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# Evaluation of Possibilities of Climate Stabilization Policy considering Different Discount Rates: Simulation Analysis using the Modified RICE Model.

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**Abstract** This study is intended to show, using simulation analyses, the validity of using a discount rate for changing combinations of the pure rate of time preference and the elasticity of marginal utility of consumption. Furthermore, this study explores climate stabilization policies, both global and Japanese, and suggests Japan's correct action based on those results. In Japan, interpretations of reducing 50% of $CO_2$ emissions by 2055 as a baseline of the BASE case, and reductions or emissions to 1995 emission levels differ greatly. In the scenarios assumed for this study, although the former reduction is infeasible, the latter is feasible but costly.

**Key words** *global warming, the modified RICE model, climate stabilization targets, social discount rate, policy evaluation*

## 1 Introduction

After Stern (2006) presented the *Stern Review on the Economics of Climate Change* in November 2006, discussions were undertaken to promote radical policies stemming from it. The Stern Review argued for urgent, immediate, and sharp reductions in greenhouse gas (GHG) emissions. In Japan, former Prime Minister Shinzo Abe stated "the long-term target to reduce 50% of emissions of the whole world from now to 2050 is suggested as a universal strategy" in his speech of "Invitation to Cool Earth 50" on May 24 2007. Many comments on the Stern Review, however, were published immediately. Nordhaus (2007c) accepts that the Stern Review contributes to selection of climate change policies with an eye to balancing economic priorities with environmental dangers, in spite of doubts about its model and economic assumptions. The NIES AIM team (2007) praises the path-breaking effort of the Stern Review in bringing in a new dimension in environmental economics and its assertion that large-scale climate policies possess a high degree of economic rationality based only on the scientific knowledge available at this time. On the other hand, Nordhaus (2006), (2007b), (2007c) criticizes the argument of the Stern Review in terms of its use of a low discount rate that does not match data. Tol (2006) makes the criticism that the Stern Review overestimates the impacts of climate change: the Stern Review selects pessimistic studies of the impacts of climate change in addition to applying a low discount rate. The NIES AIM team (2007) explores the Stern Review in terms of four such major points as assumptions of the discount rate, cost estimation of climate change impacts, cost estimation of climate mitigation, and the advantages of early action.

Considering that "If we don't act, the overall costs and risks of climate change will be equivalent to losing at least 5% of global GDP (Gross Domestic Production) each year, now and forever. If a wider range of risks and impacts is taken into account, the estimates of damage could rise to 20% of GDP or more." in the Stern Review, the argument of the Stern Review depends on estimates of the damage of the impacts of climate change and cost to mitigate global warming; these estimates depend on assumptions of the discount rate and the methodology to estimate damage and costs related to global warming. Especially, when long-term costs and benefits such as those associated with global warming must be taken into account, evaluation of the intertemporal costs and benefits becomes a subject of discussion. Because using even a marginally different discount rate, which is used to convert the future values into present values, engenders starkly different evaluations, the assumed the discount rate must be examined carefully. In these comments on the Stern Review, Nordhaus (2007c) analyzes the same simulations as the Stern Review using his Dynamic Integrated model of Climate and the Economy (DICE). Because both the PAGE model by Hope (2002), used in the Stern Review, and the DICE-2007 model by Nordhaus (2007c) treat the global level, no attempts have been made at evaluating the regional reduction needed to achieve stabilizing targets of GHG concentrations and the feasibility of these targets.

This study is intended to show, by simulation analyses with some scenarios, the validity of the discount rate by sensitivity analyses with various combinations of the pure rate of time preference (PRTP) and the elasticity of the marginal utility of consumption. It also describes the global and Japanese possibilities of climate stabilization policies and what action Japan should take. The remainder of this report is organized as follows. Chapter 2 presents a review of the relevant literature. Chapter 3 presents assumptions of model structure and scenarios used in this study. Chapter 4 describes simulations of climate stabilization targets for the whole world and Japan used in the assumed scenarios with subsequent presentation of sensitivity analyses of the combination of the PRTP and the elasticity of marginal utility of consumption in Chapter 5. Finally, Chapter 6 concludes this paper.

## 2 Literature review

In terms of the features on which climate change has a long-term impact, assumptions of the discount rate become a serious subject of some discussions to evaluate the intertemporal costs and benefits. Nordhaus (1994) shows that discount rate assumptions strongly affect estimates of some policy simulations because values of the PRTP are uncertain. As one approach to conduct a discount rate, the optimal growth model, called the Ramsey model, is used in economics. In this optimal growth framework, Eq.(1), the Ramsey equation, is obtained in the steady state in which the growth rate of per capita consumption is constant, and when constant population is normalized to unity for simplicity.

$$r^* = \rho + \alpha g^* \tag{1}$$

Therein, $r^*$ is the real return on capital (real interest rate) in a steady state, $\rho$ represents the value of PRTP, $\alpha$ is the elasticity of marginal utility of consumption, and $g^*$ is the growth rate of per capita consumption in a steady state. Sakata and Hayashiyama (2002) present this approach as a prescriptive approach obtained from the marginal rate of substitution of intertemporal consumption. The right-hand side of Eq.(1) shows the social discount rate in the public project. Therefore, it is apparent that the discount rate used in converting damage and costs on global warming into the present value is applied not to the PRTP $\rho$ but to the real return on capital $r^*$ in in Eq.(1). By this theory, a discount rate summarizes the factors such as the PRTP, the elasticity of marginal utility of consumption and the consumption growth rate.

Next, to calculate the elasticity of marginal utility of consumption, which is a factor to determine a discount rate, the utility function is formulated. In this optimal growth model, the utility function of the constant relative rate of risk aversion indicated in Eq.(2) is often used.

$$U[c(t)] \begin{cases} = \dfrac{c(t)^{1-\alpha}}{1-\alpha}, & (\alpha > 0, \alpha \neq 1) \\ = \ln c(t), & (\alpha = 1) \end{cases} \tag{2}$$

In this equation, $\alpha$ is the consumption elasticity and $c$ is the per capita consumption. Equation (1) and Eq.(2) show that the consumption elasticity $\alpha$ does not depend on consumption. Furthermore, for a consumption grow rate $g^*$ that is constant in a steady state, the discount rate is obtained from the combination of the PRTP $\rho$ and the consumption elasticity $\alpha$.

In the Stern Review, the combination of the PRTP $\rho$ and the consumption elasticity $\alpha$ is applied to $\rho$=0.1% and $\alpha$=1 (a logarithm utility function). The Stern Review recommended urgent, immediate and sharp reductions, such as a carbon tax for carbon dioxide ($CO_2$) of 311 \$ per ton carbon. In contrast to the Stern Review, Nordhaus (2007c) examines three assumptions of the combination of the PRTP and the consumption elasticity used in the DICE2007 model. Nordhaus (2007c) presents the criticism that results of the Stern Review are unacceptable on the same assumptions as the Stern Review because its discount rate does not match data on real investment and savings. In the Stern Review, under the assumption of 1.3% consumption growth, the combination of $\rho$=0.1% and $\alpha$=1 yields a discount rate of 1.4% using Eq.(1) and these parameters. The Stern Review uses a very low discount rate of 1.4%.

Earlier studies by Manne *et al.* (1995), Nordhaus (1994), Nordhaus and Boyer (2000), Peck and Teisberg (1994) and Tol (1997) conclude that lesser reductions in GHG emissions in the near term and greater reductions in the future are reasonable. On the other hand, Cline (1992) argues for urgent, immediate and sharp reductions like those recommended by the Stern Review, with a lower discount rate. Cline (1992) recommends that an appropriate discount rate should be 0% to 0.5% and that a lower discount rate should be adopted. In contrast to these arguments, Nordhaus (1994) points out that although adoption of a lower discount rate is desirable from an ethical standpoint, it is unrealistic from a theoretical and empirical point because of inconsistency with real investment and savings. Manne (1994) shows that a given lower discount rate causes an unrealistic increase of the savings rate using some simulation analyses. Toth (1994) indicates that an ethically desirable assumption of a low discount rate not only has little economic basis, it is also inconsistent with results of the cost-benefit analysis.

As Sakata and Hayashiyama (2002) point out, because no concrete measure is acceptable objectively with the PRTP and the consumption elasticity, and because some subjective decisions to measure a discount rate distort ethical value judgments such as intergenerational equity, there is little agreement on an appropriate discount rate. It can be considered, however, that an exploration of validity of assumed discount rates and policy evaluation under some different assumptions of discount rates are meaningful to avoid inappropriate evaluations and decisions related to global warming with uncertainty.

