Non-productive expenditures			Panel (c) Non-dis. Taxation and non-prod. Expenditures		
	Est.	s.d.	t-val.	Est.	s.d.	t-val.	Est.	s.d.	t-val.
1. Lending minus repayments	0.417	0.229	1.820	0.380	0.178	2.130	0.384	0.176	2.180
2. Other revenues	0.154	0.190	0.810	0.117	0.104	1.120	0.118	0.104	1.130
3. Other expenditures	0.315	0.158	2.000	0.279	0.115	2.420	0.289	0.105	2.750
4. Budget surplus	0.446	0.160	2.790	0.410	0.089	4.600	0.416	0.084	4.930
5. Distortionary taxation	0.446	0.160	2.790	0.410	0.097	4.210	0.410	0.094	4.370
6. Non-distortionary taxation	-	-	-	-0.037	0.161	-0.230	-	-	-
7. Productive expenditures	0.290	0.146	1.980	0.253	0.130	1.950	0.268	0.110	2.430
8. Non-productive expenditures	0.037	0.161	0.230	-	-	-	-	-	-

Note: The entries in columns s.d. and t-val. are standard deviations and t-values, respectively.

Table 2. Derived estimates

	Omitted Fiscal Variable	1.	2.	3.	4.	5.	6.	7.	8.
1.	Lending minus repayments	-	0.263	0.101	-0.030	-0.030	0.417 (1.82)	0.127	0.380 (2.13)
2.	Other revenues	-0.263	-	-0.162	-0.293	-0.293	0.154 (0.81)	-0.136	0.117 (1.12)
3.	Other expenditures	-0.101	0.162	-	-0.131	-0.131	0.316 (2.00)	0.026	0.279 (2.42)
4.	Budget surplus	0.030	0.293	0.131	-	0.000	0.447 (2.79)	0.157	0.410 (4.60)
5.	Distortionary taxation	0.030	0.293	0.131	0.000	-	0.447 (2.79)	0.157	0.410 (4.21)
6.	Non-distortionary taxation	-0.417	-0.154	-0.316	-0.447	-0.447	-	-0.290	-0.037 (0.23)
7.	Productive expenditures	-0.127	0.136	-0.026	-0.157	-0.157	0.290 (1.98)	-	0.253 (1.95)
8.	Non-productive expenditures	-0.380	-0.117	-0.279	-0.410	-0.410	0.037 (0.23)	-0.253	-

Note: The t-statistics for columns 6 and 8 are in parentheses. The t-statistics for the other columns are unavailable.

Table 3. Approximate ranges of t-values for testing $\gamma_{j,n} = 0$ against $\gamma_{j,n} \neq 0$:

Budget surplus is omitted

		Est.	t(j, n: 0)	t(j, n: 1)
1.	Lending minus repayments	-0.030 (b)	-0.266	-0.151
2.	Other revenues	-0.293 (a)	-5.348	-2.141
3.	Other expenditures	-0.131 (c)	-2.140	-0.901
4.	Budget surplus	-	-	-
5.	Distortionary taxation	0.000 (b)	0.000	0.000
6.	Non-distortionary	-0.447 (a)	-4.590	-2.430
7.	Productive expenditures	-0.157 (c)	-2.187	-0.997
8.	Non-productive expenditures	-0.41	-	-

Note: Symbols (a), (b) and (c) respectively denote significant, insignificant, and undetermined at the 5% level.