# 3 Outline of model and scenarios

## 3.1 Model assumptions

This study extends the Regional dynamic Integrated model of Climate and the Economy (RICE), an integrated assessment model developed by Nordhaus and Boyer (2000), and constructs the modified RICE. RICE is a dynamic optimal model based on DICE developed by Nordhaus (1994). It consists of an economic model described in economic activities and GHG emissions and climate model represented in atmospheric GHG concentrations, temperature increases, and negative feedback of temperature increases on economic activities. Although DICE is a regional model, RICE divides the world into eight regions. Although RICE can numerically evaluate the regional effects of countermeasures against global warming, it does not consider the regional trading of goods and services. Compared to similar regional dynamic optimal models as MERGE, developed by Manne *et al.* (1995) and MARIA by Mori (2000), its regional relations reflecting only emission trading are insufficient. Fussel (2007) criticizes methodological and empirical inconsistencies of DICE. Considering these features that RICE and DICE have a transparency of model structure and are easy to deal with, and that this study investigates criticism of Nordhaus (2007) to the Stern Review used in DICE, however, to use RICE is more effective than some complex models.

Because the regional classification of RICE is based on World Development Indicators of the World Bank, and because Japan is classified into Other High Income (OHI), the present study uses nine regions, separating Japan from the eight RICE regions. **Table.1** shows details of nine regions in this modified RICE model. Regions "ANNEX I", "DVLPED" and "WLD" in **Table.1** have $CO_2$ emission regulations imposed in our simulation analysis. Also, WLD includes all regions, ANNEX I means countries including the Annex I in the Kyoto Protocol, and DVLPED corresponds with countries adding Brazil, Russia, India, and China (BRICs) to ANNEX I.

Settings of major numerical values of the parameters by separation of Japan from the OHI region in RICE are shown below. As economic values of population, GDP, and $CO_2$ emission are based on Nordhaus and Boyer (2000); the same data sets are used for details of these data sets, see Nordhaus and Boyer (2000). The values of the coefficient of damage function are used as values of RICE before the RICE-99 model. Because the share parameters of $CO_2$ emissions in the production function are the same values for Japan and OHI region in RICE before the RICE-99 model, they are set to the same values. For the parameter, markup, of energy costs in Japan, this value is estimated using sensitivity analysis. After values of markup in Japan are changed from 0 to 1000 by 50 in the Base case, the minimum deviation from the observed value in 1995 for production and $CO_2$ emissions is applied as markup in Japan. For equations of the modified RICE used in this study, see Appendix and Nordhaus and Boyer (2000).

**Table. 1: Definition of regional classification in the modified RICE model.**

| Code | Region |
|---|---|
| USA | Unitede States |
| OHI | Other High Income OECD except Japan (Canada, Australia, etc.) |
| EUROPE | OECD Europe |
| EE | Russia &Eastern Europe (the Formerly Centrally Planned Economies) |
| MI | Middle Income (Brazil, South Korea, Argentina, Malaysia, high-income OPEC) |
| LMI | Lower Income (Mexico, Turkey, South Africa, South America etc.) |
| CHINA | China |
| LI | Low Income (South & Southeast Asia, India, Subsaharan Africa, Latin America etc.) |
| JPN | Japan |
| ANNEX I | USA, OHI, EUROPE, EE, JPN |
| DVLPED | USA, OHI, EUROPE, EE, MI, CHINA, JPN |
| WLD | USA, OHI, EUROPE, EE, MI, LMI, CHINA, LI, JPN |

### 3.2 Scenario assumptions

This study examines some scenarios centered upon the climate stabilization targets discussed in IPCC (2007a), (2007b), (2007c). Recently, numerical evaluations of danger attributable to global warming and stabilization level of GHG concentration have been specifically addressed. Matsuoka (2005) describes that "compared to preindustrial temperatures, increase in global mean temperature over 2 ℃ would induce some serious impacts" through his review of a level of dangerous climate change and climate stabilization of GHG concentration. In addition, Hijioka (2005) shows that, to avoid a global mean temperature increase to greater than +2 ℃, a stabilization target for GHG concentrations of less than 500 ppm (by volume) is needed. In view of these discussions, 11 scenarios are defined: Base scenario, Optimal scenario, three temperature stabilization scenarios, three $CO_2$ concentration scenarios, and three emission control scenarios. **Table.2** presents details of scenarios used in this study.

(1) Base scenario (BASE)
This scenario is the baseline scenario in this study: no policies are taken to slow climate change.

(2) Optimal scenario (OPT)
Economically efficient policies are taken to slow climate change. Nordhaus and Boyer

(2000) take a uniform carbon tax as an example of such optimal policies. The policy finds a Pareto optimal trajectory that balances current abatement costs against future environmental benefits of carbon abatement.

(3) Temperature stabilization scenario (TMPxx)
Stabilizing global mean temperature at +2.0 ℃, +2.5 ℃, and +3.0 ℃, respectively pursued. As described above, according to some findings of review of Matsuoka (2005), the possibilities of some $CO_2$ emission reduction policies are examined in cases of settings of +2.0 to +3.0 ℃.

(4) $CO_2$ concentration stabilization scenario (Cxxx)
Stabilizing atmospheric $CO_2$ concentrations at 450, 550, and 650 ppm, respectively is taken. Above all, $CO_2$ concentrations at 550 ppm are about twice their preindustrial levels, which corresponds with the $CO_2$-doubling Scenario used in some studies that have evaluated countermeasures against global warming economically, as in Cline (1992), Nordhaus (1994), and Fankhauser (1995) in the early 1990s. Then, note that the concentration unit used is not the $CO_2$ equivalent concentration, but the $CO_2$ concentration.

(5) $CO_2$ emission regulation scenario (E05xxx)
Limiting $CO_2$ emissions to half of 2005 emission levels is pursued after 2055. By changing regions with imposed regulation that $CO_2$ emission is controlled to half of the 2005 level in each scenario: three scenarios are examined. Although the scenario in which $CO_2$ emissions in ANNEX I region are limited to half of the 2005 level is more stringent than those of the policies that limit $CO_2$ emissions from an industrial sector in the high-income countries belonging to the Annex I of the Kyoto Protocol, it can be considered that it is an allowable scenario to judge the assertions of the Stern Review and Abe (2007) described above.

**Table. 2: Definitions of alternative scenarios.**

| Code | Description |
|---|---|
| BASE(a) | Business-as-usual scenario |
| OPT(a) | Emissions and carbon prices set at Pareto optimal levels to slow climate change. |
| TMP20 | Temperature rise from pre-industrial level is limited to +2.0 ℃. |
| TMP25 | Temperature rise from pre-industrial level is limited to +2.5 ℃. |
| TMP30 | Temperature rise from pre-industrial level is limited to +3.0 ℃. |
| C450 | Atmospheric $CO_2$ concentrations are stabilized at 450 ppm. |
| C550 | Atmospheric $CO_2$ concentrations are stabilized at 550 ppm. |
| C650 | Atmospheric $CO_2$ concentrations are stabilized at 650 ppm. |
| E95WLD | Emissions in all regions are limited to the half of 2005 level after 2055. |
| E95ANX | Emissions in the Annex I regions are limited to the half of 2005 level after 2055. |
| E95DVL | Emissions in the developed regions are limited to the half of 2005 level after 2055. |

(a) These scenarios are the same as those of Nordhaus and Boyer (2000).

# 4 Simulation analysis

## 4.1 Sensitive analysis of discounting rates

In this section, sensitivity analysis of the validity of the combination of the PRTP and the elasticity of marginal utility in the Stern Report and Nordhaus (2007) is performed using the modified RICE. To make this sensitivity analysis, important lessons on the appropriate range of these parameters from Nordhaus (2007) are shown as follows. These are: 1) the PRTP should start at 2% to 3% per year and decline to 0.5% or 1%, respectively, per year in 300 years; 2) the elasticity of marginal utility of consumption should be 1 to 2 to satisfy Eq.(1); 3) the growth of per capita consumption should be 0.5% to 1.5%; and 4) the combination of values of parameters described above should be selected such that the resulting values of social discount rate would be 3% to 5% to satisfy Eq.(1). Based on these lessons, this study assumes that the growth rate of per capita consumption ($g$) in Eq.(1) sets the value of 1.3%, and examines five optimal cases changing the combination of the PRTP and the elasticity of marginal utility $(\rho, \alpha)$. Then, these fice cases are defined as follows.

(1) RJP1 case: $(\rho, \alpha) = (3.0 - 1.39,\ 1)$
RJP1 case has the standard combination used in RICE and the same combination as the RUN1 case of a study used in the DICE2006 model; the elasticity value of 1 signifies the logarithmic utility function.

(2) RJP2 case: $(\rho, \alpha) = (3.0 - 1.0,\ 1)$
RJP2 case has the combination of the PRTP declining from 3.0% to 1.0% per year in 300 years; the elasticity value of 1 signifies the logarithmic utility function.

(3) RJP3 case: $(\rho, \alpha) = (2.0 - 0.5,\ 1)$
RJP3 case has the combination of the PRTP declining from 2.0% to 0.5% per year in 300 years; the elasticity value of 1 signifies the logarithmic utility function.

(4) NORD case: $(\rho, \alpha) = (1.5 - 1.49,\ 2)$
NORD case has the same combination as RUN1 case of a study used in DICE2007 model; the elasticity value of 2 does not signify the logarithmic utility function.

(5) STERN case: $(\rho, \alpha) = (0.1,\ 1)$
STERN case has the same combination as the Stern Review; the elasticity value of 1 signifies the logarithmic utility function.

**Table.3** shows results of the Stern Review, Nordhaus (2006), Nordhaus (2007) and our sensitivity analysis used in the modified RICE. In this table, results of Nordhaus (2006) are indicated in the row of "DICE2006" and those of Nordhaus (2007) in the row of "DICE2007", both cases assumed three cases changing in the combinations of the PRTP and the elasticity of marginal utility of consumption. In addition, results in this study are shown in the row of"DICE_JPN".

In **Table.3**, a comparison with DICE2006 and DICE2007 is presented, although values of the optimal carbon price and the optimal emission reduction in RJP1 case are low, **Figure.1** and **Figure.2** show that the trajectories of temperature increase and $CO_2$ emissions in RJP1

**Table. 3: Comparison of our sensitive analyses to the Stern Review and the Nordhaus.**

| Model | Case | PRTP (%) | Elasticity | Discounting[a] (%) | Optimal Cabon Price[b] ($/t-C) | | | | Optimal Emission Reduction[c] (%) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 2005 | 2015 | 2050 | 2100 | 2005 | 2015 | 2050 | 2100 |
| Stern | | 0.1 | 1 | 1.4 | 311 | | | | | | | |
| DICE2006 | | | | | | | | | | | | |
| | RUN 1 | 3.0-1.39 | 1 | 2.69-4.3 | 17.1 | | 84 | 270 | 6 | | 14 | 25 |
| | RUN 2 | 0.1 | 1 | 1.4 | 159.0 | | | | | 50 | | |
| | RUN 3 | 0.1 | 2.25 | 3.025 | 19.6 | | | | | | | |
| DICE2007 | | | | | | | | | | | | |
| | RUN 1 | 1.5 | 2 | 4.1 | | 35 | 85 | 206 | | 14 | 25 | 43 |
| | RUN 2 | 0.1 | 1 | 1.4 | | 360 | | | | 53 | | |
| | RUN 3 | 0.1 | 3 | 4 | | 36 | | | | | | |
| RICE_JPN | | | | | | | | | | | | |
| | RJP 1 | 3.0-1.39 | 1 | 2.69-4.3 | 8.1 | 12.1 | 27.9 | 60.7 | 5.1 | 6.0 | 8.1 | 10.2 |
| | RJP 2 | 3.0-1.0 | 1 | 2.3-4.3 | 9.3 | 13.9 | 32.9 | 74.8 | 5.9 | 6.9 | 9.3 | 12.8 |
| | RJP 3 | 2.0-0.5 | 1 | 1.8-3.3 | 23.8 | 33.3 | 67.0 | 137.4 | 12.3 | 12.6 | 15.3 | 22.3 |
| | NORD | 1.5-1.49 | 2 | 4.09-4.1 | 18.7 | 26.1 | 51.5 | 90.4 | 8.1 | 8.6 | 9.9 | 12.6 |
| | STERN | 0.1 | 1 | 1.4 | 149.1 | 181.1 | 255.4 | 368.8 | 48.4 | 47.4 | 45.1 | 45.5 |

(a) The growth of per capita consumption and population growth are assumed to be 1.3 % and 0%, respectively, per year.
(b) While DICE2007 uses 2005 U.S. dollars price, RICE_JPN uses 1990 U.S. dollars price.
(c) Emissions reduction rate is relative to a baseline case in each model.

case are about the same as that of NORD case. Compared to the optimal carbon price of NORD case, however, that of RJP1 case is about half. The assumption that the combinations of the PRTP and the elasticity in RJP2 case are about the same as those of RJP1 case engenders the same results related to the optimal carbon price and emission reduction of RJP2 case as those of RJP1 case. On the other hand, although values of the optimal carbon and emission reduction in RJP3 case and RUN1 of DICE2006 are the same, these values of RJP3 case diverge from those of RUN1 of DICE2006 after 2050.

Next, as shown in **Table.3**, values of the optimal carbon price and emission reduction in NORD case are lower than those of RUN1 of DICE2007 in spite of their similar assumptions. In addition, **Figure.1** and **Figure.2** indicate that, compared to RJP3 case, which obtains similar results of RUN1 of DICE2007, paths of temperature increase and $CO_2$ emission in NORD case are followed above those of RJP3 case. In this modified RICE model, it is considered that the NORD case provides a different scenario to RUN1 of DICE2007.

Third, in the STERN case of **Table.3**, the optimal carbon price of 145 $ per ton C in 2005 and of 181 $ per ton C in 2015, respectively, are about the half of those of RUN2 of DICE2007 and Stern. On the other hand, the optimal emission reduction of STERN case has about the same result of RUN2 of DICE2006 and RUN2 of DICE2007. **Figure.1** and **Figure.2** show that $CO_2$ emissions of the STERN case in 2055 are less than those of TMP20 scenario limiting temperature increase to 2.0 ℃; those of STERN case in 2125 are less than those of C550 scenario, stabilizing at 550 ppm of $CO_2$ concentrations. Therefore, these results means that $CO_2$ emissions require more reductions from the business-as-usual scenario (BASE) and the argument of the Stern Review, "urgent, immediate, and sharp reductions in GHG emissions". Considering these results, though it is likely that the optimal policy used in setting for discount rate of STERN case achieve to stabilize temperature increase within 2.0 ℃ in this century and $CO_2$ concentrations within 550 ppm in the next century, the optimal policy of the STERN case does not always ensure these stabilization targets.

Finally, in **Table.3**, two findings related to the validity of the combinations of the PRTP and the consumption elasticity are presented in all results of this sensitivity analysis in RICE_JPN. First, if the elasticity of marginal utility of consumption and the consumption growth rate are constant, then as the PRTP declines, the optimal carbon price and emissions reduction increase and more stringent and earlier reduction is required. Therefore, it is apparent that the recommendation of the Stern Review, that is "urgent, immediate, and sharp reductions in GHG emissions", depends on a low time discount rate. Secondly, compared to the RJP1 case and RJP3 case, although the optimal carbon price and $CO_2$ emission reduction of RJP1 case are about half of those of the RJP3 case and there is a difference in $CO_2$ emissions between the RJP1 case and RJP3 case, both trajectories of temperature increase are about the same. Consequently, in terms of a climate stabilization policy, if the elasticity of marginal utility of consumption is the same because selecting the PRTP depends on the growth rate of consumption or the real return on capital, more stringent reduction recommended by the Stern Review, which sets a low time discount rate that is inappropriate for the real world, can not be advocated from our simulation results.

Recently, evaluation of public projects in developed countries uses a discount rate of 3% to 4%. IPCC (2007b) summarizes discount rates of 3% to 10% in the major studies, and IPCC (2007c) uses a discount rate of 5% in calculating the net present value. In this section,

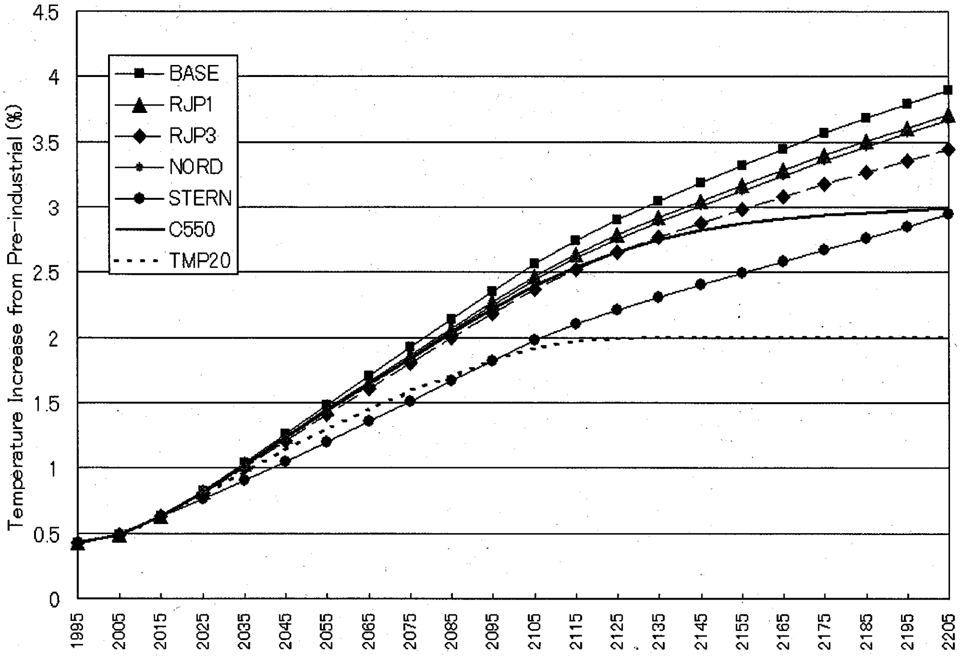


**Figure. 1: Projected temperature increase in sensitive analysis.**

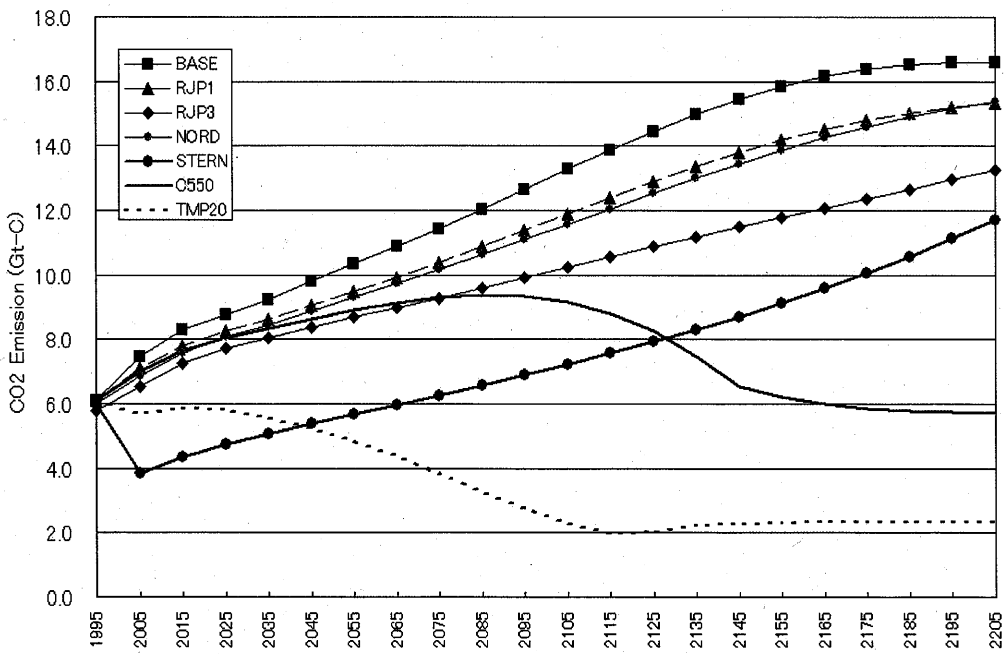


**Figure. 2: Projected $CO_2$ emissions in sensitive analysis.**

discount rates used in these cases, RJP1, RJP2, RJP3 and NORD case, are 3% to 4%. Therefore, although there is a slight difference among these results, it is apparent that the projected optimal trajectories show similar trends in each case. In the next section, using different combinations of the PRTP and the consumption elasticity, $(\rho, \alpha) = (3.0 - 1.39,\ 1)$ of RJP1 case and $(\rho, \alpha) = (0.1,\ 1)$ of the STERN case, this study undertakes simulation analyses of all scenarios defined in 3.2.

## 4.2 Analysis of alternative global policies

In this section, using the modified RICE model, simulations of scenarios assumed in 3.2 are analyzed. For the combination of the pure rate of time preference and the elasticity of marginal utility of consumption, $(\rho, \alpha) = (3.0 - 1.39,\ 1)$ of RJP1 case and $(\rho, \alpha) = (0.1,\ 1)$ of STERN case are examined in each scenario.

First, **Figure.3** shows that the maximum temperature levels of all scenarios in RJP1 case are 1.49 ℃ in 2055 (BASE), 2.56 ℃ in 2105 (BASE) and 3.90 ℃ in 2205 (BASE), whereas the minimum are 1.30 ℃ in 2055 (TMP20), 1.91 ℃ in 2055 (TMP20) and 2.00 ℃ in 2205 (TMP20). In the RJP1 case, the only scenario in which the temperature increase is limited to +2.0 ℃ is only TMP20 scenario. Temperature changes of all scenarios except TMP20 scenario reach the range of +2.0 ℃ to 2.5 ℃ in 2105. In addition, a temperature increase of the C550 scenario corresponding to the $CO_2$-doubling scenario surpasses +2.0 ℃ in 2085, and reaches +3.0 ℃ during 2105 to 2205. **Figure.4** shows that $CO_2$ concentrations should be at 878 GtC to stabilize the temperature increase at +2.0 ℃ because of stabilization of $CO_2$ concentrations at that level in the TMP20 scenario after 2105. This level corresponds to less than about 420 ppm. For the OPT scenario, E05ANX scenario and E05DVL scenario, increasing $CO_2$ concentrations of these scenarios after 2205 engenders a temperature increase greater than +3.0 ℃. **Figure.5** shows that, although $CO_2$ emissions of six scenarios, C450, C550, TMP20, TMP25, TMP30, and E05WLD, increase or decrease in 2105, they are the same as or below the 1995 emissions level in 2205. Temperature changes of the six scenarios are stabilized at less than +3.0 ℃. On the other hand, to limit the temperature increase to +2.0 ℃, $CO_2$ emissions reduction in TMP20 scenario in 2055 requires 5.49 GtC from level of BASE scenario or 1.25 GtC from 1995 level. Consequently, in spite of identical targets, it is apparent that the different level of baseline means a different level needed to achieve the target. In these results, the $CO_2$ emissions reduction target as the baseline of the BASE scenario would be stringent, and to achieve the reduction target as baseline of the half of 1995 emissions level would be impossible in these assumed scenarios. **Table.4** shows $CO_2$ emissions reduction needed in 2055 in each scenario. AAdditionally, although $CO_2$ emissions of two emission regulation scenarios, E05ANX and E05DVL, also increase and temperature increase of these scenarios are, respectively, +3.5 ℃ and +3.2 ℃ after 2055, it can be considered that these results are attributable to $CO_2$ emissions increase in developing countries.

Next, for the STERN case, **Figure.6** shows the maximum and minimum temperature levels in all scenarios. The maxima are +1.31 ℃ in 2055 (E05WLD), +2.17 ℃ in 2105 (BASE) and +3.34 ℃ in 2205 (BASE); the minimum are +1.05 ℃ in 2055 (TMP20), +1.59 ℃ in 2105 (TMP20) and +1.94 ℃ in 2205 (TMP20). In the STERN case, although the temperature level of the E05WLD scenario in 2055 shows the greatest increase, its level is stabilized at +2.0 ℃ after 2105 and is the same as that of the TMP20 scenario in 2205. In addition, the temperature increase of the TMP20 scenario and the E05WLD scenario can be limited to +2.0 ℃. Furthermore, that of these scenarios is less than +2.0 ℃ in 2205. Although temperatures of all scenarios except the TMP20 scenario in RJP1 case are greater than +2.0 ℃ in 2105, that of all scenarios except the BASE scenario in STERN case is less than +2.0 ℃ in 2105. That of C550 scenario corresponding to the $CO_2$-doubling scenario is greater than +2.5 ℃ in 2205. **Figure.7** shows $CO_2$ concentrations of the TMP20 scenario. The temperature increase of its scenario can be stabilized at +2.0 ℃; the E05WLD scenario stabilizes at about 875 GtC (corresponding to about 420 ppm) in 2205; it is apparent that $CO_2$ concentrations should be below about 420 ppm to stabilize the temperature at +2.0 ℃ as well as RJP1 case. **Figure.8** shows that $CO_2$ emissions of all scenarios except the BASE scenario are less than 1995 emissions level in 2055 and those of only four scenarios, C450, TMP20, TMP25, and E05WLD, are below 1995 level in 2205. Compared to $CO_2$ emissions in the RJP1 case, although emissions of C550 scenario and TMP30 scenario are above those of 1995 level, lower total emissions of these scenarios in 2205 result in a lower temperature increase of these scenarios in the STERN case than those of the same scenarios in RJP1 case. Furthermore, even for the temperature increase of OPT scenario, the $CO_2$ emission is much greater than that of 1995 level rise of less than +3.0 ℃ in 2205. In **Table.4**, it is apparent that $CO_2$ emissions reduction that is necessary to achieve the policy targets yields a positive value in all scenarios, even if both the BASE scenario and 1995 level are taken as a baseline. In the STERN case, the TMP20 scenario or E05WLD scenario must be selected to achieve the target as $CO_2$ emission of the half of 1995 level in 2055.

Finally, compared to the RJP1 case and STERN case, although near-term $CO_2$ emission reductions in all scenarios except the E05ANX scenario and E05DVL scenario in RJP1 case are low, more stringent reductions are required in the future. As before, this result implies that emission reduction with advanced future technologies is more reasonable than a sharp reduction with low technology today. On the other hand, in the STERN recommendation, because a sharp reduction is taken in the near term, little reduction is needed in the future. The desirability of these results stems from different discount rates used in the RJP case and STERN case. Considering that the larger discount rate means a lower present value in converting the future costs to the present value, in the STERN case, with very low discount rates, the present sharp reduction is reasonable because the present value of the future damage caused by the present marginal emissions is very high. Because Nordhaus (2007c) criticizes the Stern Review, with a low discount rate of 1.4% and, as described above, public projects in developed countries use discount rates of 3% to 4%, it can be considered that the discount rate of RJP1 used in this study is appropriate.

### 4.3 Analysis of alternative policies for Japan

In this section, simulations of climate stabilization policies in Japan are analyzed. Here, the combination of the PRTP and the elasticity of marginal utility of consumption in the RJP1 case and in the STERN case are examined in each scenario, as in the previous section.

First, for the RJP1 case, **Figure.9** shows that $CO_2$ emissions in all scenarios decrease after

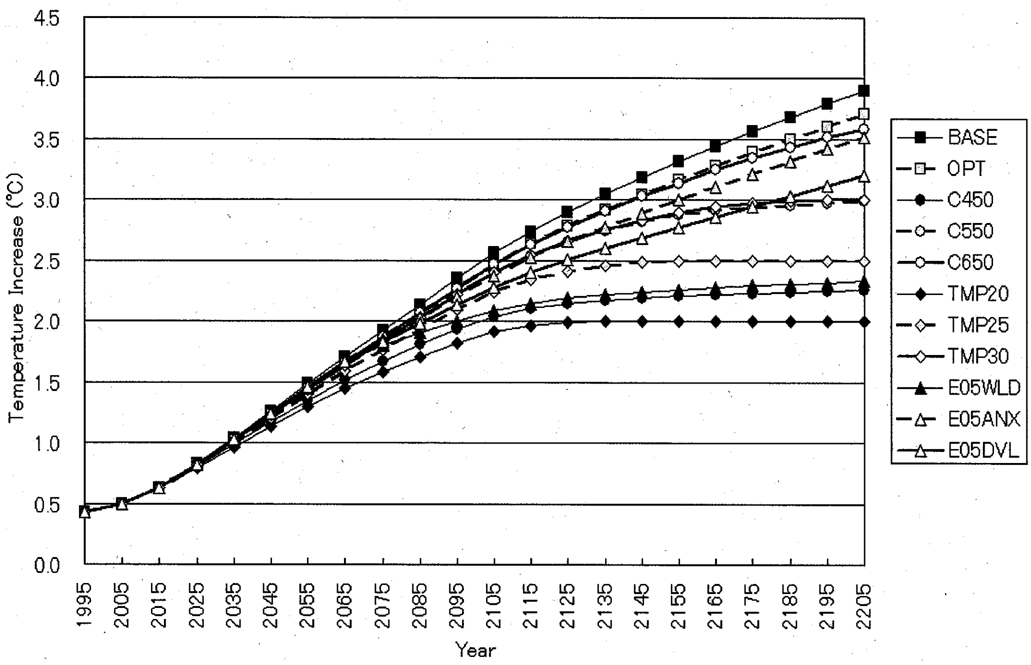


**Figure. 3: Projected temperature increase in alternative scenarios (RJP1 case).**

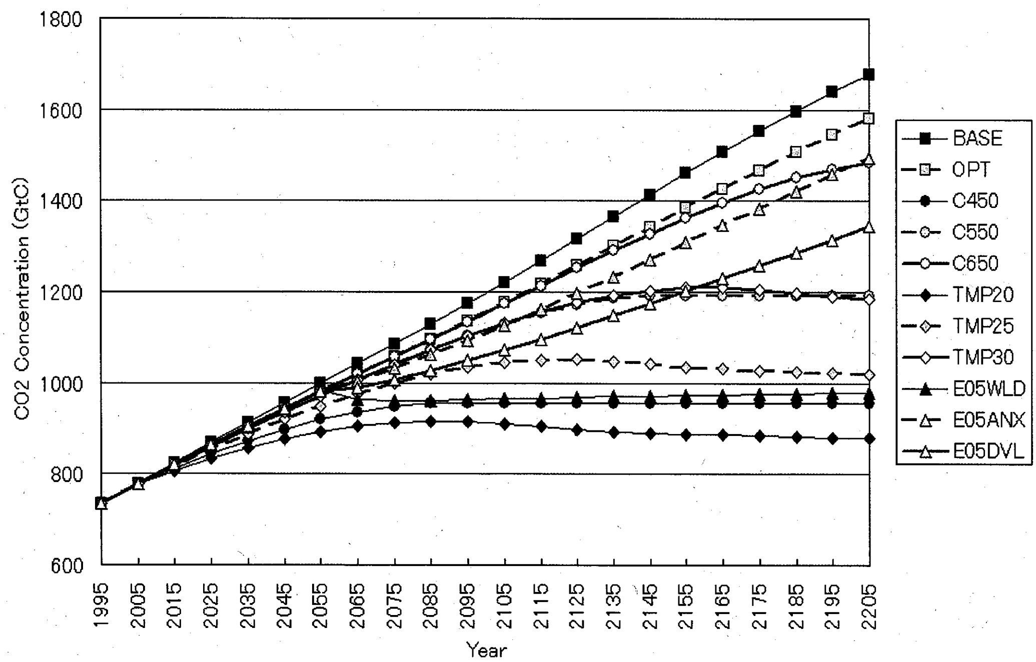


**Figure. 4: Projected $CO_2$ concentrations in alternative scenarios (RJP1 case).**

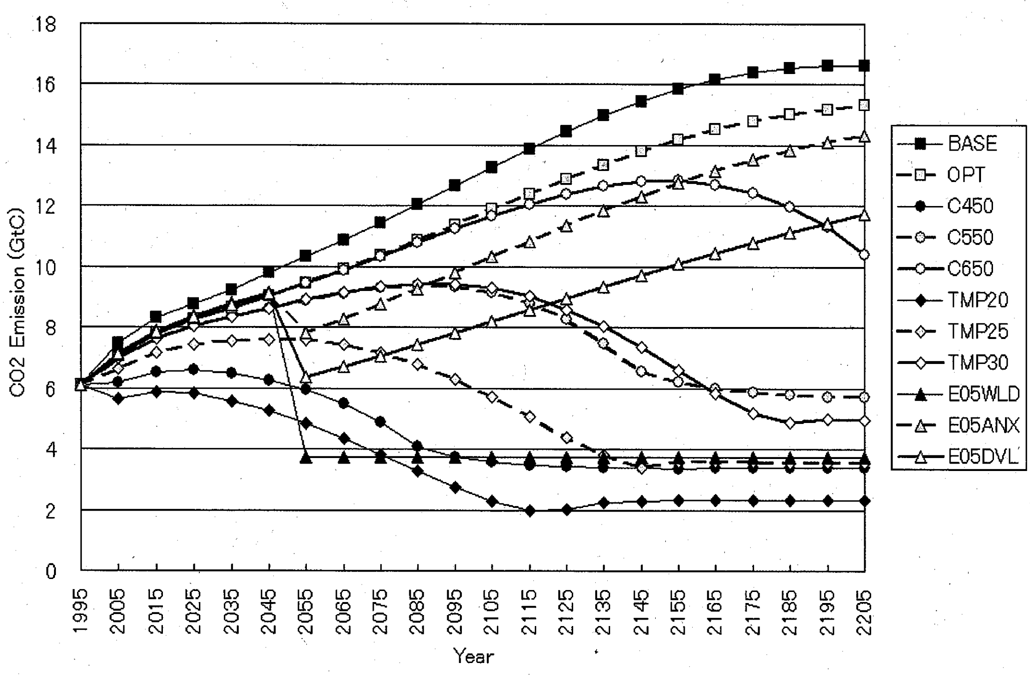


**Figure. 5: Projected $CO_2$ emissions in alternative scenarios (RJP1 case).**

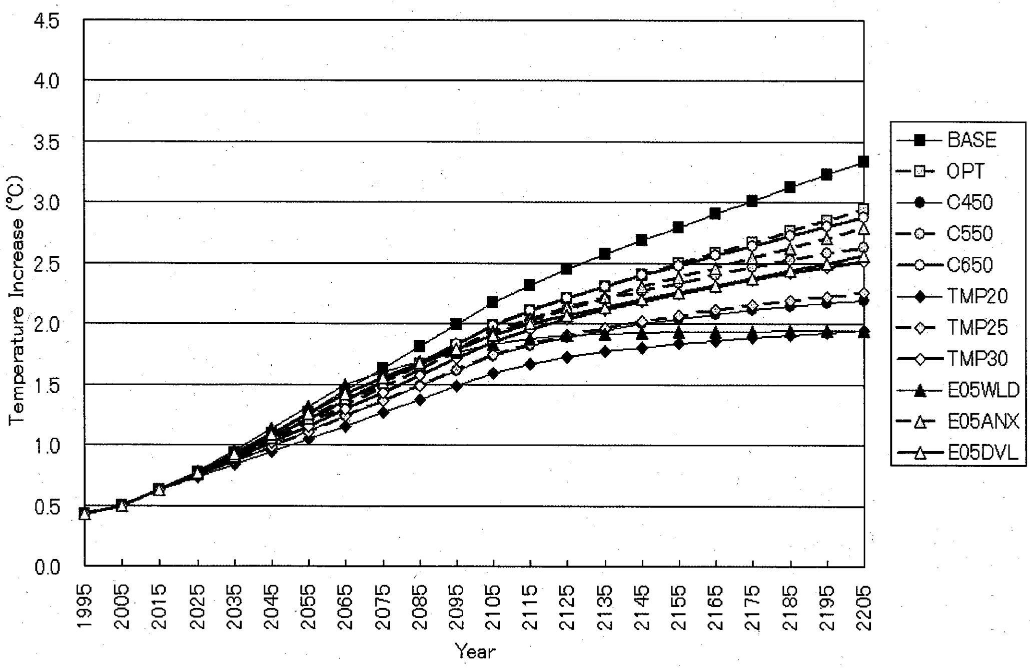


**Figure. 6: Projected temperature increase in alternative scenarios (STERN case).**

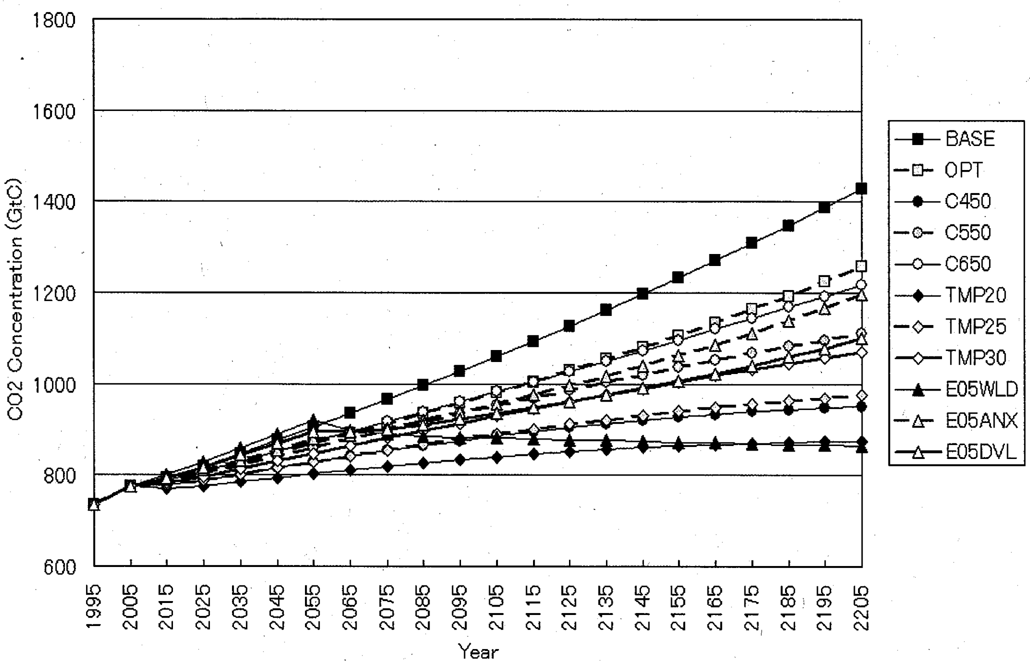


**Figure. 7: Projected $CO_2$ concentrations in alternative scenarios (STERN case).**

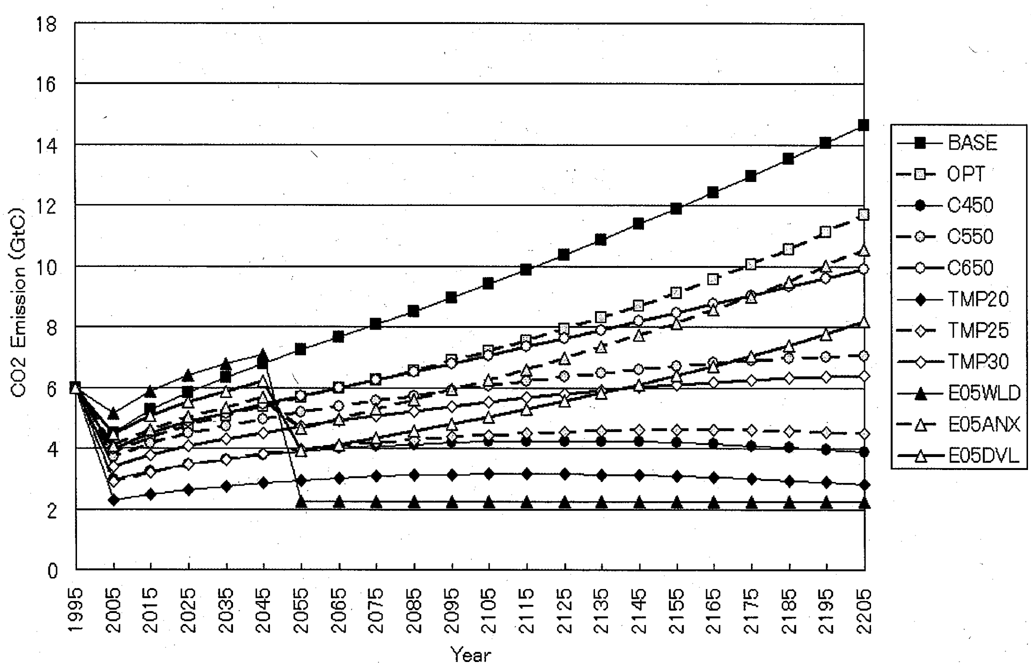


**Figure. 8: Projected $CO_2$ emissions in alternative scenarios (STERN case).**

**Table. 4: Global $CO_2$ emissions reductions in 2055(GtC)**

| | | BASE | OPT | C450 | C550 | C650 | TMP20 | TMP25 | TMP30 | E05WLD | E05ANX | E05DVL |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| RJP1 | BASE | 0.00 | 0.87 | 4.39 | 1.44 | 0.90 | 5.49 | 2.77 | 1.43 | 6.61 | 2.53 | 3.96 |
| | 1995 | −4.24 | −3.37 | 0.15 | −2.80 | −3.34 | 1.25 | −1.47 | −2.81 | 2.37 | −1.71 | −0.28 |
| STERN | BASE | 0.00 | 1.57 | 3.35 | 2.07 | 1.51 | 4.29 | 3.29 | 2.52 | 4.98 | 2.58 | 3.33 |
| | 1995 | −1.26 | 0.31 | 2.09 | 0.81 | 0.25 | 3.03 | 2.03 | 1.26 | 3.72 | 1.33 | 2.07 |

Note: While positive values show emissions reduction, negative values show emissions increase.

2025. Although $CO_2$ emissions of three emission control scenarios —E05WLD, E05ANX, and E05DVL— are markedly limited in 2055, those of the three scenarios are constant after 2055. Then, **Table.5** shows that $CO_2$ emissions reduction is necessary to achieve each scenario as the baseline of the BASE scenario and 1995 emission level. In the baseline of the BASE scenario, half of the $CO_2$ emissions reduction from the BASE case in 2055 would be impossible to achieve in any scenario. On the other hand, in the baseline of the 1995 emission level, half of the reduction from 1995 emission levels in 2055 would be feasible with the TMP scenario and three emission control scenarios. It can be considered, however, that these feasible scenarios require enormous costs to reduce $CO_2$ emissions. Compared with $CO_2$ emissions in each region, **Figure.10** shows that $CO_2$ emissions in Japan in 2055 are very low in all scenarios. Because $CO_2$ emissions in Japan are low and $CO_2$ emissions reductions are high, irrespective of the baseline, a potential reduction in Japan would be small. Furthermore, limitation or reduction in high-emission regions, such as USA, LMI, CHINA and LI, would be necessary to reduce emissions globally and efficiently. Considering the above, Japan could more efficiently reduce $CO_2$ emissions with CDM and technological transfers to regions with insufficient technological, such as LMI, CHINA, and LI, than to make vigorous reductions in the country.

Next, in the STERN case, **Figure.11** shows that although $CO_2$ emissions in Japan decrease significantly in 2005, they increase slightly in 2025 and decrease again after 2025. In addition, **Table.5** shows that, in 2055, although half of the $CO_2$ emissions reduction as baseline of BASE scenario would be impossible to achieve in any scenario, half as baseline of the 1995 emission level would be feasible with the TMP20 scenario, TMP25 scenario, C450 scenario and three emission control scenarios. These feasible scenarios, however, would impart high costs, as would the RJP1 case. Comparing $CO_2$ emissions in each region, **Figure.12** shows that the same trends as emissions of the RJP1 case, in spite of absolutely lower emissions, engenders the same implications as the RJP1 case in future projects in Japan.

In Japan, although $CO_2$ emissions and those trajectories depend on discount rates, interpretations to reduce 50% of $CO_2$ emission in 2055 as a baseline of BASE case and 1995 emission level differ greatly in both the RJP1 case and STERN case. In scenarios assumed for this study, although the former reduction is infeasible, the latter is feasible but would be costly to achieve. Moreover, because the potential reduction in Japan is low, its reduction with CDM and technological transfers in other regions would be globally and domestically more efficient than active reduction within the country.

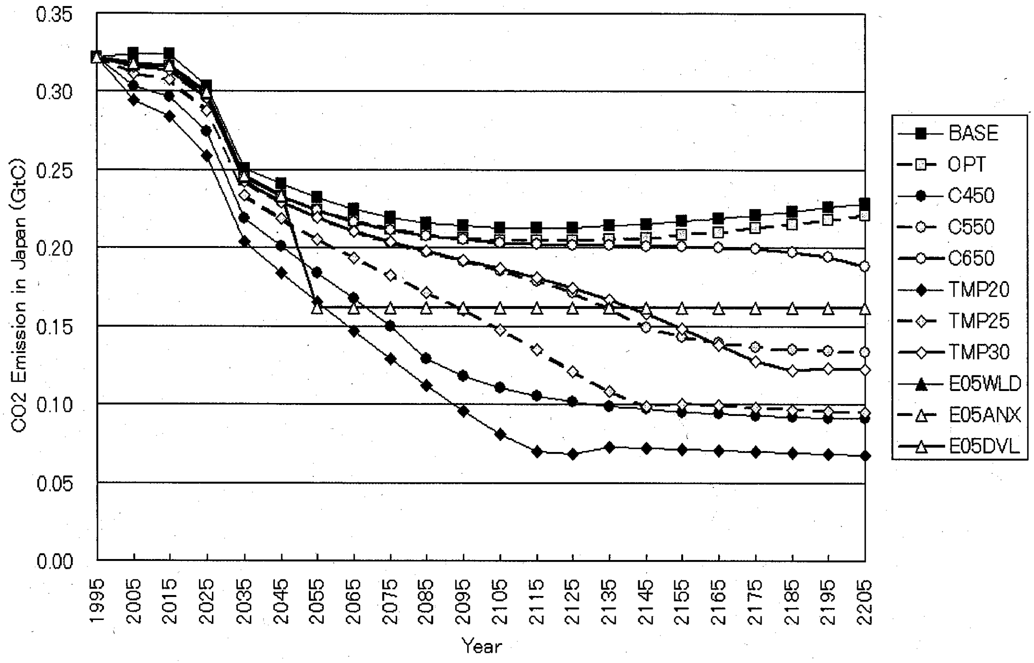


**Figure. 9: Projected $CO_2$ emissions in Japan in alternative scenarios (RJP1 case).**

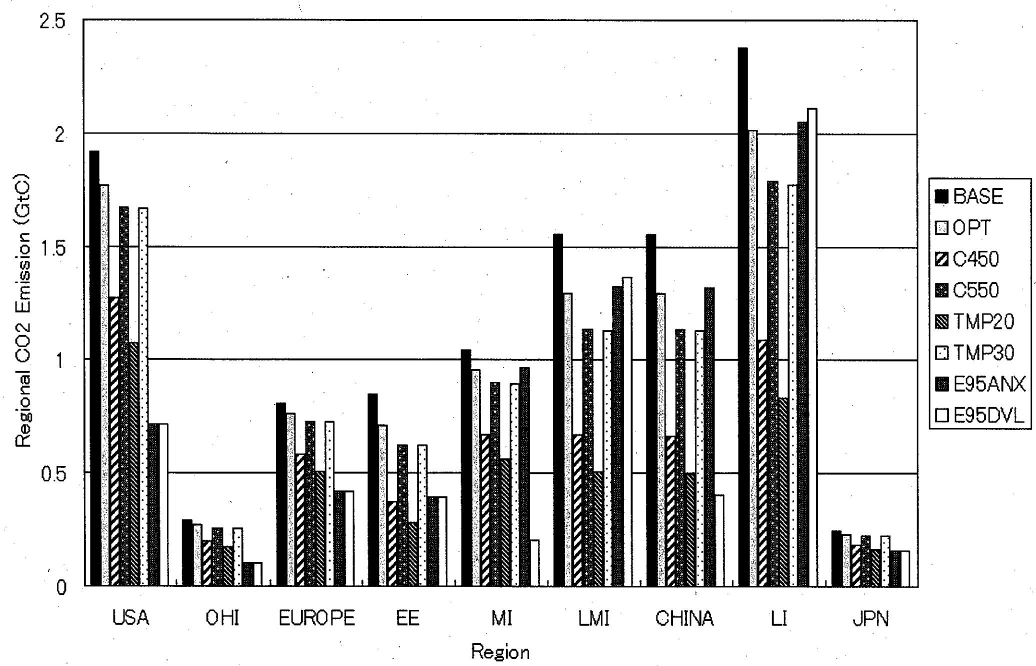


**Figure. 10: Regional $CO_2$ emissions in alternative scenarios (RJP1 case).**

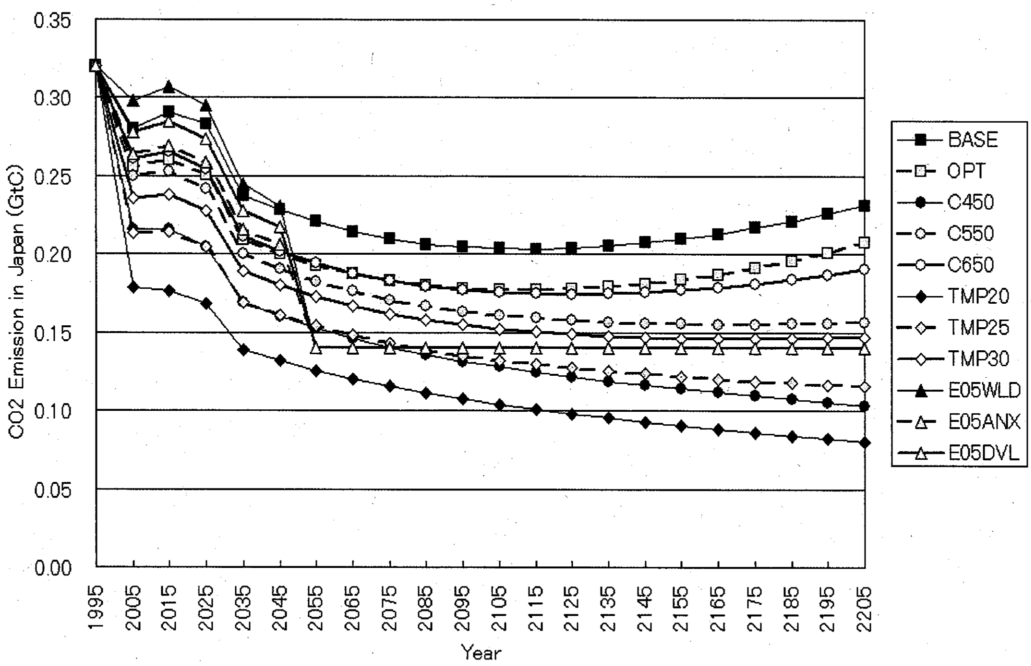


**Figure. 11: Projected $CO_2$ emissions in Japan in alternative scenarios (STERN case).**

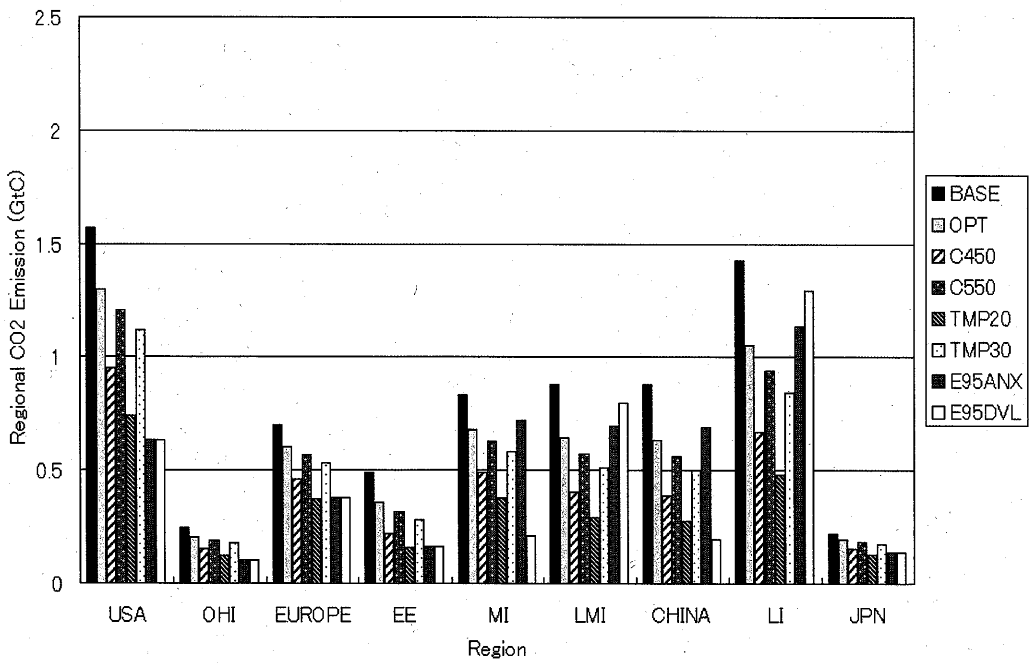


**Figure. 12: Regional $CO_2$ emissions in alternative scenarios (STERN case).**

Table. 5: $CO_2$ emissions reductions in 2055 in Japan (GtC)

| | | BASE | OPT | C450 | C550 | C650 | TMP20 | TMP25 | TMP30 | E05WLD | E05ANX | E05DVL |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| RJP1 | BASE | 0.000 | 0.013 | 0.059 | 0.021 | 0.013 | 0.081 | 0.039 | 0.022 | 0.079 | 0.079 | 0.079 |
| | 1995 | 0.076 | 0.088 | 0.135 | 0.097 | 0.089 | 0.156 | 0.115 | 0.097 | 0.154 | 0.154 | 0.154 |
| STERN | BASE | 0.000 | 0.028 | 0.068 | 0.038 | 0.027 | 0.095 | 0.067 | 0.048 | 0.081 | 0.081 | 0.081 |
| | 1995 | 0.099 | 0.127 | 0.168 | 0.137 | 0.126 | 0.195 | 0.166 | 0.147 | 0.180 | 0.180 | 0.180 |

Note: While positive values show emissions reduction, negative values show emissions increase.

# 5 Concluding remarks

This study has demonstrated the global and Japanese possibilities of climate stabilization policies and what actions Japan should take by simulation analyses with some scenarios and sensitivity analyses of the combinations of the PRTP and the elasticity of marginal utility of consumption.

First, conclusions from sensitivity analyses of discount rates are summarized as follows.

(1) If the elasticity of marginal utility of consumption and the consumption growth rate are constant, because the lower PRTP engenders a lower discount rate, the argument of the Stern Review for "urgent, immediate, and sharp reductions in GHG emissions", is reliant on a low discount rate.

(2) In terms of a climate stabilization policy, if the elasticity of the marginal utility of consumption is the same because selecting the PRTP depends on the growth rate of consumption or the real return on capital, more stringent reduction recommended by the Stern Review, which sets a low time discount rate that is inappropriate for the real world, can not be advocated from our simulation results.

(3) Because discount rates in assumed cases in this study are 3% to 4% and the projected optimal trajectories show similar trends in all cases, in spite of a slight difference among these results, discount rates of 3% to 4%, such as the combination of the PRTP and the elasticity of marginal utility of consumption, are appropriate.

Secondary conclusions from global simulation analyses using different combinations of the PRTP and the consumption elasticity are summarized as follows.

(1) For setting an appropriate discount rate, it is clear that although near-term $CO_2$ emission reductions are low, more stringent reductions are required in the future. This result reflects that emission reduction with advanced technology in the future is more reasonable than a sharp reduction with low technology today.

(2) In the case setting for a very low discount rate, because a sharp reduction is taken in the near term, little reduction is needed in the future. This result reflects that the present sharp reduction of $CO_2$ emissions is reasonable because of very low discount rates.

Other conclusions related to the possibilities of climate policies in Japan from results of the same simulations are summarized as follows.

(1) Although $CO_2$ emissions and their trajectories depend on discount rates, interpretations to reduce 50% of $CO_2$ emission in 2055 as the baseline of the BASE case and 1995 emission level differ greatly in both the RJP1 case and STERN case. In scenarios assumed for this study, although the former reduction is infeasible, the latter is feasible but would be costly to achieve.

(2) 2. Because the potential reduction of $CO_2$ emissions in Japan is low, its reduction with CDM and technological transfers in other regions would be globally and domestically more efficient than active reduction within the country.

Although much remains to be done along this avenue of study, three points particularly demand further consideration. First, because regional relationships are considered only for emission trading, in spite of a multi-regional model, trade of goods and services should be incorporated into this model structure. Secondly, parameter estimation based on scientific findings is necessary to refine this model. Thirdly, because damage estimation of climate change is an important factor, methodologies to estimate damage and to set damage functions must be considered.

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# Appendix: Equations of the modified RICE model.

This model is based on RICE model. See Nordhaus and Boyer (2000).

$$\max_{c} W = \sum_{t}\sum_{j} u_j\Big(c_j(t), L_j(t)\Big)R(t) \tag{A.1}$$

$$R(t) = \prod_{v=0}^{t} \Big[1+\rho(v)\Big]^{-v} \tag{A.2}$$

$$u_j\Big(c_j(t), L_j(t)\Big) = L_j(t)\frac{c_j(t)^{1-\alpha}}{1-\alpha} \tag{A.3}$$

$$Q_j(t) = \Omega_j(t)\left[A_j(t)K_j(t)^{\gamma}L_j(t)^{1-\gamma-\beta_j}ES_j(t)^{\beta} - c_j^E(t)ES_j(t)\right] \tag{A.4}$$

$$ES_j(t) = \varsigma_j(t)E_j(t) \tag{A.5}$$

$$Q_j(t) + \tau_j(t)\Big[\Pi_j(t) - E_j(t)\Big] = C_j(t) + I_j(t) \tag{A.6}$$

$$\tau_j(t) = \tau_b(t), \text{ for } \forall j \in b$$

$$\sum_{j\in b}\Pi_j(t) \geq \sum_{j\in b} E_j(t) \tag{A.7}$$

$$\sum_{j\in b}\Pi_j(t) = \sum_{j\in b} E_j(t), \text{ if } \tau_b(t) > 0$$

$$\tau_b(t) \geq 0$$

$$c_j(t) = C_j(t)/L_j(t) \tag{A.8}$$

$$K_j(t) = (1-\delta_k)K_j(t-1) + I_j(t-1) \tag{A.9}$$

$$c_j^E(t) = q(t) + markup_j^E \tag{A.10}$$

$$CumC(t) = CumC(t-1) + E(t) \tag{A.11}$$

$$E(t) = sum_j E_j(t)$$

$$q(t) = \xi_1 + \xi_2\Big(CumC(t)/CumC^*\Big)^{\xi_3} \tag{A.12}$$

$$M_{AT}(t) = ET(t-1) + \phi_{11}M_{AT}(t-1) + \phi_{21}M_{UP}(t-1) \tag{A.13}$$

$$ET(t) = \sum_j (E_j(t) + LU_j(t))$$

$$M_{UP}(t) = \phi_{22}M_{UP}(t-1) + \phi_{12}M_{AT}(t-1) + \phi_{32}M_{LO}(t-1) \tag{A.14}$$

$$M_{LO}(t) = \phi_{33}M_{LO}(t-1) + \phi_{23}M_{UP}(t-1) \tag{A.15}$$

$$F(t) = \eta\left[\log\Big(M_{AT}(t)/M_{AT}^{PI}\Big)/\log(2)\right] + F^{EX} \tag{A.16}$$

$$T(t) = T(t-1) + \sigma_1\Big[F(t) - \lambda T(t-1) - \sigma_2\left(T(t-1) - T_{LO}(t-1)\right)\Big] \tag{A.17}$$

$$T_{LO}(t) = T_{LO}(t-1) + \sigma_3\Big[T(t-1) - T_{LO}(t-1)\Big] \tag{A.18}$$

$$D_j(t) = \theta_{1j}T(t) + \theta_{2j}T(t)^2 \tag{A.19}$$

$$\Omega_j(t) = 1/\Big(1 + D_j(t)\Big) \tag{A.20}$$